# Matrix Fraction Description of the Lossless Positive Real Property

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***Abstract*—Suppose a matrix fraction description of a real rational square transfer function matrix is known. A condition is given involving the coefficient matrices in this matrix fraction decomposition which is necessary and sufficient for the transfer function matrix to be lossless positive real.**

## I. Introduction

Let $W(s)$ be a real rational matrix transfer function with a left matrix fraction decomposition $W(s)=A^{-1}(s)B(s)$, [1]. Thus $A(\cdot),B(\cdot)$ are polynomial matrices. In this paper, we characterize the lossless positive real property [2], [3] for $W(s)$ in terms of properties of a matrix—a generalized Bezoutian matrix [4]—formed from the coefficient matrices in $A(\cdot),B(\cdot)$. We mention here our notational convention: lower case letters (other than $s$) denote scalar quantities, upper case letters denote matrices, $\Sigma$ denotes a signature matrix—a diagonal matrix with entries of either $+1$ or $-1$ on the diagonal—superscript prime denotes matrix transposition, superscript asterisk denotes complex conjugation.

To put the result in context, we recall the following scalar results. Let $w(s)=q(s)/p(s)$ with $p(\cdot),q(\cdot)$ scalar polynomials such that degree $[p]>$ degree $[q]$. A Bezoutian matrix $D$ associated with $p(\cdot)$ and $q(\cdot)$ may be defined as that square matrix of dimension degree $[p]$ with $i$–$j$ entry the real scalar $d_{ij}$, which represents the coefficient of $x^{i-1}y^{j-1}$ in $\gamma(x,y)$, where

$$\gamma(x,y)=\frac{p(x)q(-y)+q(x)p(-y)}{x-y}. \tag{1}$$

The rational function $w(s)$ is termed lossless positive real if it represents the driving point impedance of an $LC$ one-port network, i.e., it is a reactance function. Then, as shown in, e.g., [5], $w(s)$ is lossless positive real if and only if $\Sigma D$ is nonnegative definite, where $\Sigma=\text{diag}\{1,-1,1,-1,\cdots\}$. (The matrix $\Sigma D$ is, in effect, the Hermite matrix associated with $f(s)=p(s)+q(s)$, [6]). The natural question arises as to whether there might be a matrix version of this result, involving the recently studied generalized Bezoutian matrix associated with matrix polynomials.

We remind the reader that a real rational $r\times r$ matrix $W(s)$ is termed lossless positive real if it represents the driving-point impedance of an $LC$ $r$-port network. Analytically, this may be characterized by [2], [3]

$$W(s)+W'(-s)=0 \tag{2a}$$

$$W(s)+W'(s^*)\geqslant 0, \qquad \text{Re}\,[s]>0. \tag{2b}$$

Another description of the lossless positive real property for a $W(s)$ with $W(\infty)=0$ is provided by the Kalman–Yakubovic lemma [3], [7], [8]: if one has a state variable description of $W(s)$ as $W(s)=H'(sI-F)^{-1}G$ with $[F,G]$ completely reachable and $[F,H]$ completely observable, then $W(s)$ is lossless positive real if and only if there exists a positive definite symmetric $P$ such that

$$PF+F'P=0 \qquad PG=H. \tag{3}$$

In the next section, we shall use this result to give an interpretation of the lossless positive real property in terms of a Hankel matrix. This interpretation will be used later to prove the main result.

## II. Hankel Matrix Description of the Lossless Positive Real Property

Let our $r\times r$ real rational matrix transfer function $W(s)$ have the Laurent series expansion $W(s)=W_0s^{-1}+W_1s^{-2}+W_2s^{-3}+\cdots$, and let $H_{nn}$ denote the truncated block Hankel matrix

$$H_{nn}=\begin{bmatrix} W_0 & W_1 & \cdots & W_{n-1}\\ W_1 & W_2 & \cdots & W_n\\ \cdot & \cdot & & \cdot\\ \cdot & \cdot & & \cdot\\ \cdot & \cdot & & \cdot\\ W_{n-1} & W_n & \cdots & W_{2n-2}\end{bmatrix}. \tag{4}$$

We immediately have the following lemma:

*Lemma 1:* Let $W(\cdot)$ be an $r\times r$ rational matrix with $W(\infty)=0$ and with $H_{nn}$ the associated $nr\times nr$ block Hankel matrix. Also let $\Sigma_n$ be the $nr\times nr$ signature matrix diag $[I_r,-I_r,I_r,-I_r,\cdots]$. Then $W(s)+W'(-s)=0$ if and only if $\Sigma_nH_{nn}$ is symmetric for all $n$.

*Proof:* By (4), $\Sigma_nH_{nn}$ is symmetric if and only if $W_{2i}=W_{2i}'$ and $W_{2i+1}=-W_{2i+1}'$ for all $i\geqslant 0$; i.e., if and only if $W(s)+W'(-s)=0$.

Then we can describe the lossless positive real property using the Hankel matrix.

*Theorem 1:* Let $W(\cdot)$ be an $r\times r$ real rational transfer function matrix with $W(\infty)=0$, with associated truncated block Hankel matrices $H_{nn}$, $n=1,2,\cdots$, and let $\Sigma_n=\text{diag}\,[I_r,-I_r,I_r,-I_r,\cdots]$. Then if $W(\cdot)$ is lossless positive real, $\Sigma_nH_{nn}$ is nonnegative definite symmetric for all $n$.

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Conversely, if $\Sigma_n \boldsymbol{H}_{nn}$ is nonnegative definite symmetric for some $n$ greater than or equal to $\nu$, the degree of the denominator polynomial matrix of some matrix fraction decomposition of $W(s)$, then $W(\cdot)$ is lossless positive real.

*Proof:* Let $H'(sI-F)^{-1}G=W(s)$ define a state variable realization of $W(s)$ such that $[F,G]$ is completely reachable and $[F,H]$ completely observable, i.e., $\dim F=\delta[W(s)]$, the McMillan degree of $W(s)$ [2], [3].

Suppose $W(s)$ is lossless positive real and let $P$ satisfy (3). Then $P$ is positive definite symmetric by the Kalman–Yakubovic lemma. Observe then that if $V=[G\ \ FG\ \ \cdots\ \ F^{n-1}G]$ for arbitrary positive integer $n$,

$$PV=\left[PG\quad -F'PG(F')^2PG\quad -(F')^3PG\cdots(-1)^{n-1}(F')^{n-1}PG\right]$$
$$=\left[H\quad -F'H(F')^2H\quad -(F')^3H\cdots(-1)^{n-1}(F')^{n-1}H\right]$$

and so

$$V'PV=\begin{bmatrix} G'H & -G'F'H & G(F')^2H & -G'(F')^3H\cdots(-1)^{n-1}G'(F')^{n-1}H \\ G'F'H & -G'(F')^2H & & \\ G'(F')^2H & -G'(F')^3H & & \\ \vdots & \vdots & & \end{bmatrix}$$

or, by transposing,

$$V'PV=\begin{bmatrix} W_0 & -W_1 & W_2 & -W_3 & \cdots \\ W_1 & -W_2 & W_3 & -W_4 & \cdots \\ W_2 & -W_3 & W_4 & -W_5 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \end{bmatrix}$$
$$=\boldsymbol{H}_{nn}\Sigma_n.$$

Thus $\Sigma_n\boldsymbol{H}_{nn}\geqslant 0$.

Conversely, suppose $\Sigma_n\boldsymbol{H}_{nn}$ is nonnegative definite symmetric for some $n\geqslant\nu$. Let $\bar{P}$ satisfy

$$\bar{P}[\,G\ FG\ \cdots\ F^{m-1}G\,]$$
$$=\left[H\ \ -F'H(F')^2H\ \cdots\ (-1)^{m-1}(F')^{m-1}H\right]$$

where $m=\dim F$. That $\bar{P}$ exists follows from the complete reachability of the pair $[F,G]$. Moreover, it is easily checked that $\bar{P}F+F'\bar{P}=0$, $\bar{P}G=H$, and that $V'\bar{P}V=\boldsymbol{H}_{mm}\Sigma_m$ where $V$ has rank $m$.

Let $\nu_0$ be the smallest degree taken by the denominator polynomial matrix in the set of left matrix fraction decompositions of $W(\cdot)$. Then $\nu_0\leqslant\{m,n\}$. Further, rank $\boldsymbol{H}_{\nu_0+k,\nu_0+k}=$ rank $\boldsymbol{H}_{\nu_0\nu_0}$ for all integer $k\geqslant 0$, by [4], while $\Sigma_{\nu_0}\boldsymbol{H}_{\nu_0\nu_0}$ is clearly a principal submatrix of both $\Sigma_n\boldsymbol{H}_{nn}$ and $\Sigma_m\boldsymbol{H}_{mm}$. Therefore, $\Sigma_n\boldsymbol{H}_{nn}\geqslant 0$ implies $\Sigma_{\nu_0}\boldsymbol{H}_{\nu_0\nu_0}\geqslant 0$ which in turn implies $\Sigma_m\boldsymbol{H}_{mm}\geqslant 0$.

Because $\bar{P}=[(VV')^{-1}V\Sigma_m]\Sigma_m\boldsymbol{H}_{mm}[(VV')^{-1}V\Sigma_m]'$, we see that $\bar{P}\geqslant 0$. Using the observability of $[F,H]$ it follows that $\bar{P}$ is nonsingular. So $\bar{P}$ is positive definite, and by the Kalman–Yakubovic lemma, $W(\cdot)$ is lossless positive real.

## III. Generalized Bezoutian Matrix and a Preliminary Symmetry Result

With $W(s)$ a real rational transfer function matrix, let $W(s)=A^{-1}(s)B(s)=D(s)C^{-1}(s)$ define a left and a right matrix fraction description of $W(s)$. Thus $A(\cdot)$, $B(\cdot)$, $C(\cdot)$, and $D(\cdot)$ are polynomial matrices with $A(\cdot)$ and $C(\cdot)$ nonsingular. The associated generalized Bezoutian form is [4]

$$\Gamma(x,y)=\frac{A(x)D(y)-B(x)C(y)}{x-y}. \tag{5}$$

Note that $\Gamma(x,y)$ is integral, not rational in $x$ and $y$. From $\Gamma$, the generalized Bezoutian matrix $\Delta$ can be defined, with max (deg $A$, deg $B$) block rows, max (deg $C$, deg $D$) block columns, and $i$–$j$ block entry $\Gamma_{ij}$, the coefficient matrix of $x^{i-1}y^{j-1}$ in $\Gamma(x,y)$. Different matrix fraction descriptions of the one $W(s)$ give different $\Gamma$, $\Delta$ with certain related properties, see [4].

Notice also that, as shown in [4], $\Gamma_{ij}$ is easily definable in terms of the coefficient matrices of $A(s), B(s), C(s), D(s)$.

A necessary condition for the lossless positive real property to hold for $W(s)$ is that (2a) hold or, equivalently,

$$B(s)A'(-s)+A(s)B'(-s)=0. \tag{6}$$

This condition is easily expressible in terms of the coefficients of $A,B$. It also suggests that there is a natural right matrix fraction description to associate with the pair $A,B$ of the left matrix fraction description, viz., $C(s)=-A'(-s)$, $D(s)=B'(-s)$. In this case we have:

*Lemma 2:* Suppose that $W(s)=A^{-1}(s)B(s)$ is a left matrix fraction description with $A(s)$ of degree $n$ of an $r\times r$ real rational $W(s)$ for which $W(s)+W'(-s)=0$. Then $C(s)=-A'(-s)$, $D(s)=B'(-s)$ is a right matrix fraction description and $\Sigma_n\Delta$ is symmetric, where $\Sigma_n=\operatorname{diag}[I_r,-I_r,I_r,-I_r,\cdots]$.

*Proof:* We have

$$\Gamma(x,y)=\frac{A(x)B'(-y)+B(x)A'(-y)}{x-y}=\Gamma'(-y,-x).$$

Therefore, $\Gamma_{ij}=(-1)^{i+j}\Gamma'_{ji}$, or $\Sigma_n\Delta$ is symmetric. (Had a different right matrix fraction decomposition of $W(\cdot)$

been used in forming $\Gamma$ and $\Delta$, the symmetry of $\Sigma_n\Delta$ would not necessarily have held.)

We also require the following result of [4].

*Lemma 3:* Let $W(\cdot)$ be a real rational matrix with $W(\infty)=0$ and with left and right matrix fraction decompositions $A^{-1}(s)B(s)$ and $D(s)C^{-1}(s)$ where degree $A=n$, degree $C=m$. Let $\Delta$ be the generalized Bezoutian matrix associated with $\{A,B,C,D\}$ and $\boldsymbol{H}$ and $\boldsymbol{H}_{ij}$ be the infinite and truncated Hankel matrices associated with $W(\cdot)$. Then rank $\Delta=$rank $\boldsymbol{H}_{nm}=$rank $\boldsymbol{H}=\delta[W(s)]$, the McMillan degree of $W(\cdot)$.

## IV. Main Result

In this section, we prove a criterion for the lossless positive real property of a matrix $W(s)$ which is not necessarily zero at $s=\infty$.

*Theorem 2:* Suppose that $W(s)=A^{-1}(s)B(s)$ is a left matrix fraction decomposition with $A(s)$ of degree $n$ of an $r\times r$ real rational $W(s)$ for which $W(s)+W'(-s)=0$. With $C$, $D$, $\Delta$, and $\Sigma_n$ as in Lemma 2, $W(s)$ is lossless positive real if and only if $\Sigma_n\Delta$ is nonnegative definite symmetric.

*Proof:* We shall temporarily assume that $W(\infty)=0$. With the matrices $W_i$ and $\boldsymbol{H}_{nn}$ as defined in Section II, and with $n$ equal to the degree of the highest power of $s$ in $A(s)$, we have the following formula in [4]:

$$\Delta=\begin{bmatrix} A_{n-1} & A_{n-2} & \cdots & A_1 & A_0\\ A_{n-2} & A_{n-3} & \cdots & A_0 & 0\\ \vdots & \vdots & & \vdots & \vdots\\ A_0 & 0 & \cdots & 0 & 0\end{bmatrix}\boldsymbol{H}_{nn}\begin{bmatrix} C_{n-1} & C_{n-2} & \cdots & C_1 & C_0\\ C_{n-2} & C_{n-3} & \cdots & C_0 & 0\\ \vdots & \vdots & & \vdots & \vdots\\ C_0 & 0 & \cdots & 0 & 0\end{bmatrix}$$

where $A(s)=A_0s^n+A_1s^{n-1}+\cdots+A_n$ and $C(s)=C_0s^n+\cdots+C_n$. Because $C(s)=-A'(-s)$, we have $C_0=(-1)^{n-1}A'_0$, $C_1=(-1)^{n-2}A'_1,\cdots$. Then observe that

$$\Sigma_n\Delta=\begin{bmatrix} A_{n-1} & A_{n-2} & \cdots & A_1 & A_0\\ -A_{n-2} & -A_{n-3} & \cdots & -A_0 & 0\\ \vdots & \vdots & & \vdots & \vdots\\ (-1)^{n-1}A_0 & 0 & \cdots & 0 & 0\end{bmatrix}\boldsymbol{H}_{nn}\begin{bmatrix} A'_{n-1} & -A'_{n-2} & \cdots & (-1)^{n-2}A'_1 & (-1)^{n-1}A'_0\\ -A'_{n-2} & A'_{n-3} & \cdots & (-1)^{n-1}A'_0 & 0\\ \vdots & \vdots & & \vdots & \vdots\\ (-1)^{n-1}A'_0 & 0 & \cdots & 0 & 0\end{bmatrix}$$

$$=\begin{bmatrix} A_{n-1} & -A_{n-2} & \cdots & (-1)^{n-2}A_1 & (-1)^{n-1}A_0\\ -A_{n-2} & A_{n-3} & \cdots & (-1)^{n-1}A_0 & 0\\ \vdots & \vdots & & \vdots & \vdots\\ (-1)^{n-1}A_0 & 0 & \cdots & 0 & 0\end{bmatrix}\Sigma_n\boldsymbol{H}_{nn}\begin{bmatrix} A'_{n-1} & -A'_{n-2} & \cdots & (-1)^{n-2}A'_1 & (-1)^{n-1}A'_0\\ -A'_{n-2} & A'_{n-3} & \cdots & (-1)^{n-1}A'_0 & 0\\ \vdots & \vdots & & \vdots & \vdots\\ (-1)^{n-1}A'_0 & 0 & \cdots & 0 & 0\end{bmatrix}. \tag{7}$$

At once, we see that $\Sigma_n\boldsymbol{H}_{nn}\geqslant 0$ implies $\Sigma_n\Delta\geqslant 0$. Now the choice of $n$ made above ensures, from lemma 3, that rank $\Delta=\delta[W(s)]=$rank $\boldsymbol{H}_{nn}$. Then (7) implies that $\Sigma_n\boldsymbol{H}_{nn}\geqslant 0$ if $\Sigma_n\Delta\geqslant 0$. Using Theorem 1 we see that the result holds in case $W(\infty)=0$.

To study $W(\infty)\neq 0$, suppose first that $W(\infty)=J<\infty$. Because $W(s)=-W'(-s)$, $J$ is skew. If

$$W(s)=A^{-1}(s)B(s)=B'(-s)\left[-A'(-s)\right]^{-1}$$

then

$$V(s)=W(s)-J=A^{-1}(s)\left[B(s)-A(s)J\right]$$
$$=\left[B'(-s)+JA'(-s)\right]\left[-A'(-s)\right]^{-1}.$$

Moreover, $V(\infty)=0$. Now observe the following connection of the generalized Bezoutian forms:

$$\Gamma_V=\frac{A(x)\left[B'(-y)+JA'(-y)\right]+\left[B(x)-A(x)J\right]A'(-y)}{x-y}$$
$$=\Gamma_W.$$

If $W(s)$ is lossless positive real, so also is $V(s)$, as is easily checked on using the definition; then $\Sigma_n\Delta_V=\Sigma_n\Delta_W\geqslant 0$. Conversely, if $\Sigma_n\Delta_W=\Sigma_n\Delta_V\geqslant 0$, then $V(s)$ is lossless positive real and consequently so is $W(s)$.

Next, suppose $W(\infty)$ is not finite. For almost all skew $J$, $V(s)=[W(s)+J]^{-1}$ will exist and have $V(\infty)$ finite while $V(s)+V'(-s)=0$. If

$$W(s)=A^{-1}(s)B(s)=B'(-s)\left[-A'(-s)\right]^{-1}$$

then

$$V(s)=\left[B(s)+A(s)J\right]^{-1}A(s)$$
$$=A'(-s)\left[-B'(-s)+JA'(-s)\right]^{-1}.$$

We then have

$$\Gamma_V = \frac{[B(x)+A(x)J]A'(-y)+A(x)[B'(-y)-JA'(-y)]}{x-y}$$
$$=\Gamma_W.$$

If $W(s)$ is lossless positive real, so also is $V(s)$ (again this is easily checked); then $\Sigma_n\Delta_V=\Sigma_n\Delta_W \geqslant 0$. Conversely, if $\Sigma_n\Delta_W=\Sigma_n\Delta_V \geqslant 0$, then $V(s)$ is lossless positive real and consequently $W(s)$ is. This completes the proof of the theorem.

## V. Example

Consider the $2\times2$ real rational matrix

$$W(s)=[(s^2+4)I_2]^{-1}\begin{bmatrix} 3s^3+14s & s^3+2s^2+6s+11 \\ s^3-2s^2+6s-11 & 2s^3+12s \end{bmatrix}$$
$$=A^{-1}(s)B(s).$$

Clearly, $W(s)+W'(-s)=0$. Consequently, we consider the generalized Bezoutian matrix $\Delta$ corresponding to the quadruple $\{A(s),B(s),-A'(-s),B'(-s)\}$. The $i$-$j$ block entry of $\Delta,\Gamma_{ij}$ may be easily found from the formulas of [4]:

$$\Gamma_{ij}=\sum_{k\geqslant 0} A_{n-i-k}D_{n-j+1+k}-B_{n-i-k}C_{n-j+1+k}$$

where

$$A(s)=-C'(-s)=A_0s^n+A_1s^{n-1}+\cdots+A_n$$
$$B(s)=D'(-s)=B_0s^n+B_1s^{n-1}+\cdots+B_n.$$

(In this case, $n=3$) This yields

$$\Delta=\begin{bmatrix} 56 & 24 & 0 & -3 & 12 & 4 \\ 24 & 48 & 3 & 0 & 4 & 8 \\ 0 & -3 & -2 & -2 & 0 & 0 \\ 3 & 0 & -2 & -4 & 0 & 0 \\ 12 & 4 & 0 & 0 & 3 & 1 \\ 4 & 8 & 0 & 0 & 1 & 2 \end{bmatrix}$$

and

$$\Sigma_3\Delta=\begin{bmatrix} I_2 & 0_2 & 0_2 \\ 0_2 & -I_2 & 0_2 \\ 0_2 & 0_2 & I_2 \end{bmatrix}$$

$$\Delta=\begin{bmatrix} 56 & 24 & 0 & -3 & 12 & 4 \\ 24 & 48 & 3 & 0 & 4 & 8 \\ 0 & 3 & 2 & 2 & 0 & 0 \\ -3 & 0 & 2 & 4 & 0 & 0 \\ 12 & 4 & 0 & 0 & 3 & 1 \\ 4 & 8 & 0 & 0 & 1 & 2 \end{bmatrix}$$

which is positive definite symmetric. Hence we conclude that $W(s)$ is lossless positive real.

This may be verified by writing

$$W(s)=\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}+s\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}+\frac{s\begin{bmatrix} 2 & 2 \\ 2 & 4 \end{bmatrix}+\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}}{s^2+4}.$$

However, partial fraction decompositions are not always suited to computer utilization and the advantage of the Bezoutian method may be appreciated.

## VI. Conclusion and Remarks

We have shown that it is possible to check whether a square real rational transfer function matrix is lossless positive real by examining properties of a left matrix fraction description. (Obviously, a right matrix fraction would do just as well.) One has to check two conditions on the coefficients of the two matrix polynomials. One is the condition inherent in (6), and the other the condition inherent in Theorem 2 in requiring the modified Bezoutian matrix to be nonnegative symmetric. Note that formulas are available [4] expressing the entries of this matrix directly in terms of the coefficients of the matrix polynomials of the left and associated right matrix fraction description. These conditions cover the case when $W(\infty)$ is zero, finite, or infinite.

There are a number of ways in which the results can be extended. For example, it is almost trivial to obtain a corresponding result for lossless bounded real matrices [2], [3].

These results have also been derived in the framework of a matrix Cauchy index [10] which was constructed to examine properties of the pole locations of real rational matrices.

Both the lossless positive real property and the matrix Cauchy index have been employed in the study of the stability of matrix polynomials [11].

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