Since $\delta L = I_D + I_{\partial D}$, the necessary conditions are

$$\frac{\partial \lambda}{\partial t} + \frac{\partial H}{\partial x} + \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{\partial^2}{\partial z_i \partial z_j} \left[\frac{\partial H}{\partial \left(\frac{\partial^2 x}{\partial z_i \partial z_j} \right)} \right] - \sum_{i=1}^{m} \frac{\partial}{\partial z_i} \left(\frac{\partial H}{\left(\frac{\partial x}{\partial z_i} \right)} \right) = 0 \quad (3.10a)$$

$$\lambda(t_f, z) = \frac{\partial j_f}{\partial x} \quad (3.10b)$$

$$\frac{\partial j_{\partial D}}{\partial x} + \frac{\partial H}{\partial \left(\frac{\partial x}{\partial z} \right)} \eta + (\eta^T \eta)^{-1} \left(\frac{\partial g}{\partial x} \right) \sum_{i=1}^{m} \eta_i [H_{iz}] \eta - \sum_{i=1}^{m} \frac{\partial}{\partial z_i} [H_{iz}] \eta = 0 \quad (3.10c)$$

$$\frac{\partial H}{\partial u} = 0 \quad (3.10d)$$

$$\frac{\partial j_{\partial D}}{\partial v} + (\eta^T \eta)^{-1} \left(\frac{\partial g}{\partial v} \right) \left(\sum_{i=1}^{m} \eta_i [H_{iz}] \right) \eta = 0 \quad (3.10e)$$

$$(\eta^{\perp T} \eta^{\perp})^{-1} \left(\sum_{i=1}^{m} \eta_i [H_{iz}] \right) \eta^{\perp} = 0. \quad (3.10f)$$

IV. Conclusion

Necessary conditions are derived for a class of distributed parameter systems subject to Neumann boundary conditions.

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Inverse Liénard–Chipart Problem

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***Abstract*—An inverse Lienard–Chipart problem is posed. It is shown that, in contrast to the inverse Hurwitz problem, it is not solvable using rational formulas.**

The inverse Hurwitz problem is as follows. Given n quantities $\Delta_1, \Delta_2, \cdots, \Delta_n$ determine a real monic polynomial of degree n, $f(s) = s^n + a_1 s^{n-1} + \cdots + a_n$ such that Δ_i is the $i \times i$ leading principal minor of the Hurwitz matrix

$$H = \begin{bmatrix} a_1 & a_3 & a_5 & \cdots & a_{2n-1} \\ 1 & a_2 & a_4 & \cdots & a_{2n-2} \\ 0 & a_1 & a_3 & \cdots & a_{2n-3} \\ 0 & 1 & a_2 & \cdots & a_{2n-4} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \end{bmatrix}. \quad (1)$$

Here, $a_m = 0$ for $m > n$. The problem is discussed in, for example, [1]. In case all Δ_i are nonzero, the problem is readily solved, the a_j being expressible as rational expressions in the Δ_i. In case some Δ_i are zero, the problem may or may not be solvable.

By analogy, we are led to formulate an inverse Lienard–Chipart problem, requiring reversal of the calculations required in carrying out the Liénard–Chipart stability test [2]. The problem could be stated as follows: given quantities $\Delta_{n-1}, \Delta_{n-3}, \cdots, \Delta_2$ or Δ_1 and $a_n, a_{n-2}, a_{n-4}, \cdots$ determine $a_{n-1}, a_{n-3}, \cdots$ such that Δ_i is, as above, the $i \times i$ leading principal minor of H.

In the next paragraph, we observe that the inverse problem is not solvable by rational operations for $n = 4$; it seems easy to draw this conclusion for any specific larger value of n; so presumably the inverse problem is not solvable by rational operations for any $n \geqslant 4$.

When $n = 4$, one checks that $\Delta_1 = a_1$, $\Delta_3 = a_1 a_2 a_3 - a_1^2 a_4 - a_3^2$. With Δ_1, Δ_3, a_2, and a_4 prescribed, a_3 cannot be expressed as a rational form in the prescribed quantities, since it is defined as the solution of an equation of the form $a_3^2 + \alpha a_3 + \beta = 0$ where α and β are known.

Similar reasoning applies to another problem. Let $g(z)$ be a rational function given by

$$g(z) = \frac{b_0 z^m + b_1 z^{m-1} + \cdots + b_m}{c_0 z^m + c_1 z^{m-1} + \cdots + c_m} = g_0 z^{-1} + g_1 z^{-2} + \cdots$$

and let D_i denote the ith principal minor of the infinite Hankel matrix

$$\begin{bmatrix} g_0 & g_1 & g_2 & \cdot \\ g_1 & g_2 & \cdot & \cdot \\ g_2 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix}.$$

The question arises as to whether given $D_1, D_2, \cdots, D_m$ and $c_0, c_1, \cdots, c_m$ one can recover the b_i by rational operations. The fact that this problem is essentially the inverse Liénard–Chipart problem follows from the observation (see [2, p. 214]) that $\Delta_{2p} = c_0^{2p} D_p$ where

$$\Delta_{2p} = \det \begin{bmatrix} c_0 & c_1 & c_2 & \cdots & c_{2p-1} \\ b_0 & b_1 & b_2 & \cdots & b_{2p-1} \\ 0 & c_0 & c_1 & \cdots & c_{2p-2} \\ 0 & b_0 & b_1 & \cdots & b_{2p-2} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \end{bmatrix}$$

and c_n, b_n are zero for $n > m$. It follows that the b_i are not recoverable by rational operations.

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Manuscript received December 16, 1975. This work was supported by the Australian Research Grants Committee and the Joint Services Electronics Program under Contract F44620-71-C-0087 (to the University of California, Berkeley, CA).

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Model Matching in Time-Delay Control Systems

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***Abstract*—The problem of matching a linear time-delay system to a given time-delay model is considered and solved in state-space form. The system under control is assumed in its phase-variable form. A two-dimensional example is worked out to illustrate the solution method.**

Manuscript received December 26, 1975.

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