The coefficients of the power series can be determined by long division, and one can easily write

\[ c_k = \frac{1}{b_0} \left( a_k - \sum_{i=1}^{k} b_i c_{k-i} \right) \]  

(4)

where

\[ a_k = 0 \quad \text{for} \quad k > n - 1 \]

and

\[ b_k = 0 \quad \text{for} \quad i > n. \]

The relations (3) and (4) can be used to determine the moments of the impulse response of the system from its transfer function with significant computational advantage.

\[ A_k \]

Authors' Reply

M. LAL AND R. MITRA

We would like to thank Drs. Bajwa and Khatwani for their comments. The algorithm suggested by them, which also appears in [1], does not seem to offer any computational advantage. The procedure is essentially the same as given by us with the only difference being that whereas in our procedure all the coefficients \( A_{k,1} \) in the third and subsequent arrays in (6) are explicitly computed and stored, in the method suggested by them these coefficients are repeatedly computed afresh every time a new moment is evaluated.

References


Authors' Reply

KATSUHISA FURUTA AND JEAN-GUY PAQUET

We thank S. P. Bingulac for pointing out errors in our paper. It is not strange that our results are correct in spite of the fact that \( u(0) \) should read \(-1\), since we did not use \( u(0) \) as Budin [1] did in the example; actually we used \( b_3(1) \) of [1] to calculate \( P \).

References


Comments on “Conditions for a Feedback Transfer Matrix to be Proper”

R. W. SCOTT AND B.D.O. ANDERSON

Abstract—When constant output feedback is applied around a linear system with a rational transfer function matrix which may be improper, the closed-loop transfer function matrix is generically proper.

Introduction

In a recent paper, conditions were examined for a transfer function matrix derived by feedback to be proper. In this note we show that the conditions referred to in that paper hold generically. (We also point out several adjustments necessary to the arguments in that paper.) More specifically, we establish the following.

Comments on “Determination of Matrix Transfer Function in the Form of Matrix Fraction from Input-Output Observations”

S. P. BINGULAC

Although the authors in a recent paper have used the computational example already treated in [1], it happens that in both papers this example is published erroneously. In the above paper on page 395, as well as in [1, p. 399], instead of

\[ u(0) = 1, \quad u(1) = 2, \quad u(2) = 3, \ldots \]

it should read

\[ u(0) = -1, \quad u(1) = 2, \quad u(2) = 3, \ldots \]

Also, in the above paper two additional misprints are encountered. 1) In (18) the element in the first row, second column should read \(-1\) instead of 1.

2) In (20) the element in the second row, second column should read \( z^2 + 2z + 2 \) instead of \( z^2 + 2z + z \).

References

**Theorem 1:** Let \( W(s) \) be a \( q \times r \) matrix of real rational transfer functions. For a generic constant \( r \times q \) matrix \( F \), \( W(s) [I + FW(s)]^{-1} \) is proper, i.e., \( \lim_{s \to \infty} W(s) [I + FW(s)]^{-1} \) exists (and is finite).

**Remark 1:** We remind the reader that to say that the property of the theorem holds for a generic \( F \) is the same as saying that it holds for \( F \), except those whose entries satisfy one or more nontrivial polynomial equalities \([1]\).

**Remark 2:** It is interesting to note that the McMillan degree \([2]\) of \( W(s) [I + FW(s)]^{-1} \) is generally the same as the McMillan degree of \( W(s) \); see \([3],[4]\). This fact together with Theorem 1 can be used as an efficient tool to develop results of \([5]\) applicable to improper transfer function matrices.

**Remark 3:** It is clear that at best a generic result can be obtained. For with \( W(s) \) improper and \( F=0_{r \times q} \), evidently \( W(s) [I + FW(s)]^{-1} \) is still improper.

**ADJUSTMENTS TO THE ORIGINAL PAPER**

The proof of Lemma 3 appears incorrect. The lemma is stated thus:

"**Lemma 1:** Given any polynomial matrix \( M(s) \) one can find a constant nonsingular matrix \( U \) such that \( M(s)U \) is column proper."

Here, if \( M(s) \) is an \( n \times n \) polynomial matrix in \( s \), and if \( a_i \) denotes the highest power of \( s \) occurring in the \( i \)th column of \( M(s) \), then \( M(s) \) is column proper if \( \det M(s) \) is a polynomial of degree \( \Sigma a_i \).

Define \( M_H \) to be the matrix whose \( i \)th column consists of the coefficients of \( s^{a_i} \) in the corresponding entries of the \( i \)th column of \( M(s) \).

The crucial step in the proof of Lemma 1 says that if one finds a matrix \( U_i \) such that some column (say the \( k \)th) of \( M H U_i \) is identically zero, then the coefficients of \( s^{a_k} \) in the \( k \)th column of \( M(s)U_i \) all zero. This is not so. The entries of the \( k \)th column of \( M H U_i \) are \( \sum_{j=1}^n m_{ij} \psi_j \), where \( m_{ij} = \{ m_{ij} \} \) and \( U_i = \{ u_{ij} \} \). The entries of the \( i \)th column of \( M(s)U_i \) are \( \sum_{j=1}^n c_{ij} \psi_{ij} \), where \( c_{ij} \) is the degree of the \((ij)\) entry of \( M(s) \) and \( \psi_{ij} \) is the coefficient of the highest order term of this entry. Clearly, unless \( p_1 = p_2 = \cdots = p_n = a_k \) in a row of \( M(s) \) with an entry of degree \( a_k \), then \( \sum_{j=1}^n m_{ij} \psi_{ij} = 0(i = 1, \cdots, n) \), where \( p_i \) is the degree of the \((ij)\) entry of \( M(s) \) and \( \psi_{ij} \) is the coefficient of the highest order term of this entry. Clearly, unless \( p_1 = p_2 = \cdots = p_n = a_k \).

**Lemma 1** must be replaced by the following theorem \([6],[p.27]\).

**Theorem 2:** For any nonsingular polynomial matrix \( M(s) \) one can always find a unimodular matrix \( U(s) \) such that \( M(s)U(s) \) has column proper form.

Taking into account the replacement of Lemma 1 with Theorem 2, Fact 4 must be replaced by the following theorem.

**Theorem 3:** Consider a rational \( q \times r \) matrix \( W(s) \) expressed as the product of the \( q \times r \) polynomial matrix \( N(s) \) and the inverse of the \( r \times q \) polynomial matrix \( D(s) \); see \([7],[4]\), that is, \( W(s) = N(s)D(s)^{-1} \). For the \( r \times q \) constant matrix \( F \) select \( U(s) \) such that \( D(s) + FN(s) \) is column proper. Then \( W(s) [I + FW(s)]^{-1} \) is proper if and only if, for \( i=1, 2, \cdots, n \), the highest power of \( s \) in the \( i \)th column of \( N(s)U(s) \) is less than or equal to that in the \( i \)th column of \( D(s) + FN(s) \).

**Demonstration that \( W(s)[I + FW(s)]^{-1} \) is Generically Proper**

Let the highest power of \( s \) in the \( i \)th column of a polynomial matrix \( A \) be denoted by \( a_i \). Express \( W(s) \) as \( N(s)D(s)^{-1} \). Then by Theorem 3, \( W(s) [I + FW(s)]^{-1} \) is proper if and only if

\[
-a_i [N(s)U(s)] < a_i [D(s) + FN(s)]U(s)
\]

where \( U(s) \) is selected to make \( D(s) + FN(s)U(s) \) column proper. The existence of \( U(s) \) is guaranteed by Theorem 2 and the fact that \( D(s) + FN(s) \) is generically nonsingular; this in turn following from the nonsingularity of \( D(s) \).

Since \( F \) is a constant matrix the elements in the \( i \)th column of \( FN(s)U(s) \) are linear combinations of the elements in the \( i \)th column of \( N(s)U(s) \). Therefore,

\[
a_i [FN(s)U(s)] = a_i [N(s)U(s)]
\]

generically. If the highest coefficients of the highest powers of \( s \) in the \( i \)th column of \( N(s)U(s) \) cancel out under the linear combination caused by premultiplication by \( F \), then

\[
a_i [FN(s)U(s)] < a_i [N(s)U(s)].
\]

However, this implies that a polynomial in the entries of \( F \) is zero. That is, (1) holds generically.

Consider now the following three cases for the \( k \)th column.

**Case 1:**

\[
a_k [D(s)U(s)] > a_k [N(s)U(s)].
\]

Equivalently,

\[
a_k [D(s)U(s)] > a_k [FN(s)U(s)].
\]

Therefore,

\[
a_k [D(s)U(s) + FN(s)U(s)] = a_k [D(s)U(s)] > a_k [N(s)U(s)]
\]

and the condition of Theorem 3 is satisfied for the \( k \)th column.

**Case 2:**

\[
a_k [D(s)U(s)] < a_k [N(s)U(s)].
\]

Equivalently,

\[
a_k [D(s)U(s)] < a_k [FN(s)U(s)].
\]

generically. Therefore,

\[
a_k [D(s)U(s) + FN(s)U(s)] = a_k [FN(s)U(s)] = a_k [N(s)U(s)]
\]

and the condition of Theorem 3 is satisfied except on the solutions of a finite number of polynomial equations.

**Case 3:**

\[
a_k [D(s)U(s)] = a_k [N(s)U(s)].
\]

Equivalently,

\[
a_k [D(s)U(s)] = a_k [FN(s)U(s)]
\]

generically. Therefore,

\[
a_k [D(s)U(s) + FN(s)U(s)] = a_k [FN(s)U(s)] = a_k [N(s)U(s)]
\]

except when the coefficients of the highest power of \( s \) in the \( i \)th columns of \( D(s)U(s) \) and \( FN(s)U(s) \) cancel out. In this situation \( a_k [N(s)U(s)] > a_k [D(s)U(s) + FN(s)U(s)] \). Then, for Case 3, the condition of Theorem 3 is satisfied for column \( i \), except if the coefficient of the highest power of \( s \), in the \( i \)th column of \( D(s)U(s) + FN(s)U(s) \), is zero. This implies that a finite number of polynomials in the entries of \( F \) are zero. So, for Case 3, the conditions of Theorem 3 are satisfied generically.

For an arbitrary column \( i \), Cases 1–3 show that the conditions of Theorem 3 are satisfied generically, and so, applying Theorem 3, \( W(s)[I + FW(s)]^{-1} \) is generically proper.

**Example:** Consider Example 2:

\[
W(s)=\begin{bmatrix}
s+1 \\
s+2 \\
-s+1 \\
s+1
\end{bmatrix}
\]

Then \( W(s)=N(s)D(s)^{-1} \) where

\[
N(s)=\begin{bmatrix}
s^2+2s+1 \\
2s^2+9s+4 \\
-s^2-s+2 \\
-s^2-5s+3
\end{bmatrix}
\]
Let

\[ D(s) = \begin{bmatrix} s^2 + 3s + 2 & 0 \\ 0 & s^2 + 7s + 12 \end{bmatrix} \]

which is column proper unless

\[ f_1 - f_1 + 1 = 0 \]

\[ f_2 - f_2 = 0 \]

or

\[ 2f_1 - 2f_2 = 0 \]

or

\[ 2f_2 - 2f_2 + 1 = 0 \]

(4)

\[ (f_1 - f_1 + 1)s^2 + (2f_1 - f_1 + 3)s + (f_1 + f_1 + 2) \]

\[ (f_1 - f_1 + 2)s^2 + (2f_1 - f_1 + 2)s + (f_1 + f_1 + 2) \]

which is column proper unless

\[ f_1 - f_1 + 2f_1 - 2f_2 + 1 = 0 \]

(5)

(6)

which is column proper unless

\[ f_1 - f_1 + 1 = 0 \]

\[ f_2 - f_2 = 0 \]

or

\[ 2f_1 - 2f_2 = 0 \]

or

\[ 2f_2 - 2f_2 + 1 = 0 \]

(4)

(5)

(6)

(4)

(5)

(6)

(4)

(5)

(6)

The above paper considers the use of Walsh functions in the design of piecewise constant gains for the linear optimal control problem with quadratic performance criteria. The use of Walsh functions is illustrated in the context of solving a vector differential equation. A Walsh series is assumed for each rate variable and the unknown Walsh coefficients are determined by solving the resulting set of simultaneous equations. In this correspondence it is shown that the same solution can be easily obtained by proceeding directly with the piecewise constant approximation for the rate variables.

Consider (15) of Chen and Hsiao's paper:

\[ C = ACP + G + BH. \]

Taking the transpose of (1),

\[ C' = P'C'A' + G' + H'B'. \]

i.e.,

\[ \{c_1, c_2, \ldots, c_r\} = P\{c_1, c_2, \ldots, c_r\} + \{h_1, h_2, \ldots, h_r\}B' \]

(3)

where \( c_i \) represents the Walsh series coefficients of the ith rate variable

\[ c_i = \{c_{i0}, c_{i1}, \ldots, c_{im-1}\}^t, \]

and \( h_i \) corresponds to the Walsh series coefficients of the ith input \( u_i \),

\[ h_i = \{h_{i0}, h_{i1}, \ldots, h_{im-1}\}^t, \]

i.e.,

\[ \dot{x}_i = c_0\phi_0 + c_1\phi_1 + \cdots + c_{im-1}\phi_{im-1} \]

\[ u_i = h_{i0}\phi_0 + h_{i1}\phi_1 + \cdots + h_{im-1}\phi_{im-1} \]

It is well known that when a function is approximated by a Walsh series of finite order \( m \) (\( m = \text{some integral power of 2} \)), the approximated function is of piecewise constant form and the magnitude of the step in each subinterval is equal to the average value of the function in that subinterval. If \( f_{ij} \) is the average value of \( x_i \) and \( g_{ij} \) is the average value of \( u_i \) in the jth subinterval, \( j = 0, 1, \ldots, (m - 1) \), then the following relations are true.

\[ c_i = \frac{1}{m} Wf \]

\[ h_i = \frac{1}{m} Wg \]

Author's Reply

M. VIDYASAGAR

I wish to thank Dr. Scott and Dr. Anderson for their interest in my paper. Their comments are correct.

I would also like to point out that in Fact 4, \( N(s) + KD(s) \) should be changed to \( D(s) + KN(s) \) throughout.

References


Author's Reply

M. VIDYASAGAR

I wish to thank Dr. Scott and Dr. Anderson for their interest in my paper. Their comments are correct.

I would also like to point out that in Fact 4, \( N(s) + KD(s) \) should be changed to \( D(s) + KN(s) \) throughout.