Presented at 18th Conf. on Info. Sciences and Sys., Princeton, NJ, March 1984

A MODEL REFERENCE ADAPTIVE CONTROLLER FORMULATED FOR LOCAL OPTIMALITY IN NON-IDEAL USE

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Abstract

The objective of this paper is the development of a model reference adaptive controller that, if locally stable in non-ideal use, offers local optimality in its "convergent" performance in non-ideal use. Our particular innovation is the introduction of low-pass filtering of the plant inputs and outputs to remove their spurious, "uncontrollable" high frequency content prior to their use in the adaptive algorithm. Global convergence in ideal use is provable. Preliminary analysis, interpretation, and simulation of reduced-order use are provided to validate the local optimality character and robustness improvement possible with this practical embellishment.

I. Introduction

A popular technique for discrete-time adaptive controller development is the model reference approach [1]-[5]. This approach is based on the description of the control objective as tracking the response of a desired (or reference) model transfer function. Its attraction to adaptive control theoreticians lies in the special relationship between the tracking error and the controller parameter estimate error, which can be exploited for controller parameter estimate adaption. This exploitation has principally occurred in the case where this tracking error can be zeroed. This requires, for instance, that the controller structure be sufficiently complex to arbitrarily regulate all modes of the dynamic plant.

Only recently has significant research effort been focused on the more practical case where the tracking error cannot be totally removed by any controller parameterization, due possibly to an insufficient controller structure [6]-[12]. Much of this effort has been devoted to examining whether or not the inability to zero the tracking error can cause adaptive controller divergence. One result is that in certain cases adaptive control system stability can be proven to be retained and the existence of a region of attraction for the adapted controller parameters established.

Note that since the tracking error drives the controller parameter adaption, the inability to drive the tracking error to zero results in the continual perturbation of the adaptive controller parameter estimates and thus their lack of convergence to a single, fixed value. Reducing the magnitude of the positive step-size multiplier converting tracking error into controller parameter correction logically reduces the size of the region of this adaptive controller parameter "steady-state jitter". The positive step-size should be non-vanishing, i.e. bounded away from zero, to retain the ability to track possible time-variations in the plant parameters. Unfortunately but typically, as the step-size decreases the rate of tracking ability undesirably diminishes while "steady-state jitter" is desirably decreased.

Having struck a balance with the step-size that leads to an adequate rate of convergence but a rather small steady-state jitter, a basic question arises: How does the "steady-state controller" chosen by the particular adaptive algorithm perform in terms of its tracking error? To focus the discussion here we will address this question for a model reference adaptive controller that is unique due to its development so as to locally minimize, after effective convergence, the squared tracking error in the restricted complexity case.

The guiding concept is that a locally optimal adaptive controller should exhibit improved, robustness especially if the cost function being minimized can be interpreted and exploited for robustness enhancement. This is precisely the case with the algorithm introduced in this paper. The next section formulates a specific model reference adaptive controller problem and a related optimization objective. Section III introduces our innovation of low-pass filtering plant input-output values before using them in the adaptive algorithm. If the portion of the plant behavior that is "uncontrollable" due to insufficient controller order is high frequency phenomena, which is a practical situation, then this low-pass filtering should protect the adaption from overreacting. As we shall see, this added filtering, selected by the control designer, also appears in a minimization interpretation of the non-ideal use of this adaptive controller. Section IV provides simulation evidence that these interpretations are viable and that robustness improvement and local optimality are realizable with appropriate selection of this added filtering.

II. A Model Reference Adaptive Control Problem

Consider the following model-following problem. The plant is described by

$$A(q^{-1})y(k) = q^{-d}B(q^{-1})u(k), \qquad (2.1)$$

where y is the output, u the input,

$$A(q^{-1}) = 1 + a_1q^{-1} + a_2q^{-2} + \ldots + a_\alpha q^{-\alpha}, \qquad (2.2)$$

$$B(q^{-1}) = b_0 + b_1q^{-1} + b_2q^{-2} + \ldots + b_\beta q^{-\beta}, \qquad (2.3)$$

$b_0 \neq 0$, and q^{-1} is the unit delay operator, i.e. $q^{-i}u(k) = u(k-i)$. This plant is to be controlled such that its response y "matches" (given the same initial conditions) the output z of the desired

C. R. Johnson, Jr. is supported by NSF Grant ECS-8119312.

model

$$C(q^{-1})z(k) = q^{-\bar{d}}D(q^{-1})r(k) \qquad (2.4)$$

where

$$C(q^{-1}) = 1 + c_1q^{-1} + c_2q^{-2} + \ldots + c_\gamma q^{-\gamma}, \qquad (2.5)$$

$$D(q^{-1}) = d_0 + d_1q^{-1} + d_2q^{-2} + \ldots + d_\delta q^{-\delta} \qquad (2.6)$$

and

$$\bar{d} \geq d \geq 1. \qquad (2.7)$$

Formally the desired objective is to (adaptively) select controller parameters that minimize

$$J = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \{H(q^{-1})[z(k)-y(k)]\}^2, \qquad (2.8)$$

where $H(q^{-1})$ is a stable transfer function that emphasizes (or de-emphasizes) the importance of various frequency components of the tracking error $z-y$. The problem is complicated by the fact that $A(q^{-1})$ and $B(q^{-1})$ are initially unknown (and possibly slowly time-varying) such that an adaptive controller is suggested.

To formulate the adaptive controller problem in the standard manner as a recursive parameter estimation problem, a controller structure is first chosen that could be adequately parameterized if A and B were time-invariant and fully known. If $A(q^{-1})$ and $B(q^{-1})$ were known and $B(q^{-1})$ had all of its roots inside the unit circle the controller structure of Figure 1 would prove satisfactory when certain relations, described below, are satisfied by A, B, C, D, M, and N. From Figure 1

$$\frac{Y(z)}{R(z)} = \frac{z^{-\bar{d}}D(z^{-1})}{M(z^{-1})A(z^{-1}) - z^{-d}N(z^{-1})}. \qquad (2.9)$$

Thus $e \to 0$, if

$$C(q^{-1}) = M(q^{-1})A(q^{-1}) - q^{-d}N(q^{-1}) \qquad (2.10)$$

can be solved for the m_i and n_i in

$$M(q^{-1}) = m_0 - m_1q^{-1} - m_2q^{-2} - \ldots - m_\mu q^{-\mu} \qquad (2.11)$$

$$N(q^{-1}) = n_1q^{-1} + n_2q^{-2} + \ldots + n_\vartheta q^{-\vartheta}. \qquad (2.12)$$

For (2.10) to be solvable for an arbitrary stable C it is necessary and sufficient that

$$\mu + \vartheta \geq \max(\mu+\alpha, \vartheta+d) \geq \gamma. \qquad (2.13)$$

Define

$$W(q^{-1}) \equiv M(q^{-1})B(q^{-1}) =$$

$$w_0 - w_1q^{-1} - \ldots - w_\omega q^{-\omega}, \qquad (2.14)$$

where $w_0 \neq 0$ and $\omega = \mu + \beta$. Then at the summing junction of Figure 1

$$W(q^{-1})u(k) = q^{-(\bar{d}-d)}D(q^{-1})r(k) + N(q^{-1})y(k), \qquad (2.15)$$

which can be solved for a causal description of u(k) as

$$u(k) = \frac{1}{w_0}\Big[\sum_{i=1}^{\omega} w_i u(k-i) + \sum_{j=1}^{\vartheta} n_j y(k-j)$$

$$+ \sum_{t=0}^{\delta} d_t r(k-t-\bar{d}+d)\Big]. \qquad (2.16)$$

The control law implicit in (2.15) and explicit in (2.16) uses a deadbeat observer as is apparent from Figure 1. Such an observer is notoriously sensitive to output measurement noise. The perturbational effect of high bandwidth output measurement noise can be reduced by a low bandwidth observer. However, as is evident from an equivalent combined state observer-controller interpretation, the observer bandwidth should exceed that defined by the desired characteristic polynomial C in order to provide a rapidly convergent state observation error. A non-unity observer polynomial can effectively be added to the control loop of Figure 1 as the numerator of the series compensator and the denominator of the feedback compensator with no effect on the r to y transfer function. The adaptive solution of the next section is unaltered with a nonunity observer polynomial except in the definition of its information vector. (See (3.2).) With the observer, the past u and y in this vector are filtered through the autoregression operator with the observer polynomial as its denominator.

The adaptive control problem now becomes one of recursively estimating the w_i and n_i in (2.16). The hope is that when J in (2.8) cannot be set to zero at adaptive controller convergence, possibly due to the restricted complexity of W and N such that ω and ϑ are inadequate to satisfy (2.13) with $\alpha = \omega-\beta$, that it is practically minimized by the nearly convergent estimates of w_i and n_i. The next section presents a candidate algorithm for this local minimization objective.

III. Our Innovation and Its Justification

This section will state a candidate algorithm for the model reference adaptive control problem of the preceding section. We will then motivate further study of this particular algorithm by supporting its possible desirable minimization and robustness characteristics via heuristic arguments. The next section provides brief simulation evidence of these desirable characteristics.

The interest in (2.16) arises from recognizing that in ideal use

$$C(q^{-1})y(k) = z^{-\bar{d}}D(q^{-1})r(k)$$

$$= W(q^{-1})u(k-d) - N(q^{-1})y(k-d) \qquad (3.1)$$

when the parameters of W and N are chosen to satisfy (2.10) and (2.14). Define

$$X(k) \equiv [u(k)\ u(k-1) \ldots u(k-\omega)\ y(k-1)$$

$$\ldots y(k-\vartheta)]^T \qquad (3.2)$$

and

$$\theta \equiv [w_0 \;\; -w_1 \ldots -w_\omega \;\; -n_1 \ldots -n_\vartheta]^T. \quad (3.3)$$

Thus

$$C(q^{-1})y(k) = X^T(k-d)\theta. \quad (3.4)$$

For the estimates of θ, which are being adapted,

$$C(q^{-1})y(k) - X^T(k-d)\hat{\theta}(k-1)$$

$$= X^T(k-d)[\theta - \hat{\theta}(k-1)], \quad (3.5)$$

which is the equation error style prediction error useful in adapting the estimates in $\hat{\theta}$.

Adaptive Control Procedure

The form of (3.5) suggests an adaptive control procedure, similar to that in [13]: (i) measure y(k), (ii) update $\hat{\theta}(k-1)$ to $\hat{\theta}(k)$ via

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{\lambda(k)X(k-d)[C(q^{-1})y(k) - X^T(k-d)\hat{\theta}(k-1)]}{\epsilon + X^T(k-d)X(k-d)}, \quad (3.6)$$

where $\epsilon > 0$ and $\lambda(k)$ is chosen as a scalar within (0,2) such that the first entry of $\hat{\theta}(k)$, i.e. $\hat{w}_0(k)$, will not be zero, (iii) using the entries of

$$\hat{\theta}(k) \equiv [\hat{w}_0(k) \;\; -\hat{w}_1(k) \ldots -\hat{w}_\omega(k) \;\; -\hat{n}_1(k) \ldots -\hat{n}_\vartheta(k)]^T \quad (3.7)$$

solve (2.16) rewritten as

$$u(k) = \frac{1}{\hat{w}_0(k)}\Big[\sum_{i=1}^{\omega} \hat{w}_i(k)u(k-i) + \sum_{j=1}^{\vartheta} \hat{n}_j(k)y(k-j) + \sum_{t=0}^{\delta} d_t r(k-t-\bar{d}+d)\Big], \quad (3.8)$$

which is equivalent to solving $q^{-\bar{d}}D(q^{-1})r(k) = \hat{\theta}^T(k-1)X(k)$ for u(k). Note that, defining

$$\Lambda(k) = \frac{\lambda(k)}{\epsilon + X^T(k-d)X(k-d)} \quad (3.9)$$

and

$$\tilde{\theta}(k) \equiv \theta - \hat{\theta}(k) \quad (3.10)$$

and using (3.5), (3.6) can be written compactly as

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \Lambda(k)X(k-d)X^T(k-d)\tilde{\theta}(k-1). \quad (3.11)$$

Introduction of Information Vector Filtering

In an attempt to provide a more robust adaptive controller minimizing (2.8) but retaining the update form of (3.11), which is provably convergent, we propose the addition of information vector filtering. If the reduced-order controller parameterization being sought is intended to "overlook" imprecisely regulated high frequency plant modes, this filtering is quite practical as noted in [11, remarks below (5.2)]. In the ideal case, this suggestion results in the replacement of (3.11) by

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \Lambda(k)Z(k-d)Z^T(k-d)\tilde{\theta}(k-1) \quad (3.12)$$

where $\Lambda(k)$ is redefined as

$$\Lambda(k) = \frac{\lambda(k)}{\epsilon + Z^T(k-d)Z(k-d)} \quad (3.13)$$

and Z(k) is a *filtered version of* X(k)

$$F(q^{-1})Z(k) = G(q^{-1})X(k), \quad (3.14)$$

where F and G both have stable roots (within $|q| < 1$),

$$F(q^{-1}) = 1 + f_1 q^{-1} + f_2 q^{-2} + \ldots + f_\rho q^{-\rho}, \quad (3.15)$$

and

$$G(q^{-1}) = g_0 + g_1 q^{-1} + g_2 q^{-2} + \ldots + g_\rho q^{-\rho} \quad (3.16)$$

where $g_0 \neq 0$. As (3.11) is a compacted, implicit version of the implemented form in (3.6), (3.12) is also implicit. Developing an implementable form of (3.12) requires construction of $Z^T(k-d)\tilde{\theta}(k-1)$ from available entities such as $X^T(k-d-i)\tilde{\theta}(k-1-i)$, $X(k-d-i)$, $Z(k-d-i)$, and $\hat{\theta}(k-1-i)$ for $i = 0,1,2,\ldots$. Direct algebraic manipulations reveal that this is *possible with*

$$Z^T(k-d)\tilde{\theta}(k-1) = g_0[C(q^{-1})y(k) - X^T(k-d)\hat{\theta}(k-1)] + \sum_{i=1}^{\rho} [g_i - g_0 f_i][r(k-i) - Y^T(k-d-i)\hat{\theta}(k-1)] \quad (3.17)$$

where

$$F(q^{-1})r(k) = C(q^{-1})y(k) \quad (3.18)$$

and

$$G^{-1}(q^{-1})Z(k) = Y(k) = F^{-1}(q^{-1})X(k). \quad (3.19)$$

Thus (3.17) provides the causally implementable form of $Z^T(k-d)\tilde{\theta}(k-1)$ to be used in (3.12).

Non-Ideal Use Improvements

The pragmatic information vector filtering in (3.14) has two related purposes/interpretations: robustness enhancement and minimized cost function manipulation. We will discuss both of these briefly.

The filtered information vector Z is used only in the adaptive algorithm of (3.12) but not in the control law of (3.8), which still uses entries from X in (3.2). If the filtering in (3.14) is appropriately chosen, e.g. as low-pass, then high-frequency, unmodelable "perturbations" in X will be attenuated in their deleterious effect on the adaption with an anticipated improvement in robustness. This should help insulate the adaption mechanism from overreacting to broadband measurement noise or to well-damped, high frequency, unmodeled plant modes that are intentionally imprecisely controlled due to restricted controller order. This low-pass cutoff frequency should not be too low. *If it were, the persistent excitation* engendered by r might be effectively lost. It would be reasonable to require the low-pass information vector filter bandwidth to be greater than that desired for the control system. Thus, our information vector filtering is an improvement motivated (and restrained) by engineering practicality rather than solely analytical tractability.

As a bonus, the information vector filtering appears to be susceptible to an analytical interpretation in terms of the optimization problem statement of the preceding section. From a projection interpretation, as in [15], the equation error form algorithm of (3.12), if convergent, locally minimizes the mean of the square of $Z^T(k-d)\theta$ as implemented on the right of (3.17), where the time index of θ has been removed to indicate the convergence of $\hat{\theta}$. Our argument is that, if (3.12) remains stable such that $\hat{\theta}$ and X remain bounded, despite the inability to zero $Z^T(k-d)\theta$ due to nonideal use, and λ is small enough that the jitter in $\hat{\theta}$ near convergence is effectively imperceptible, then upon "convergence" J in (2.8) is minimized for the $H(q^{-1})$ that equates $Z^T(k-d)\theta$ and $H(q^{-1})[z(k)-y(k)]$. Assuming that the "convergent" $\hat{\theta}$ is effectively time-invariant, permits the derivation of

$$Z^T(k-d)\theta = \frac{G(q^{-1})C(q^{-1})}{F(q^{-1})}[y(k)-z(k)]. \quad (3.20)$$

Thus, our algorithm if stably convergent, is expected to locally minimize (2.8) with H set via

$$H(q^{-1}) = \frac{G(q^{-1})C(q^{-1})}{F(q^{-1})} \quad (3.21)$$

Note that if the filtering of X in (3.14) is not utilized, i.e. $F \equiv G \equiv 1$, then (3.12) reduces to (3.11) or equivalently (3.6) and $H \equiv C$ in (3.21). This same form for $H(q^{-1})$ results if a fixed non-unity observer polynomial is used in the controller. This implies that the mean of $[C(q^{-1})(z(k) - y(k))]^2$ is being minimized. This would be inappropriate in a typical restricted complexity setting by the following reasoning. Since C represents the desired characteristic equation, C^{-1} will typically be low-pass. Thus, with H = C, J in (2.8) inappropriately emphasizes the high frequency mismatch of z and y that is assumed to be "uncontrollable" due to the restricted complexity of W and N. Assuming that the low frequency portion of z-y will generate the error we most wish to reduce suggests using F and G in (3.21) to correct the high-pass tendency of C in a more appropriate H. One such choice is F = C and G = 1, which reduces (3.21) to H = 1 so that the mean of $[z - y]^2$ is being minimized. This choice does not emphasize the high frequency content of the model-following error as does the absence of information vector filtering with F = G = 1. To further attenuate the high frequency model-following error additional roots could be added to F beyond those of C. Note that it is reasonable to require that the cutoff frequency of these additional roots should not be less than the bandwidth of the low-pass C.

In summary, the proposed information vector filtering appears to offer practical performance improvements with appropriate selection of F and G guided by a minimization interpretation. The traditional initial analytical effort for theoretical support of confidence in application of adaptive controllers is proof of global asymptotic stability in ideal use. Such a proof is possible based on the approach of [13]. Since the form of (3.12)-(3.13) matches that of (3.9) and (3.11), the additional difficulty arises in the justification of the conditions in the linear boundedness lemma pivotal to the global stability proof in [13]. These details can be found in [14].

IV. Simulations

To succinctly summarize key observations from our current simulation experience we will focus first on an example of adaptive model-following behavior described in [2] and interpreted in [7] to illustrate the robustness improvement possibilities of our algorithm. The actual, continuous-time, third order plant has the transfer function

$$\frac{Y(s)}{U(s)} = \frac{458}{(s+1)(s^2+30s+229)}. \quad (4.1)$$

A first order model is assumed, i.e. $\alpha = 1$, $\beta = 0$, and d = 1 is presumed in (2.1)-(2.3), and the desired discrete-time (sample period = 0.1 seconds) reference model transfer function is chosen as

$$\frac{Z(z)}{R(z)} = \frac{1}{z-0.8}, \quad (4.2)$$

where $\bar{d} = \gamma = \delta = 1$. With $\mu = \vartheta = 1$ in (2.10)-(2.12) so $\omega = 1$ in (2.14), (2.13) is satisfied with $2 \geq \max(2,2) \geq 1$. In [2], the reference input, a unit step plus a 2 unit peak-to-peak sinusoid with frequency ω_r, resulted in instability with $\omega_r \geq 16.09$. With $\omega_r = 16.09$, F = G = 1 in (3.14), i.e. without dynamic information vector filtering, and $\lambda = \varepsilon = 1$ in (3.13), the adaptive algorithm of (3.12) resulted in instability. The initial controller parameter estimates were chosen from solution of the first-order model-following problem for the zero-order-hold equivalent of the zero frequency input model of (4.1) from [7] (yielding $\hat{v}_0(0) = 0.169$, $\hat{w}_1(0) = 0.01964$, and $\hat{n}_1(0) = -0.106256$). This controller initialization, if unchanged, leads to a stable closed-loop system when applied to the zero-order-hold equivalent of (4.1), which has its lowest frequency pole at $z = e^{-T} = 0.9$. Thus, adaption without information vector filtering destabilizes an initially stable controller. With information vector filtering of $F(q^{-1}) = (1-0.8q^{-1})(1-0.75q^{-1})$ and $G(q^{-1}) = 1$, this instability does not occur as shown in Figure 2. Figure 2 also shows the response of the reference model, which the adaptive controller quickly tracks. Note the relative attenuation of the "high" frequency sinusoid, which caused the instability when information vector filtering was not present. Also, note that the selection of these information vector filtering polynomials, our first choice for this example, was based on our development and interpretation of (3.21).

We also ran simulations with our adaptive controller parameters initialized totally arbitrarily as all zero and all unity. The adaptive controller with information vector filtering still maintained asymptotic stability, though with drastically increased transients in the plant output. The stable behavior for these alternative controller parameter initializations indicates that the region of attraction for this improved scheme is quite significant.

In [2] and [7] the destabilizing effect of output measurement noise on the adaptive control system step response was also noted. We observed this unstable behavior with a white zero-mean guassian output measurement noise of 0.2 standard deviation. This occurred in the unit step response in an attempt to follow (4.2) without using information vector filtering. Large (and growing) oscillations appeared in the output after approximately *1.5 seconds. With the addition of the information* vector filtering used before in conjunction with (4.2), stable behavior resulted as shown in Figure 3. This response was observed for 60 seconds (600 samples) with the concern that instability may have only been delayed, but no such undesirable behavior appeared. Note that the desired, noise-free, steady-state response is an output of five units.

To examine the local minimization of (2.8) given (3.21) by our proposed algorithm we simulated adaptive control of the discrete-time plant with the transfer function

$$\frac{Y(z)}{U(z)} = \frac{z^{-1}(1+0.9z^{-1})}{(1-0.1z^{-1})(1-0.85z^{-1})(1-0.9z^{-1})} \quad (4.3)$$

assuming a second-order model and using the second-order desired reference model with transfer function

$$\frac{Z(z)}{R(z)} = \frac{z^{-2}}{(1-0.5z^{-1})(1-0.5z^{-1})}. \quad (4.4)$$

We considered three choices for the information vector filtering polynomials: (i) $F(q^{-1}) = G(q^{-1}) = 1$, (ii) $F(q^{-1}) = C(q^{-1}) = (1-0.5q^{-1})(1-0.5q^{-1})$, $G(q^{-1}) = 1$, and (iii) $F(q^{-1}) = C(q^{-1})(1-0.3q^{-1})$, $G(q^{-1}) = 1$. The reference input used was $r(k) = 1 + \sin(0.3k) + 1.5 \sin(0.7k) + 0.7 \sin(1.5k)$. The initial controller parameterization was chosen by removing the highest frequency pole from (4.3), i.e. $z = 0.1$, rescaling the transfer function for the correct DC gain, and solving (2.10) and (2.14) for $W(q^{-1})$ and $N(q^{-1})$. For each simulation using one of the three choices for $F(q^{-1})$ the model-following error $z-y$ was measured for the last 100 iterations of a 5000 iteration simulation. For each of these three "convergent" model-following error sequences the cost function of (2.8) was computed for various $H(q^{-1})$. The results are summarized in Table 1. Note that the smallest cost function in each column of Table 1 arises with the $F(q^{-1})$ that would generate via (3.21) the $H(q^{-1})$ used in (2.8), as would be expected. In these simulations we obtained nearly convergent controller estimates by dropping the step-size multiplier λ in (3.13) to a very small value over the last 1000 iterations. We also perturbed the controller that yielded the smallest cost function J in Table 1 and simulated the control system with this new parameterization fixed (i.e. not adapting). These controller parameterization perturbations always increased the cost function value. These tests, though far from conclusive, suggest that the adaptive controller exhibits local optimality. Thus our simulation experience currently supports and encourages further study of our claims of improved robustness and local optimality for our proposed adaptive controller.

V. Conclusion

This paper has raised the problem of developing a model reference adaptive controller that, in non-ideal use, will essentially converge to a *locally optimum controller parameterization.* A causal candidate algorithm was proposed. It was argued and demonstrated in simple simulation examples that, if stable in non-ideal use, this candidate algorithm could exhibit improved robustness and be locally optimal. Further study has been amply justified.

VI. Acknowledgement

The authors would like to thank M. L. Cauldwell of Cornell University for performing the simulations reported in Section V.

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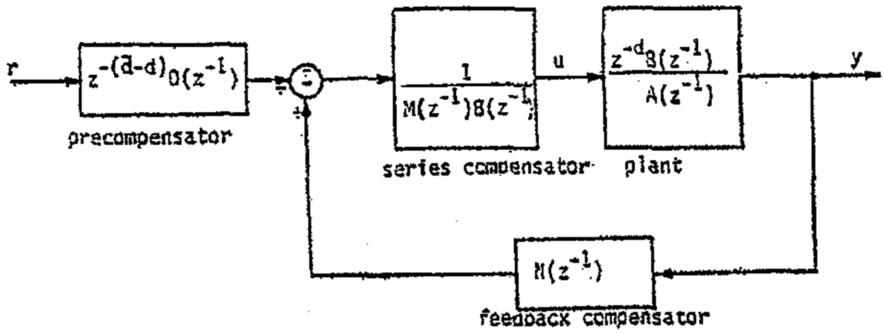

Figure 1: Numerator-Cancelling, Model-Following Controller Structure

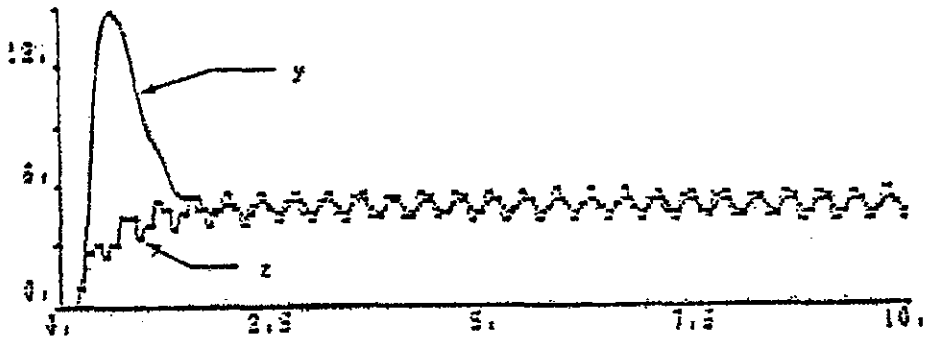

Figure 2: First-Order MRAC Using Information Vector Filtering Attempting to Cause Third-Order Platn in (4.1) to Track Step Plus Sinusoid Response of Reference Model in (4.2)

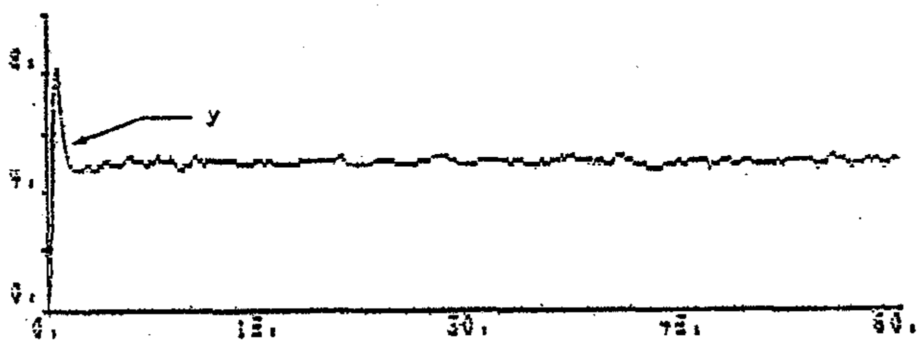

Figure 3: First-Order MRAC Using Information Vector Filtering Attempting to Cause Third-Order Plant in (4.1) to Track Step Response of Reference Model in (4.2) in the Presence of Plant Output Measurement Noise

	Cost Function Filtering		
	$H(q^{-1})=C(q^{-1})$	$H(q^{-1})=1$	$H(q^{-1})=1/(1-0.3q^{-1})$
Information Vector Filtering			
$F(q^{-1})=G(q^{-1})=1$	0.118	0.059	0.329
$F(q^{-1})=C(q^{-1}), G(q^{-1})=1$	0.199	0.035	0.375
$F(q^{-1})=C(q^{-1})(1-0.3q^{-1}), G(q^{-1})=1$	0.122	0.054	0.208

Table 1: Evaluation of Cost Function in (2.8) for Various $H(q^{-1})$ for Last 100 Iterations of MRAC with Various *Information Vector Filtering* $F(q^{-1})$ *and* $G(q^{-1})$.