2016 IEEE 55th Conference on Decision and Control (CDC)
ARIA Resort & Casino
December 12-14, 2016, Las Vegas, USA

# Formation feasibility on coordination control of networked heterogeneous systems with drift terms

Zhiyong Sun and Brian D. O. Anderson

*Abstract*— **In this paper, we discuss a general problem of formation feasibility for multi-agent coordination control when individual agents have kinematics constraints modelled by affine nonlinear control systems with possible drift terms. All agents need to work cooperatively to maintain a global formation task described by edge constraints. For such multi-agent group, we assume that different agents may have totally different dynamics, which brings the problem of coordination control of networked *heterogeneous* systems. Based on concepts of (affine) distribution and codistribution, we propose a unified framework and an algebraic condition to determine the existence of feasible motions under both kinematic constraints and formation constraints. In the case that feasible motions exist, we propose a systematic procedure to obtain an equivalent dynamical system which generates all types of feasible motions. Examples involving coordination control of constant-speed agents are provided to demonstrate the application of this coordination control framework.**

## I. INTRODUCTION

Collective coordination control of networked multi-agent systems has received considerable attention in recent years, partly motivated by its applications in many areas [1], [2]. A particular class of cooperative tasks for multi-agent coordination is formation control, in which the control objective is to form or maintain a prescribed geometric relationship for a group of spatially distributed agents [3]. Maintaining a formation is important in multi-agent coordination and in some cases would be a premise for agents to perform additional tasks such as surveillance, coverage, target detection, etc ([4], [5]).

Given a predefined formation task assigned to a group of distributed agents, a fundamental problem is to determine whether there exists a feasible trajectory for such agent group to maintain the formation task subject to both kinematics and formation constraints. Formation feasibility was firstly discussed by P. Tabuada et. al. in [6]. They employed tools in differential geometry and derived an elegant criterion to analyze whether a networked agent group has feasible motions to maintain the formation constraint. Later papers along this direction include [7], which proposed a similar geometric approach to discuss some real-time formation control problems, including radar deception, formation keeping and formation reconfiguration. As revealed in [6] and [7], the central idea in the formation feasibility analysis is the interplay of agents' kinematic constraints and formation task constraints in controlling a formation and in generating a feasible trajectory for all the agents.

The concept of formation feasibility in this paper builds on the analysis in [6], while here we present several extensions and generalizations to this fundamental networked control problem. On the one hand, compared with the control system model discussed in [6], here we consider agents' dynamics modelled by affine nonlinear control systems with possibly additional *drift terms*. This is motivated by the fact that most real-life control systems have drift terms. We note that affine nonlinear control systems with drift terms are very general in system modelling and are also popular choices for nonlinear control system analysis in the control community [8]. One motivating example to consider agent dynamics with drift terms is the coordination control of *constant-speed* agents. Coordination control and collective circular motion with unit-speed agents or non-identical constant-speed agents was reported in e. g. [9], [10]. More recently, the tracking control problem for multiple constant-speed agents was discussed in [11] which also revealed its performance limitations. However, a general analysis on performance feasibility and its relationship to the speed constraint is still lacking. The results in this paper will shed new insights to this coordination control problem.

On the other hand, we further consider the case that individual agents in the networked system may have totally different dynamics, a possibility which includes fully actuated agents, under-actuated agents, or agents with a nonholonomic constraint and/or drift terms. Under this framework, we develop a fairly checkable condition for the existence of feasible motions by including both heterogeneous agent dynamics and formation constraints. Thus, the coordination control framework developed in this paper is general enough to encompass many coordination control problems and presents a unified approach in the coordination feasibility analysis of networked *heterogeneous* systems. When the feasibility condition is satisfied and there exist feasible motions, a natural problem arises as how to obtain such feasible motions. We then propose a systematic approach to derive feasible motions in terms of an equivalent control system.

The rest of this paper is organized as follows. Section II introduces preliminary concepts on differential geometry and the problem formulation with a motivating example. Section III presents the main result on formation feasibility condition by incorporating both heterogeneous kinematic constraint and formation constraint into a unified form. Section IV discusses its application in coordination control of networked

Z. Sun and B. D. O. Anderson are with Data61-CSIRO (formerly NICTA) and Research School of Engineering, The Australian National University, Canberra ACT 2601, Australia. B. D. O. Anderson is also with School of Automation, Hangzhou Dianzi University, Hangzhou 310018, China. {zhiyong.sun, brian.anderson}@anu.edu.au

agents with constant-speed dynamics. In Section V, we discuss the motion generation problem using the formation feasibility analysis. Finally, Section VI concludes this paper.

## II. Preliminaries, problem formulation and motivating examples

### A. Preliminaries on differential geometry and system equation

Some standard notions from differential geometry (especially the concepts of distribution/codistribution) will be introduced here. More background can be found in [8, Chapter 1] and [12, Chapter 7].

A distribution $\Delta(x)$ on $\mathbb{R}^n$ is an assignment of a linear subspace of $\mathbb{R}^n$ at each point $x$. Given a set of $k$ vector fields $X_1(x), X_2(x), \cdots, X_k(x)$, we define the distribution as

$$\Delta(x) = \text{span}\{X_1(x), X_2(x), \cdots, X_k(x)\}. \tag{1}$$

A vector field $X$ belongs to a distribution $\Delta$ if $X(x) \in \Delta(x)$, $\forall x \in \mathbb{R}^n$. Here we assume all distributions have constant rank.

A codistribution assigns a subspace to the dual space, denoted by $(\mathbb{R}^n)^\star$. Given a distribution $\Delta$, for each $x$ consider the annihilator of $\Delta$, which is the set of all covectors that annihilates all vectors in $\Delta(x)$ (see [8, Chapter 1] or [13, Section 3.7 of Chapter 3])

$$\Delta^\perp = \{\omega \in (\mathbb{R}^n)^\star | \langle \omega, X\rangle = 0, \ \forall X \in \Delta\}. \tag{2}$$

In this paper, we assume that each individual agent's dynamics are described by the following general form (i.e. affine nonlinear control system)

$$\dot{p}_i = f_{i,0} + \sum_{j=1}^{l} f_{i,j} u_{i,j} \tag{3}$$

where $p_i \in \mathbb{R}^{n_i}$ is the state of agent $i$, $n_i$ is the number of the state, $f_{i,0}$ is a smooth drift term, and $u_{i,j}$ is the *scalar* control input associated with the smooth vector field $f_{i,j}$. Such an affine nonlinear control system (3) with a drift term [1] is a very general tool to describe many different types of real-life control systems, including control systems with an underactuation property or nonholonomic constraints.

### B. Motivating examples

The paper [6] introduced the concept of motion feasibility problem for multi-agent formations. The discussions in [6] were restricted to the coordination control of *drift-free* control systems (i.e. $f_{i,0} = 0$) in the form of

$$\dot{p}_i = \sum_{j=1}^{l} f_{i,j} u_{i,j} \tag{4}$$

However, the above model is not general enough to describe many real-life nonlinear control systems. In contrast, the control system model in (3) encompasses a larger number of practical models and is modelling the most popular nonlinear control system [8]. As an example, the unicycle-type agent with constant-speed constraints is one such nonlinear control system with drift terms that can be described by (3) but not by (4). Such system dynamics can be described as

$$\begin{aligned} \dot{x}_i &= v_i \cos(\theta_i) \\ \dot{y}_i &= v_i \sin(\theta_i) \\ \dot{\theta}_i &= u_i \end{aligned} \tag{5}$$

where $x_i \in \mathbb{R}, y_i \in \mathbb{R}$ are the coordinates in the real plane and $\theta_i$ is the heading angle for agent $i$. The agent has a fixed cruising speed $v_i > 0$, which could be different for distinct agents; $u_i$ is the control input to be designed for steering the orientation.

Introducing the vector fields as

$$f_{i,0} = \begin{bmatrix} v_i\cos(\theta_i) \\ v_i\sin(\theta_i) \\ 0 \end{bmatrix}, f_{i,1} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \tag{6}$$

we can rewrite the system (5) as

$$\dot{p}_i = [\dot{x}_i, \dot{y}_i, \dot{\theta}_i]^\top = f_{i,0} + f_{i,1} u_i \tag{7}$$

which has the form of (3).

### C. Problem formulation: motion feasibility with kinematics and formation constraints

In this paper, we assume a networked multi-agent control system modelled by an undirected graph $\mathcal{G}$, in which we use $V$ to denote its vertex set and $E$ to denote the edge set. The vertices consist of $n$ heterogeneous agents each modelled by the general dynamical equation (3). The graph consists of $m$ edges, each with an inter-agent formation constraint.

A family of formation constraints $C$ is indexed by the edge set, denoted as $C_E = \{c_{ij}\}_{(v_i, v_j)}$ with $(v_i, v_j) \in E$. For each edge $(v_i, v_j)$, $c_{ij}$ is a vector function defining the formation constraints between agents $i$ and $j$ and the constraint is enforced if $c_{ij}(p_i, p_j) = 0$. Such formation constraints can be used to describe very general coordinate control problems, such as formation shape control, formation tracking, coverage control, etc. For example, in formation shape control, the constraint vector function $c_{ij}$ can be functions of desired relative position, or desired bearings, or desired distances between agents $i$ and $j$ that can be used to describe a target formation (for example, see [3]).

The formation feasibility problem is stated as follows:

***Problem 1:*** Given a formation graph $F = [V, E, C]$, determine whether there are feasible trajectories $p_i(t)$ (or equivalently, feasible agents' motions $\dot{p}_i(t)$) for all agents whose kinematics are modelled by (3) with possible drift terms, such that the trajectories $p_i(t)$ also meet formation constraints $C_E = \mathbf{0}$, where $C_E = [\cdots, c_{ij}^\top, \cdots]^\top$ for all $(v_i, v_j) \in E$ and $t \in I$.

In the case that there exist feasible motions, we further consider the motion generation problem formulated as below.

***Problem 2:*** Given a formation graph $F = [V, E, C]$ with feasible agents' motions, determine an equivalent control system that generates feasible motions for the networked heterogeneous multi-agent system.

[1] A statement by Roger Brockett in his recent survey paper [14]: "almost all real systems have a drift term...".

***Remark 1:*** A premise of solving the above motion feasibility problem is that the formation constraint for the distributed edge set should be non-conflicting, and the overall constraint for all the edges should be realizable at least in the full Euclidean space. For example, if the formation is described by relative position vectors which are conflicting, then the formation is unachievable and this may lead to unexpected flocking motion [15]. As another example, if the formation is described by distance constraints to realize a rigid shape in a 3-agent group, then the set of formation distances should satisfy the triangle inequality [16]. Thus, in order to well define the feasibility problem, we need to first assume that the formation constraints are non-conflicting and realizable.

## III. Conditions for feasible motion

### A. Constraints on inter-agent formations

For ease of notation, we collect all of the agents' states together, denoting them by the composite state vector $P = [p_1^\top, p_2^\top, \cdots, p_n^\top]^\top$. We collect the formation constraints for all the edges and define an overall formation constraint denoted by

$$C_E = [\cdots, c_{ij}^\top, \cdots]^\top = \mathbf{0}. \quad (8)$$

Formation feasibility means that the constraints are satisfied along the formation trajectories. Thus, one can obtain

$$\frac{\mathrm{d}}{\mathrm{d}t} C_E = \mathcal{L}_P C_E + \frac{\partial C_E}{\partial t} = 0, \quad (9)$$

where $\mathcal{L}_P C_E$ denotes the Lie derivative of $C_E$ along $P$.

If we group all the constraints for all the edges by writing down a compact form

$$T_F = -[\cdots, (\frac{\partial c_{ij}}{\partial t})^\top, \cdots]^\top, \quad (10)$$

we can reexpress equation (9) as

$$\Omega_F(\dot{P}) = T_F. \quad (11)$$

Thus, the vector field $\dot{P}$ defined by the above equation represents possible motions for all the agents that respect the formation constraint.

### B. Constraints on individual agents' kinematics

The kinematics constraints of the drift-free agent model (4) can be described in an equivalent form

$$\omega_{i,j}(p_i)\dot{p}_i = 0, j = 1, \cdots, n_i - l. \quad (12)$$

The above transformation is based on the idea that a distribution generated by vector fields of a nonlinear control system can be equivalently defined by its annihilating codistribution. Note that each $\omega_{i,j}(p_i)$ in (12) is a *row* covector in the dual space $(\mathbb{R}^{n_i})^\star$.

We consider a group of $n$ heterogeneous agents, each modelled by the general control system in the form of (3). Note that an affine nonlinear control system (3) with drifts can be equivalently described by the following affine distribution

$$\Delta_i = f_{i,0} + \text{span}\{f_{i,1}, f_{i,2}, \cdots, f_{i,l}\}. \quad (13)$$

As opposed to the system constraint transformation shown in (12) for drift-free systems, for the system (3) with drifts, one can obtain a corresponding transformation with equivalent constraints via the construction of covectors:

$$\omega_{i,j}(p_i)\dot{p}_i = q_{i,j}, j = 1, \cdots, n_i - l, \quad (14)$$

where the parameter $q_{i,j}$ is due to the existence of the drift term $f_{i,0}$. We collect all the row covectors $\omega_{i,j}$ as

$$\Omega_{K_i} = \begin{bmatrix} \omega_{i,1} \\ \omega_{i,2} \\ \vdots \\ \omega_{i,n_i-l} \end{bmatrix}, \quad (15)$$

and similarly define

$$T_{K_i} = [q_{i,1}, q_{i,2}, \cdots, q_{i,n_i-l}]^\top. \quad (16)$$

By doing this, one can rewrite (14) in a compact form as follows

$$\Omega_{K_i}\dot{p}_i = T_{K_i}. \quad (17)$$

From the above, we collect all the kinematics constraints for all the agents in a composite form

$$\Omega_K = [\Omega_{K_1}^\top, \Omega_{K_2}^\top, \cdots, \Omega_{K_n}^\top]^\top, \quad (18)$$

and

$$T_K = [T_{K_1}^\top, T_{K_2}^\top, \cdots, T_{K_n}^\top]^\top. \quad (19)$$

Thus, the overall kinematic constraint for all the agents can be stated in a compact equation

$$\Omega_K(\dot{P}) = T_K. \quad (20)$$

***Remark 2:*** Note that if the system $i$ has a drift (as in (3)) but the drift satisfies

$$f_{i,0} \in \text{span}\{f_{i,1}, f_{i,2}, \cdots, f_{i,l}\} \quad (21)$$

then one can choose a control $u_i$ to cancel the drift. In such case the system with drifts (3) can be transformed to be the drift-free system (4), and the codistribution can be defined by using the covector $\omega_{i,j}$ in (12). Note that the constant-speed agent model (5) does not meet the above assumption (21).

### C. Main results on motion feasibility

By including both kinematic constraints and formation constraints for all the agents into a unified expression, the motion feasibility problem then becomes one of establishing existence of a solution to the following equation

$$\Omega(\dot{P}) = T \quad (22)$$

where

$$\Omega = \begin{bmatrix} \Omega_F \\ \Omega_K \end{bmatrix}, T = \begin{bmatrix} T_F \\ T_K \end{bmatrix}. \quad (23)$$

We remark that the expression of the codistribution $\Omega$ is coordinate-free and is also independent of the enumeration of edge sets.

The formation feasibility condition is shown in the following theorem.

***Theorem 1:*** The undirected formation has feasible motions if the above equation (22) has solutions, or equivalently if $T$ belongs to the range of $\Omega$.

***Remark 3:*** As we remarked before, the condition is applicable for networked heterogeneous systems as the model of (3) includes different types of agent dynamics. Also, to determine and calculate the condition, symbolic toolboxes from certain popular software (e.g. *Matlab* or *Mathematica*) are readily available. Note that the condition stated in (22) of Theorem 1 can be reexpressed in an equivalent way in terms of the rank condition of the form as below

$$\mathrm{rank}(\Omega) = \mathrm{rank}([\Omega, T]). \tag{24}$$

This alternative condition could facilitate calculations using available software.

***Remark 4:*** For the time-invariant formation case we have $\frac{\partial C_E}{\partial t} = 0$ and thus $T_F = \mathbf{0}$; in the drift-free model case there holds $T_{K,i} = 0$ for all $i$ and thus $T_K = \mathbf{0}$. Under these two special cases, one has

$$T = \begin{bmatrix} T_F \\ T_K \end{bmatrix} = \mathbf{0}, \tag{25}$$

and the feasibility condition in Theorem 1 reduces to the condition statement in Theorem 4.1 of [6].

## IV. Application to Coordination Control of Multiple Constant-Speed Agents

Coordination and formation control of constant-speed agents (e.g. fixed-wing UAVs with constant cruising speeds) has been discussed in [9], [17], [10], [18], [11], etc. However, a complete solution to such a control problem is not yet available. In this section, this coordination control problem is revisited with insights obtained from the above formation feasibility analysis.

### *A. Affine distribution and codistribution generated by constant-speed dynamics*

Following similar notations as in Section II, we denote each agent's state as $p_i = [x_i, y_i, \theta_i]^\top$, with the system dynamics described in (5) and (7). The vector fields shown in (6) generate the following affine distribution:

$$\Delta_i = f_{i,0} + \mathrm{span}(f_{i,1}) \tag{26}$$

We now construct the covectors for the codistribution[2] $\Delta_i^\perp$. Note that the dimension of the affine distribution $\Delta_i$ is 1, which implies that the dimension of the codistribution is 2. We denote two linearly independent covectors of the codistribution as $\omega_{i,1}$ and $\omega_{i,2}$. Direct calculation shows the following:

$$\begin{bmatrix} \sin(\theta_i) & -\cos(\theta_i) & 0 \\ \cos(\theta_i) & \sin(\theta_i) & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} 0 \\ v_i \end{bmatrix} \tag{27}$$

which is an instance of system constraint transformation shown in (14). Thus, according to (17), we let $T_{K_i} = [q_{i,1}, q_{i,2}]^\top = [0, v_i]^\top$ in the construction of the corresponding codistribution. Based on the constraint transformation concept presented in Section III-B, the above (27) presents an equivalent way to describe the kinematics constraint for a constant-speed agent. Note that a basis for describing any covector in the dual space is $(\mathrm{d}x_i, \mathrm{d}y_i, \mathrm{d}\theta_i)$, which gives the following expression of the covectors

$$\begin{aligned} \omega_{i,1} &= \sin(\theta_i)\mathrm{d}x_i - \cos(\theta_i)\mathrm{d}y_i \\ \omega_{i,2} &= \cos(\theta_i)\mathrm{d}x_i + \sin(\theta_i)\mathrm{d}y_i \end{aligned} \tag{28}$$

By denoting

$$\Omega_{K,i} = \begin{bmatrix} \omega_{i,1} \\ \omega_{i,2} \end{bmatrix}, \tag{29}$$

there holds $\Omega_{K,i} f_{i,1} = 0$ and $\Omega_{K,i} f_{i,0} = T_{K_i}$. These covectors and their equivalent forms in formulating distributions/codistributions will be useful in later analysis on coordination control with constant-speed agent groups.

### *B. Example: rigid formation maintenance and coordination by constant-speed agents*

In this example, we suppose the formation is defined by a certain set of inter-agent distances (as the problem formulation in [20]) and the control task is to coordinate a group of constant-speed agents while maintaining the rigid target formation. We consider a 3-agent group for such control task. The inter-agent distances are denoted as $d_{ij}$ with $i, j = 1, 2, 3, i \neq j$, and the constraint function for edge $(i, j)$ is defined as

$$c_{ij} = \frac{1}{2}(x_i - x_j)^2 + \frac{1}{2}(y_i - y_j)^2 - d_{ij}^2. \tag{30}$$

According to the analysis in Section III-A and Section IV-A, the matrices $\Omega_F$ and $\Omega_K$ for characterizing codistributions can be calculated as shown in (31) and (32), respectively. [3] The overall constraint matrix $\Omega$ is now constructed as $\Omega = [\Omega_F^\top, \Omega_K^\top]^\top \in \mathbb{R}^{9\times 9}$.

$$\Omega_K = \begin{bmatrix} \sin(\theta_1)\mathrm{d}x_1 - \cos(\theta_1)\mathrm{d}y_1 \\ \cos(\theta_1)\mathrm{d}x_1 + \sin(\theta_1)\mathrm{d}y_1 \\ \sin(\theta_2)\mathrm{d}x_2 - \cos(\theta_2)\mathrm{d}y_2 \\ \cos(\theta_2)\mathrm{d}x_2 + \sin(\theta_2)\mathrm{d}y_2 \\ \sin(\theta_3)\mathrm{d}x_3 - \cos(\theta_3)\mathrm{d}y_3 \\ \cos(\theta_3)\mathrm{d}x_3 + \sin(\theta_3)\mathrm{d}y_3 \end{bmatrix} \tag{31}$$

[2] The codistribution that is dual to an affine distribution is termed as $t$-codistribution in [19].

[3] We would like to clarify that $\Omega_F$ and $\Omega_K$ are matrices and not vectors, with $\Omega_F \in \mathbb{R}^{6\times 9}$ and $\Omega_K \in \mathbb{R}^{3\times 9}$. Note that we have used the standard basis $(\mathrm{d}x_1, \mathrm{d}y_1, \mathrm{d}\theta_1, \cdots, \mathrm{d}x_3, \mathrm{d}y_3, \mathrm{d}\theta_3)$ in the dual space to write down each row covector in $\Omega_F$ and $\Omega_K$.

$$\Omega_F = \begin{bmatrix} (x_1 - x_2)\mathrm{d}x_1 + (y_1 - y_2)\mathrm{d}y_1 + (x_2 - x_1)\mathrm{d}x_2 + (y_2 - y_1)\mathrm{d}y_2 \\ (x_1 - x_3)\mathrm{d}x_1 + (y_1 - y_3)\mathrm{d}y_1 + (x_3 - x_1)\mathrm{d}x_3 + (y_3 - y_1)\mathrm{d}y_3 \\ (x_2 - x_3)\mathrm{d}x_2 + (y_2 - y_3)\mathrm{d}y_2 + (x_3 - x_2)\mathrm{d}x_3 + (y_3 - y_2)\mathrm{d}y_3 \end{bmatrix} \tag{32}$$

$$R = \begin{bmatrix} (x_1 - x_2) & (y_1 - y_2) & (x_2 - x_1) & (y_2 - y_1) & 0 & 0 \\ (x_1 - x_3) & (y_1 - y_3) & 0 & 0 & (x_3 - x_1) & (y_3 - y_1) \\ 0 & 0 & (x_2 - x_3) & (y_2 - y_3) & (x_3 - x_2) & (y_3 - y_2) \end{bmatrix} \tag{33}$$

The matrix $T$ is calculated as

$$T = [0, v_1, 0, v_2, 0, v_3, 0, 0, 0]^\top \in \mathbb{R}^{9\times 1}. \tag{34}$$

Thus, the motion feasibility problem is now transformed to the problem of determining whether there exists a vector field $\dot{P}$ that satisfies $\Omega\dot{P} = T$. After these matrices are constructed with the specified parameters $v_i$ and $d_{ij}$ describing both kinematics constraints and formation constraints, available computation approaches could then be employed to assist the analysis of determining the solution issue.

For the aim of demonstration, in the following analysis we assume that $d_{12} = d_{23} = d_{31} = d$, that is, the target rigid formation is an equilateral triangle. We then consider different sets of cruising speeds to determine the solution property. In the case that all agents have identical cruising speeds, i.e. $v_1 = v_2 = v_3$, a simple calculation shows that the motion solution is that either

$$\dot{\theta}_1 = \dot{\theta}_2 = \dot{\theta}_3 = 0, \theta_1 = \theta_2 = \theta_3 \tag{35}$$

or

$$\dot{\theta}_1 = \dot{\theta}_2 = \dot{\theta}_3, \theta_1 = \theta_2 + \frac{2\pi}{3} = \theta_3 + \frac{4\pi}{3} \tag{36}$$

which correspond to (i) a translational motion with the same direction or (ii) a rotational motion around the triangle centroid. In the case that $v_1 = 0, v_2 = v_3$, a feasible motion exists such that agents 2 and 3 rotate around agent 1 with the same angular velocity. These two types of feasible formations are shown in Figure 1. In the case that all agents have non-identical cruising speeds, there usually do not exist a solution to the feasibility condition except for some special cases, which agrees with our intuition that maintaining a rigid shape by using non-identical constant-speed agents is generally impossible.

We also remark that the codistribution matrix $\Omega_F$ for characterizing the formation constraint can be interpreted as a **rigidity matrix** $R$ (see [21] for detailed definitions). For ease of demonstration, $R$ is written in (33). A reinterpretation of the above motion feasibility result is that any feasible velocity $(\dot{x}_i, \dot{y}_i)$ should satisfy

$$[\dot{x}_1, \dot{y}_1, \dot{x}_2, \dot{y}_2, \dot{x}_3, \dot{y}_3]^\top \in \text{null}(R) \tag{37}$$

where null$(R)$ denotes the null space of the rigidity matrix $R$ associated with a realization of the target formation. For a 2-D infinitesimally rigid formation, the dimension of the null space of the associated rigidity matrix $R$ is 3 (see [21]), corresponding to rotational and translational motions. Thus, the motion solution of a feasible translational motion mentioned above is that

$$[\dot{x}_1, \dot{y}_1, \dot{x}_2, \dot{y}_2, \dot{x}_3, \dot{y}_3]^\top \in \text{span}(\mathbf{1} \otimes I_2) \tag{38}$$

and the feasible rotational motion corresponds to the rotational part in the null space of $R$. This gives another viewpoint to confirm the above analysis.

## V. Motion generation for networked heterogeneous agents with feasible formations

We mention the fact that the theory developed in the previous sections is not limited to any specific type of agent dynamics, and it provides a unified analysis and framework to the coordination control with distinct agent dynamics. Actually, agent dynamics considered here include, but are not limited to, the following types:

- fully actuated agents;
- agents with drift-free, nonholonomic dynamics;
- agents with nonholonomic dynamics and drift terms.

Thus, the general framework developed in previous sections can be applied in the coordination control problem for a multi-agent group with heterogeneous dynamics. We then consider Problem 2, i.e., how to derive feasible motions for networked heterogeneous agents.

Suppose the formation feasibility condition is satisfied, i.e. there holds rank$(\Omega)$ = rank$([\Omega, T])$. One can first calculate a special solution to the feasibility equation (22), denoted by $\bar{K}$. Further we suppose the null space of $\Omega$ is spanned by $\kappa$ vectors, denoted by $K_1, K_2, \cdots, K_\kappa$. Then any feasible motion for the overall heterogeneous networked system can be generated by the following equivalent dynamical system

$$\dot{P} = \bar{K} + \sum_{l=1}^{\kappa} K_l w_l \tag{39}$$

where $w_l$ is an input that activates the associated vector field $K_l$.

Due to the space limit, we do not provide a full and detailed analysis on the application of the above motion generation theory in coordination control of networked heterogeneous agents under both kinematic constraints and formation constraints. These will be provided in the full version of this paper.

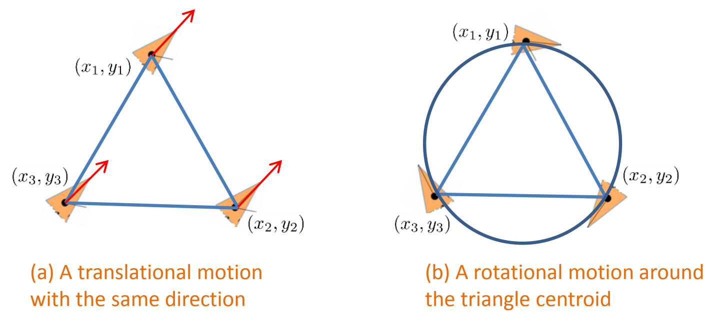


Fig. 1. Two feasible formations with a group of constant-speed agents.

## VI. Concluding Remarks

In this paper, we have discussed the formation feasibility problem in the coordination control of multi-agent systems whose dynamics are heterogeneous and may contain drift terms. Based on concepts from elementary differential geometry and tools of (affine) distribution and codistribution, we formulate the constraints from the formation aspect and kinematics aspect into a unified form, and propose an algebraic condition to determine the existence of feasible motions. Furthermore, in the case that the feasibility condition holds, we propose a systematic approach to obtain an equivalent dynamical system that can generate all feasible motions. Through several examples, we have shown how this general and unified framework can be applied to coordination control of constant-speed agent groups with nonholonomic constraints in achieving a feasible formation.

We remark some future directions based on this work. One extension is the development of formation feasibility theory with *inequality* constraints. This is motivated by the coordination control problem of multiple agents with almost constant speeds (as in [17]). Also, the motion generation algorithm reported in Section V assumes that all agents are initially at the feasible trajectory, which would be a restrictive condition in practice. It will be interesting to develop control approaches to stabilize all agents' motions to one of the feasible motions if they exist.

## Acknowledgement

The work of B. D. O. Anderson and Z. Sun was supported by NICTA, which is funded by the Australian Government through the ICT Centre of Excellence program, and by the Australian Research Council under grant DP130103610 and DP160104500. Z. Sun is also supported by the Prime Ministers Australia Asia Incoming Endeavour Postgraduate Award from Australian Government.

## References

[1] Y. Cao, W. Yu, W. Ren, and G. Chen, "An overview of recent progress in the study of distributed multi-agent coordination," *Industrial Informatics, IEEE Transactions on*, vol. 9, no. 1, pp. 427–438, 2013.

[2] S. Knorn, Z. Chen, and R. Middleton, "Overview: Collective control of multi-agent systems," *IEEE Transactions on Control of Network Systems*, p. In press, 2015.

[3] K.-K. Oh, M.-C. Park, and H.-S. Ahn, "A survey of multi-agent formation control," *Automatica*, vol. 53, pp. 424–440, 2015.

[4] M. Egerstedt and X. Hu, "Formation constrained multi-agent control," *Robotics and Automation, IEEE Transactions on*, vol. 17, no. 6, pp. 947–951, 2001.

[5] M. Mesbahi and M. Egerstedt, *Graph theoretic methods in multiagent networks*. Princeton University Press, 2010.

[6] P. Tabuada, G. J. Pappas, and P. Lima, "Motion feasibility of multi-agent formations," *Robotics, IEEE Transactions on*, vol. 21, no. 3, pp. 387–392, 2005.

[7] D. Maithripala, D. Maithripala, and S. Jayasuriya, "Unifying geometric approach to real-time formation control," in *Proc. of American Control Conference*, pp. 789–794, IEEE, 2008.

[8] A. Isidori, *Nonlinear control systems*. Springer Science & Business Media, 1995.

[9] R. Sepulchre, D. Paley, N. E. Leonard, *et al.*, "Stabilization of planar collective motion with limited communication," *Automatic Control, IEEE Transactions on*, vol. 53, no. 3, pp. 706–719, 2008.

[10] G. S. Seyboth, J. Wu, J. Qin, C. Yu, and F. Allgower, "Collective circular motion of unicycle type vehicles with nonidentical constant velocities," *Control of Network Systems, IEEE Transactions on*, vol. 1, no. 2, pp. 167–176, 2014.

[11] Z. Sun, G. S. Seyboth, and B. D. O. Anderson, "Collective control of multiple unicycle agents with non-identical constant speeds: Tracking control and performance limitation," in *Proc. of 2015 IEEE Multi-Conference on Systems and Control (MSC)*, pp. 1361–1366, IEEE, 2015.

[12] R. M. Murray, Z. Li, and S. S. Sastry, *A mathematical introduction to robotic manipulation*. CRC press, 1994.

[13] F. Bullo and A. D. Lewis, *Geometric control of mechanical systems: modeling, analysis, and design for simple mechanical control systems*, vol. 49. Springer Science & Business Media, 2004.

[14] R. Brockett, "The early days of geometric nonlinear control," *Automatica*, vol. 50, no. 9, pp. 2203–2224, 2014.

[15] D. V. Dimarogonas and K. J. Kyriakopoulos, "A connection between formation infeasibility and velocity alignment in kinematic multi-agent systems," *Automatica*, vol. 44, no. 10, pp. 2648–2654, 2008.

[16] Z. Sun, S. Mou, U. Helmke, and B. D. O. Anderson, "Convergence analysis for rigid formation control with unrealizable shapes: The 3 agent case," in *Proc. of the 4th Australian Control Conference (AUCC)*, pp. 1–6, IEEE, 2014.

[17] B. D. O. Anderson, B. Fidan, C. Yu, and D. Walle, "UAV formation control: theory and application," in *Recent advances in learning and control*, pp. 15–33, Springer, 2008.

[18] S. Guler, N. Koksal, and B. Fidan, "Adaptive control of a three-agent surveillance swarm with constant speed constraint," in *Proc. of the 9th Asian Control Conference (ASCC)*, pp. 1–6, IEEE, 2013.

[19] V. I. Elkin, *Reduction of nonlinear control systems: a differential geometric approach*, vol. 472. Springer Science & Business Media, 2012.

[20] L. Krick, M. E. Broucke, and B. A. Francis, "Stabilisation of infinitesimally rigid formations of multi-robot networks," *International Journal of Control*, vol. 82, no. 3, pp. 423–439, 2009.

[21] B. Hendrickson, "Conditions for unique graph realizations," *SIAM Journal on Computing*, vol. 21, no. 1, pp. 65–84, 1992.