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t-Fault $t/2$-Step Sequentially Diagnosable Systems

KEWAL K. SALUJA AND BRIAN D. O. ANDERSON

Abstract—Fault diagnosis is considered for systems comprising n units $u_0, u_1, \ldots, u_{n-1}$ in which u_i tests u_{i+1} and u_{i+2}. A necessary and sufficient condition for the diagnosability of t faults in $t/2$ or $t/2 + 1/2$ steps is stated, the condition taking the form of an inequality relating n and t.

I. Introduction

Since the introduction of graph theoretic models for system level diagnosis by Preparata *et al.* [1], a number of other models have been developed (see [2] for references) and analyzed. In this letter we will use Preparata's model for a special class of systems and state necessary and sufficient conditions for fault diagnosability on the number of nodes n as a function of the number of faults t in the system. Full proofs will be given elsewhere.

The system is composed of n units $u_0, u_1, \ldots, u_{n-1}$ where each unit is capable of testing other units. The system is represented by a directed graph where nodes correspond to units and arcs to test links. The outcome of the test applied by u_i to u_j is denoted by a binary variable a_{ij} where

$$a_{ij} = 1, \text{ if } u_i \text{ is fault free and } u_j \text{ is faulty}$$
$$a_{ij} = 0, \text{ if } u_i \text{ is fault free and } u_j \text{ is fault free}$$
$$a_{ij} = d, \text{ if } u_i \text{ is faulty, where } d \in \{0, 1\}.$$

Necessary and sufficient conditions for $D_{\delta t}$ systems [1] to be one step diagnosable and single loop systems [1], [3] to be sequentially diagnosable are known. In this correspondence we give necessary and sufficient conditions for $D_{\delta 2}$ systems to be $t/2$-step or $(t+1)/2$-step sequentially diagnosable. In the following section, the model is defined and in Section III the results are presented.

II. Model

A system with n-units $u_0, u_1, \ldots, u_{n-1}$ is said to be $D_{\delta 2}$ if its graph has the following property: there is a test link from u_i to u_j if and only if $j - i = \delta m \pmod n$ for $m = 1, 2$. It can easily be shown that if δ and n are relatively prime, a $D_{\delta 2}$ system is isomorphic to a D_{12} system (e.g., see [1], [4]). A D_{12} system with $n = 9$ is shown in Fig. 1. Therefore, we shall only study D_{12} systems. In a D_{12} system,certain fault patterns have the property that at most two faults can be diagnosed in one step. (Consider, for example, a fault pattern in which precisely $u_i, u_{i+1}, \ldots, u_{i+k}$ are faulty. Even if u_i and u_{i+1} can be diagnosed as faulty, there is no way, if $a_{\alpha\beta} = 0$ for $\alpha \in [i, i+k]$, that any of $u_{i+2}, \ldots, u_{i+k}$ could be diagnosed as being fault free or faulty.)

It follows that to diagnose t faults, we might require at least $\lceil t/2 \rceil$ steps (where $\lceil x \rceil$ denotes the smallest integer $\geq x$), it being understood that at each step, two faulty units are diagnosed and replaced by fault-free units. This observation gives rise to the following problem statement.

Find the smallest number of units n such that a D_{12} design of a system with n units is sequentially $\lceil t/2 \rceil$-step t-fault diagnosable.

Clearly such a system will be optimal with respect to the number of links and number of units n.

III. Results

For a given value of t, we define

$$n_1 = \lceil t/4 \rceil (t - 2\lceil t/4 \rceil) + t + 3$$

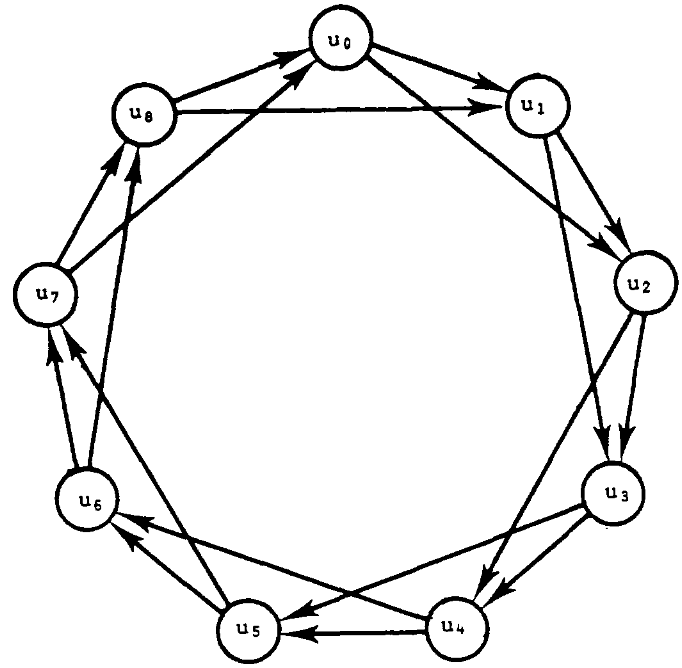

Fig. 1. A D_{12} system with $n = 9$.

TABLE I
Minimum Value of n for Single Loop and D_{12} Systems

t	Single loop system	D_{12} system n_1	n_2	$n \geq \max(n_1, n_2)$
2	5	5	5	5
3	7	7	6	7
4	10	9	9	9
5	13	10	11	11
6	17	13	13	13
7	21	16	15	16
8	26	19	19	19
9	31	21	22	22
10	37	25	25	25

and

$$n_2 = \lfloor t/4 \rfloor (t - 2\lfloor t/4 \rfloor) + t + 3$$

where $\lfloor x \rfloor$ is the largest integer $\leq x$.

The following theorem gives the necessary and sufficient conditions for a D_{12} system to be sequentially $\lceil t/2 \rceil$-step t-fault diagnosable. The proof of the theorem is quite long and complex and is not included here.

Theorem 1: A D_{12} system with n units is sequentially $\lceil t/2 \rceil$-step t-fault diagnosable, for $t \geq 2$, if and only if

$$n \geq \max(n_1, n_2).$$

In view of our earlier comment we can say that the result of Theorem 1 also applied to $D_{\delta 2}$ systems with coprime δ and n and also to systems which may contain $D_{\delta 2}$ systems as subsystems. Table I lists the minimum value of n for different $t (\leq 10)$ for D_{12} and single loop systems (where t-step diagnosis is being considered [1]).

It is evident from Table I that for small systems ($n \leq 13$), n meets the absolute necessary minimum bound (i.e. $n \geq 2t + 1$, see [1]) for t-fault diagnosability.

IV. Conclusions

The main result as stated in the theorem requires of itself little comment. What, though, of extensions? An obvious direction to extend

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the ideas is to consider conditions for $\lceil t/m \rceil$-step diagnosability for arbitrary m in $[1, t]$. The authors have developed a necessity condition which they conjecture to be sufficient. They have also studied 2-step diagnosability, and are able to strengthen in a minor way a result of [4]. These results will be reported separately.

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Correction to "Computerized Geophysical Tomography"

K. A. DINES AND R. J. LYTLE

In the above article,[1] equations (12) and (18) contain misprints. The correct equations are

$$\Delta f_{ijk}^{q} = \frac{(\Delta s_{ijk})^{w} \Delta g_k}{\sum_{i}^{I} \sum_{j}^{J} (\Delta s_{ijk})(\Delta s_{ijk})^{w}} \tag{12}$$

and

$$\Delta f_{ij}^{q} = \frac{\sum_{k=1}^{K} \Delta f_{ijk}^{q} / N_k^4}{\sum_{k=1}^{K} \operatorname{sgn}(\Delta s_{ijk}) / N_k^4} \tag{18}$$

where N_k is the number of picture cells intercepted by the kth ray.

In addition, equation (11) is not labeled.

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On True 3-D Object Reconstruction from Line Integrals

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Abstract—An algorithm for true three-dimensional reconstruction from line integrals (projections onto planes) is discussed and a three-dimensional filter equivalence theorem is formulated. A Radon-type equation is obtained, and the global character of the reconstruction is established. The relation of the algorithm to the conventional slice by slice reconstruction is examined.

In a recent communication Nalcioglu and Cho [1] reported on a 3-D analog of the 2-D Fourier synthesis operator for object reconstruction from its line integrals. The purpose of this communication is to relate one of their results to a filter equivalency theorem in the Fourier synthesis and the filtered summation-image methods. The global nature of the reconstruction algorithm, as well as the redundancy in the experimental data required by the algorithm, are discussed.

The Filter Equivalency Theorem

Barrett *et al.* [2], [3] proved a theorem on the identity of the functional forms of the one- and two-dimensional filters in 2-D object reconstruction by the Fourier-synthesis and filtered summation-image methods. Their corresponding equations are

$$\hat{f}_1(x,y) = \beta_2 \{F_1^{-1} [H_1(|\rho|) \cdot F_1[g]]\} \tag{1}$$

$$\hat{f}_2(x,y) = F_2^{-1} [H_2(\rho) \cdot F[\beta_2 \{g\}]] . \tag{2}$$

Here $F_1[\ ]$ and $F_2[\ ]$ indicate one- and two-dimensional Fourier transforms, $\beta_2\{\ \} = \int_0^\pi d\theta \{\ \}$ is the two-dimensional linear superposition operator that yields the two-dimensional summation image, g is a line-integral of a two-dimensional object, and ρ is the spatial frequency in (1) and the length of the radius-vector of a point in the Fourier plane in (2). Barrett's theorem states that if the image estimates $\hat{f}_1$ and $\hat{f}_2$ are equal, then

$$H_1(|\rho|) = H_2(\rho) . \tag{3}$$

In the special case of a purely mathematical reconstruction, i.e., without additional filtering, $H_1 = |\rho|$ and $H_2 = \rho$.

The equations analogous to (1) and (2) in the 3-D case are

$$\hat{f}_1(x,y,z) = \beta_3 \{F_2^{-1} [K_2(\rho_1, \rho_2) \cdot F_2[h]]\} \tag{4}$$

$$\hat{f}_2(x,y,z) = F_3^{-1} [K_3(\rho) \cdot F_3[\beta_3\{h\}]] . \tag{5}$$

Here $F_3\ [\ ]$ indicates the three-dimensional Fourier transform, $\beta_3\ \{\ \} = \int_0^\pi \int_0^\pi \sin\theta\, d\theta\, d\varphi\ \{\ \}$ is the three-dimensional linear superposition operator that yields the three-dimensional summation image, h is a line-integral over a three-dimensional object along a line parallel to a direction specified by the angles θ and φ in a spherical coordinate system, ρ_1 and ρ_2 in (4) are the spatial frequency components in the Fourier plane, and ρ in (5) is the length of the radius-vector of a point in the 3-D Fourier space.

By a rather lengthy derivation for the algorithms represented by (4) and (5) it can be shown that

$$K_2(\rho_1, \rho_2) = K_3(\sqrt{\rho_1^2 + \rho_2^2}) \tag{6}$$

provided that the image estimates are equal.

Equation 6 represents a generalization of Barrett's theorem to a 3-D object reconstruction from its line integrals. For a purely mathematical reconstruction $K_3 = \rho/\pi$ and $K_2 = \sqrt{\rho_1^2 + \rho_2^2}/\pi$. The filter equivalency theorem reflects on the relationship between the Fourier-synthesis and the filtered summation-image methods and can be used for the development of specific filtering procedures, for example, those based on regularization theory for ill-posed problems [4].

Discussion

When $K_2 = \sqrt{\rho_1^2 + \rho_2^2}/\pi$, equation (4) can be written explicitly in a form that corresponds to (9) given by Nalcioglu and Cho [1].

$$f(x,y,z) = 1/\pi \int_0^\pi \int_0^\pi \left\{ \iint_{-\infty}^{\infty} \sqrt{\rho_1^2 + \rho_2^2}\, \tilde{h}(\rho_1, \rho_2; \theta, \varphi) \cdot \exp\,[2\pi i(x'\rho_1 + y'\rho_2)]\, d\rho_1\, d\rho_2 \right\} \sin\theta\, d\theta\, d\varphi \tag{7}$$

where

$$\tilde{h}(\rho_1, \rho_2; \theta, \varphi) = \iint_{-\infty}^{\infty} h(x', y'; \theta, \varphi) \exp\,[-2\pi i(x'\rho_1 + y'\rho_2)]\, dx'\, dy'. \tag{8}$$

Here x' and y' are rectangular coordinates in the plane onto which all parallel line integrals are projected. The inverse Fourier transform of $\sqrt{\rho_1^2 + \rho_2^2}$ is $-1/4\pi^2(x'^2 + y'^2)^{-3/2}$ (see [5, p. 197]. It is a generalized function and is defined as a certain functional. As a part of the definition of this generalized function the singular integral related to it has to be regularized by removing the singularity. (See [5] p. 10, 45, and further].) Substituting the inverse Fourier transform of $\sqrt{\rho_1^2 + \rho_2^2}$ into (7) and using the convolution theorem we get the following intermedi-

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