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A new distributed Zeno-free event-triggered algorithm for multi-agent consensus

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Abstract— **This paper develops a novel distributed event-triggered algorithm for achieving multi-agent consensus while guaranteeing a fully Zeno-free triggering for *all* the agents. In this framework, the controller input and event detector are based on local measurements in terms of relative states with respect to neighboring agents, and each agent updates its control input only at its own triggering instants. A bounded and convergent function is included in the event detector function for each agent, which avoids the comparison of event error term to a zero threshold in the zero-crossing scenario. We also revisit several popular event-triggered consensus algorithms reported in the literature, clarify some issues relating to Zeno triggering, consensus convergence, communication/measurement requirements, local/global coordinate frames, etc., and highlight the advantages of the event-triggered consensus algorithm proposed in this paper.**

I. INTRODUCTION

Consensus is a fundamental concept in distributed control which serves as a basis for many distributed algorithms. In the last decade, multi-agent coordination and consensus control has been a topic of active research activity in the control community [1]. Such multi-agent system may consist of digital sensors or microprocessors with limited computation/actuation capability, and thus a distributed resource-aware control is desirable for coordination control of multi-agent systems. Recent years have witnessed increasing interests on the event-triggered control [2] to address such resource challenges for linear/nonlinear control systems [3], [4], [5], [6], for decentralized/networked control systems [7], [8], [9], [10], [11], and in particular for multi-agent coordination control systems (see e.g. [12], [13]). This is because by using an event-triggered mechanism to update the controller input, instead of using a continuous updating strategy, the overall system can save resources in sensors/processors equipped with each individual agent and thus can significantly reduce computation/actuation burden for all the agents. There has been an increasing amount of literature on event-triggered control for achieving multi-agent consensus, and we refer the readers to [14], [15], [16], [17], [18], [19] for some typical event-triggered consensus algorithms.

This paper aims to revisit some popular event-triggered consensus algorithms reported in the literature, to clarify some issues relating to triggering performance and event requirements, and to propose a novel event-triggered scheme for achieving multi-agent consensus. The main contributions of this paper therefore include a novel event-triggered consensus algorithm which guarantees Zeno-free triggering for *all* the agents with a much simpler triggering mechanism, and a revisit and comparison of several existing event-triggered consensus algorithms which also highlights the advantages of the proposed event-triggered scheme.

The rest of this paper is organized as follows. Section II introduces preliminary concepts on graph theory, some notations to be used in the paper and the problem formulation. Section III proposes a novel Zeno-free event-triggered consensus algorithm and presents a detailed analysis on its triggering performance. We also provide some comparisons with several reported event-triggered consensus algorithms in this section. Finally, Section IV concludes this paper.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. Preliminaries on graph theory and notations

In this paper we consider the event-triggered multi-agent consensus problem with an undirected and connected graph. The extension of the event-triggered scheme to a directed graph setting will be reported as future work. The following gives a brief introduction on undirected graph theory and some relevant notations.

Consider an undirected graph with m edges and n vertices, denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with vertex set $\mathcal{V} = \{1, 2, \cdots, n\}$ and edge set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$. The neighbor set $\mathcal{N}_i$ of node i is defined as $\mathcal{N}_i := \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$. The matrix relating the nodes to the edges is called the incidence matrix $H_\mathcal{G} = \{h_{ki}\} \in \mathbb{R}^{m \times n}$, whose entries are defined as (with arbitrary edge orientations for an *undirected* graph considered here): $h_{ki} = 1$ if the kth edge sinks at node i, or $h_{ki} = -1$ if the kth edge leaves node i, or $h_{ki} = 0$ otherwise.

The adjacency matrix $A_\mathcal{G}$ is a symmetric $n \times n$ matrix encoding the vertex adjacency relationships, with entries $A_{ij} = 1$ if $\{i, j\} \in \mathcal{E}$, and $A_{ij} = 0$ otherwise. Another important matrix representation of an undirected graph $\mathcal{G}$ is the Laplacian matrix $L_\mathcal{G}$, which is defined as $L_\mathcal{G} = H_\mathcal{G}^T H_\mathcal{G}$. For a connected and undirected graph, one has $\text{rank}(H_\mathcal{G}) =$

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$n-1$ and null$(H_{\mathcal{G}}) = \text{span}\{\mathbf{1}_n\}$. For notational convenience, we also define $H = H_{\mathcal{G}} \otimes I_d$ and $L = L_{\mathcal{G}} \otimes I_d$, where $\otimes$ denotes the Kronecker product. In this paper, we assume that the graph modelling the interactions between agents is undirected and connected.

Given a vertex element $i \in \mathcal{V}$ we associate to it a point p_i of Euclidean space $\mathbb{R}^d$, where d is the dimension of the ambient space in which each agent is located. For example, p_i could be the position of agent i in robotic coordination control with $d \in \{1, 2, 3\}$. The column vector $p = [p_1^T, p_2^T, \ldots, p_n^T]^T \in \mathbb{R}^{nd}$ thus describes the composite state vector for all the n agents, labelled by the set of vertices of $\mathcal{G}$. For any edge $k \in \mathcal{E}$ with head j and tail i which is consistent with the construction of the matrix $H_{\mathcal{G}}$, consider the associated relative position vector defined as $z_k = p_j - p_i$. Let $z = [z_1^T, z_2^T, \cdots, z_m^T]^T \in \mathbb{R}^{dm}$ denote the associated composite relative state vector. Note that there holds

$$z = Hp \tag{1}$$

B. Problem formulation

Suppose each agent i is modelled by a single integrator described by $\dot{p}_i(t) = u_i(t)$, where $u_i(t) \in \mathbb{R}^d$ is the control input to be designed. We aim to design event-triggered controllers to drive the states of all the agents to reach consensus (e.g., a state agreement, or a rendezvous point, depending on different control contexts). In the case of continuous updating, the well-known consensus protocol is written as

$$\dot{p}_i(t) = u_i(t) = \sum_{j \in \mathcal{N}_i} (p_j(t) - p_i(t)) = -\{Lp(t)\}_i \tag{2}$$

where $\{Lp\}_i$ denotes the ith vector block (i.e. the vector block taken from the $(di - d + 1)$th to the (di)th entries) of the vector Lp.

In this paper, we will consider an event-triggered control scheme for achieving the multi-agent consensus. Denote the event time for each agent i as $t_0^i, t_1^i, \cdots, t_h^i, \cdots$. Under the event-triggered control framework, the dynamical system for agent i can be described by the following form

$$\dot{p}_i(t) = u_i(t) = u_i(t_h^i), \forall t \in [t_h^i, t_{h+1}^i) \tag{3}$$

One of the main aims of this paper concerns a Zeno-free triggering scheme for the multi-agent systems. We first give a formal definition of Zeno triggering (or termed *Zeno execution* in the study of hybrid system) [20], [21].

***Definition** 1:* A triggering is *Zeno* for agent i if

$$\lim_{h \to \infty} t_h^i = \sum_{h=0}^{\infty} (t_{h+1}^i - t_h^i) = t_\infty^i \tag{4}$$

for some *finite* t_∞^i. Such a finite time *finite* t_∞^i, if it exists, is termed the *Zeno time*. The *Zeno point* for agent i is defined as $p_i^z := p_i(t_\infty^i)$.

This paper aims to propose a novel event-triggered consensus algorithm to achieve the multi-agent consensus, which should satisfy the following requirements:

- The controller and event detector are based on *local measurements* in terms of relative states with respect to neighboring agents.
- The control input for agent i is updated at *its own event time* instants $t_0^i, t_1^i, \cdots, t_h^i, \cdots$ based on local information.
- Zeno triggering should be excluded for all the agents (i.e. each agent i should guarantee a *Zeno-free* triggering).
- For each agent, the measurements of relative states can be done via its *local coordinate frame*, which is *independent* of a global coordinate frame.

III. A NOVEL ZENO-FREE EVENT-BASED CONSENSUS SCHEME

In this section, we first discuss a multi-agent consensus system with a basic event-triggered scheme and point out its Zeno triggering issue. We then propose a modified Zeno-free triggering algorithm and present a detailed triggering behavior analysis.

A. A basic event-based consensus controller and its Zeno triggering issue

We propose and analyze the following event-based updating law for agent i

$$\begin{aligned}\dot{p}_i(t) = u_i(t) &= \sum_{j \in \mathcal{N}_i} (p_j(t_h^i) - p_i(t_h^i)) \\ &= -\{Lp(t_h^i)\}_i, \forall t \in [t_h^i, t_{h+1}^i)\end{aligned} \tag{5}$$

We note that the above event-based controller (or similar forms) has also been discussed in [22] and [15] with different event functions for triggering an event.

By defining $\delta_i(t) = \{Lp(t_h^i)\}_i - \{Lp(t)\}_i$, we can rewrite the above equation (5) as $\dot{p}_i(t) = -\{Lp(t)\}_i - \delta_i(t)$. Let $\delta = [\delta_1^T, \delta_2^T, \cdots, \delta_n^T]^T \in \mathbb{R}^{dn}$. Then a compact form for the multi-agent consensus dynamics can be written as

$$\dot{p}(t) = -Lp(t) - \delta(t) \tag{6}$$

Choose the Lyapunov function candidate as $V = \frac{1}{2}p^T Lp$. Its derivative along (6) is calculated as

$$\begin{aligned}\dot{V}(t) &= p^T(t)L\dot{p}(t) \\ &= -p^T(t)LLp(t) - p^T(t)L\delta(t) \\ &\leq -p^T(t)LLp(t) + \|p(t)^T L\delta(t)\| \\ &\leq -\sum_{i=1}^{n} \|\{Lp(t)\}_i\|^2 + \sum_{i=1}^{n} \|\{Lp(t)\}_i\|\|\delta_i(t)\|\end{aligned} \tag{7}$$

By using the inequality $\|\{Lp(t)\}_i\|\|\delta_i(t)\| \leq \frac{1}{2a_i}\|\delta(t)_i\|^2 + \frac{a_i}{2}\|\{Lp(t)\}_i\|^2$ with $a_i \in (0, 2)$, the above inequality (7) on $\dot{V}$ can be further derived as

$$\begin{aligned}\dot{V}(t) \leq &-\sum_{i=1}^{n} \|\{Lp(t)\}_i\|^2 \\ &+ \sum_{i=1}^{n} \frac{a_i}{2}\|\{Lp(t)\}_i\|^2 + \sum_{i=1}^{n} \frac{1}{2a_i}\|\delta_i(t)\|^2\end{aligned}$$

$$= -\sum_{i=1}^{n} \frac{2-a_i}{2} \|\{Lp(t)\}_i\|^2 + \sum_{i=1}^{n} \frac{1}{2a_i} \|\delta_i(t)\|^2$$

If we enforce the norm of $\delta_i(t)$ to satisfy

$$\frac{1}{2a_i} \|\delta_i(t)\|^2 \leq \gamma_i \frac{2-a_i}{2} \|\{Lp(t)\}_i\|^2 \tag{8}$$

with $\gamma_i \in (0, 1)$, we can guarantee

$$\dot{V}(t) \leq \sum_{i=1}^{n} (\gamma_i - 1) \frac{2-a_i}{2} \|\{Lp(t)\}_i\|^2 \leq 0 \tag{9}$$

This implies that one can design a local triggering function for agent i as

$$f_i(t) := \|\delta_i(t)\|^2 - \gamma_i a_i (2-a_i) \|\{Lp(t)\}_i\|^2 \tag{10}$$

and the event time t_h^i for agent i is defined to satisfy $f_i(t_h^i) = 0$ for $h = 0, 1, 2, \cdots$. For the time interval $t \in [t_h^i, t_{h+1}^i)$, the control input keeps as $u_i(t) = u_i(t_h^i)$ until the next event is triggered. Furthermore, every time when an event is triggered for agent i, the local event vector δ_i (or $\|\delta_i(t)\|$) then resets to zero.

The following lemma shows the convergence to the consensus and its triggering behavior with the triggering function (10).

***Lemma** 1:* By using the event-triggered consensus controller (5) and the triggering function (10), all agents' states reach a consensus point exponentially fast. Also, at any time instant $t > 0$, there exists at least one agent in the group that does not exhibit Zeno triggering behavior.

Proof: The exponential convergence to the consensus point has been shown by the Lyapunov argument as above. We now prove the second statement, i.e. the exclusion of Zeno triggering for at least one agent. Note that there holds $\|\delta_i(t)\|^2 \leq \|\delta(t)\|^2$ for any i. Also note that at any time t there exists an agent i^* (depending on t) such that $\|\{Lp(t)\}_{i^*}\|^2 \geq \frac{1}{n} \|Lp(t)\|^2$. Then one has

$$\frac{\|\delta_{i^*}(t)\|}{\|\{Lp(t)\}_{i^*}\|} \leq \sqrt{n} \frac{\|\delta(t)\|}{\|Lp(t)\|} \tag{11}$$

By following similar arguments and approaches as in [3, Theorem III.1] or [14, Theorem 4], one can look at the dynamics of $\|\delta(t)\|/\|Lp(t)\|$ and derive $\frac{\mathrm{d}}{\mathrm{d}t} \frac{\|\delta(t)\|}{\|Lp(t)\|}$, from which one can obtain a positive lower bound on the inter-event triggering intervals for the overall multi-agent system. This guarantees that at any time $t > 0$, there exists at least one agent (agent i^* included) for which the next inter-event interval is strictly positive. ■

***Remark** 1:* We remark that the analysis for the convergence using a Lyapunov argument as shown above can also be done via the input-state-stability (ISS) theory (see [23, Chapter 4]). Since the consensus equilibrium is exponentially stable, the above proof and the proof via ISS theory are equivalent and the latter does not necessarily lead to less conservative results (see [23, Lemma 4.6]).

***Remark** 2:* The above event-based controller design (5) follows a similar approach to the one discussed in [22] (with a different event-triggered scheme) and [15] (with a scaled triggering function). However, no rigorous analysis is available in [22], [15] to guarantee the exclusion of the Zeno behavior of the above controller. The authors in [15] claimed that if the term $\{Lp(t_k^i)\}_i \neq 0$, then there does not exist Zeno triggering for agent i for all time $t > t_k^i$. However, one should notice that the term $\{Lp(t)\}_i$ in the comparison function (10) may cross zero before the whole agent group reaches the consensus point (i.e., the point with $\{Lp(t)\}_i = 0$ for some i is not necessarily an equilibrium point). That is, there may exist a finite time $t^* < \infty$ such that $\{Lp(t^*)\}_i = 0$ for some i, and this may lead to Zeno triggering for that agent i. Overall, the analysis on the exclusion of Zeno behavior in [15] is questionable.

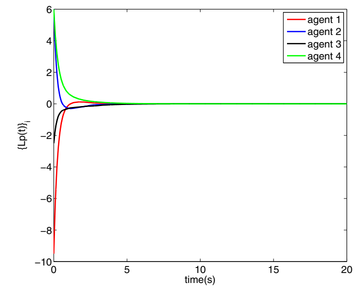

Fig. 1. Trajectories of $\{Lp(t)\}_i$ under the event-triggered function (10).

To further illustrate the Zeno triggering issue for (10) at the zero-crossing points, we show a typical simulation example. Consider a four-agent group, with the event-based controller (5) and the triggering function (10). In the simulation we choose an undirected graph whose topology is described by the adjacency matrix with non-zero entries of $A_{12} = 1$, $A_{14} = 1$, $A_{23} = 1$. We assume each agent's state is one-dimensional for ease of demonstration, and set the initial conditions for the four agents as $p_1(0) = 3$, $p_2(0) = -0.5$, $p_3(0) = 2$, $p_4(0) = -3$. The event parameters are chosen as $\gamma_1 = 0.5$, $\gamma_2 = 0.3$, $\gamma_3 = 0.7$, $\gamma_4 = 0.2$; $a_1 = 0.3$, $a_2 = 0.5$, $a_3 = 0.6$, $a_4 = 0.5$. Fig. 1 shows the trajectories of $\{Lp(t)\}_i$ of the four agents. Note that $\{Lp(t)\}_1$ (in red) and $\{Lp(t)\}_2$ (in blue) cross zeros respectively at some time instants before $t < 3s$. In Fig. 2, the corresponding triggered events of agents 1 and 2 occur densely around the zero-crossing points. The minimum inter-event time interval observed in simulations is 5×10^{-6} with the default numerical accuracy setting in Matlab. Theoretically speaking, infinitely many events are triggered in a finite time and Zeno triggering actually occurs around the zero-crossing points.

In the next subsection we aim to propose a modified triggering function to guarantee a fully Zeno-free event-triggered control for all the agents.

B. A modified, Zeno-free event-triggered consensus scheme

We propose a modified event function according to (10):

$$f_i(t) := \|\delta_i(t)\|^2 - \gamma_i a_i (2-a_i) \|\{Lp(t)\}_i\|^2 - 2a_i \varpi_i(t) \tag{12}$$

where $\varpi_i(t)$ is a function that can be chosen by agent i, which satisfies $\varpi_i(t) \in \mathcal{L}_\infty$, $\varpi_i(t) > 0$ for all $t > 0$ and

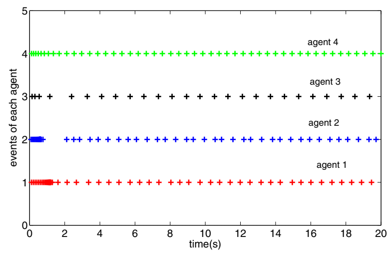

Fig. 2. Event time instants under the event-triggered function (10).

$\varpi_i(\infty) = 0$. Different agents can choose different $\varpi_i(t)$ in their triggering functions. The event time instant t_h^i for agent i is defined to satisfy $f_i(t_h^i) = 0$ for $h = 0, 1, 2, \cdots$. For the time interval $t \in [t_h^i, t_{h+1}^i)$, the control input remains constant as $u_i(t) = u_i(t_h^i)$ until the next event is triggered. Furthermore, every time when an event is triggered for agent i, the local event error variable $\|\delta_i\|$ automatically resets to zero.

The intuition to propose the above modified event comparison function (12) is to deal with the possibility of a zero-crossing occurrence of the term $\{Lp(t)\}_i$. Thus, even if $\{Lp(t)\}_i$ exhibits zero-crossing scenario at some finite time instant, the inclusion of the bounded, convergent term $\varpi_i(t)$ in the event function (12) guarantees a positive threshold value which avoids the case of comparing $\|\delta_i(t)\|^2$ with a zero threshold.

Before giving the triggering feasibility analysis, we first show that by using the above modified event function (12), the convergence to a consensus point is still guaranteed.

Theorem 1: Suppose the function $\varpi_i(t)$ is bounded and convergent to zero for all i. With the modified event function (12) and the event-triggered consensus controller (5), all agents' states converge to a consensus point.

Proof: Consider the same Lyapunov function defined as $V = \frac{1}{2}p^T Lp$. Note that $V = \frac{1}{2}p^T Lp = \frac{1}{2}p^T H^T Hp = \frac{1}{2}z^T z$ and $V = 0$ if and only if $z = 0$. The triggering condition from the modified event function (12) yields

$$\dot{V}(t) \leq \sum_{i=1}^{n} \left((\gamma_i - 1)\frac{2 - a_i}{2} \|\{Lp(t)\}_i\|^2 + \varpi_i(t) \right) \quad (13)$$

Note that there holds

$$\sum_{i=1}^{n} \|\{Lp(t)\}_i\|^2 = p^T LLp$$
$$= p^T H^T H H^T Hp = z^T H H^T z \quad (14)$$

The definition $z = Hp$ indicates that z is in the range space of H, which implies

$$z^T H H^T z \geq \lambda_{\text{min}}^{+}(HH^T)\|z\|^2 \quad (15)$$

where $\lambda_{\text{min}}^{+}(HH^T)$ is the smallest positive eigenvalue of HH^T, which is equivalent to the algebraic connectivity $\lambda_2(L_{\mathcal{G}})$. By defining $\zeta_{\text{min}} = \min_i(1-\gamma_i)\frac{2-a_i}{2}$, one can derive the following from (13) and (15):

$$\dot{V}(t) \leq -2\zeta_{\text{min}}\lambda_2(L_{\mathcal{G}})V(t) + \sum_{i=1}^{n} \varpi_i(t) \quad (16)$$

Since by construction, each $\varpi_i(t)$ is bounded (i.e. $\varpi_i(t) \in \mathcal{L}_\infty$) and $\varpi_i(\infty) = 0$, there holds

$$\Phi(t) := \sum_{i=1}^{n} \varpi_i(t) \in \mathcal{L}_\infty, \text{ and } \Phi(\infty) = 0.$$

Now we define a new dynamical system, in the form of

$$\dot{W}(t) = -2\zeta_{\text{min}}\lambda_2(L_{\mathcal{G}})W(t) + \Phi(t). \quad (17)$$

By the comparison method [23, Chapter 9.3], if $V(0) = W(0)$, there holds $V(t) \leq W(t)$. The solution to (17) can be written as

$$W(t) = e^{-2\zeta_{\text{min}}\lambda_2(L_{\mathcal{G}})t}W(0) + \int_0^t e^{-2\zeta_{\text{min}}\lambda_2(L_{\mathcal{G}})(t-\tau)}\Phi(\tau)\mathrm{d}\tau.$$

According to the property of the function $\Phi(t)$, there holds $W(\infty) = 0$, which implies $V(\infty) = 0$. In conclusion, one can obtain $z(\infty) = 0$ and thus $x_1(\infty) = x_2(\infty) = \cdots = x_n(\infty)$, i.e. the consensus is achieved when $t \to \infty$. ■

C. Triggering behavior analysis

This subsection aims to analyze some properties of the distributed event-triggered control strategy proposed above and its triggering behavior. We first consider the triggering feasibility. Generally speaking, singular triggering means no more triggering exists after a feasible triggering event, and Zeno triggering means that infinite triggering occurs at some finite time (see Definition 1). The following lemma shows the triggering feasibility with the local and distributed event-based controller (5) and event function (12) by excluding these two triggering cases.

Theorem 2: (Triggering feasibility) Consider the multi-agent consensus dynamics with the distributed event-based controller (5) and the event function (12).

- (No singular triggering) No agent will exhibit singular triggering for all $t > 0$.
- (No Zeno triggering) No agent will exhibit Zeno behavior for all $t > 0$.

The proof is omitted due to the space limit and will be provided elsewhere.

Besides the exclusion of Zeno triggering for all the agents, the modified event function (12) also enables an enlarged inter-event intervals as compared to the original one (10).

Lemma 2: Consider two identical multi-agent groups with the same dynamics and with the same event-based controller (5), but with the two groups triggered by different event functions (12) and (10), respectively. Suppose at t_k^i they have the same $\{Lp(t_k^i)\}_i$. Then the next event time given by (12) is larger than that given by (10).

The proof is omitted due to the space limit, and will be provided elsewhere. Lemma 2 also indicates that, with the additional function $\varpi_i(t)$ in the triggering function (12), the system will experience a longer inter-event time (at the price of a slower convergence to consensus) compared to that of using (10).

We finally show a lemma on the coordinate frame requirement to implement the proposed event-triggered consensus algorithm. This is a particularly important issue if the consensus controllers are applied to a group of mobile robots for achieving rendezvous, where the measurements of relative positions play a key role to implement the distributed control.

***Lemma** 3:* To implement the distributed consensus controller (5), each agent can use its own local coordinate frame to measure the relative positions to its neighbors and a global coordinate frame is not required. Furthermore, to detect the distributed triggering condition (12), a local coordinate frame is sufficient which is not required to be aligned with a global coordinate frame.

The proof of the above statements follows similar arguments to the *SE(N)* invariance principle for networked control system discussed in [24]. The proof is omitted here due to the space limit.

***Remark** 3:* (**Influences of the function** $\varpi_i(t)$) The inclusion of a bounded and convergent function $\varpi_i(t)$ in the event function (12) gives rise to a trade-off between the inter-event triggering interval and the convergence rate of the consensus process. Generally speaking, a larger value of $\varpi_i(t)$ at some interval will lead to a longer inter-event interval, but will result in a slower convergence rate to consensus. Nevertheless, the novel consensus algorithm in this section provides a general framework with great flexibility and one can choose $\varpi_i(t)$ according to specific requirements on inter-event interval and consensus convergence rate in practice. We also note that some papers have discussed the inclusion of a constant positive offset in the event function (which is termed a *mixed triggering* as in [5]). Such strategy leads to a positive lower bound on the inter-event intervals but can only guarantee practical stability or practical consensus in the context of multi-agent control.

***Remark** 4:* The triggering function designed in (12) has similarity to the one termed a *dynamic triggering* mechanism in [18]. However, the event function and comparison signals in [18] are generated by an auxiliary dynamical system with several complex terms. Here we have proposed a simpler approach to design the event function by including an additional function which is only required to be bounded and convergent to zero. Also note that different to the strategy in [18], the function $\varpi_i(t)$ in the current event-triggered framework does not necessarily need to be a $\mathcal{K}$ or $\mathcal{K}_\infty$ function of the states x_i.

D. Simulations and performance comparisons

In this subsection we show several simulations to demonstrate the performance of the consensus dynamics with the modified event function, particularly its Zeno-free triggering property. We consider a four-agent group, modelled by the single integrator with the event-based controller (5). The graph topology, initial conditions and event function parameters γ_i, and a_i are chosen as the same as those of the simulation settings in Section III.A. The additional functions $\varpi_i(t)$ in each agent's event function are set as $\varpi_1 = 2e^{-0.6t}$, $\varpi_2 = 2e^{-0.5t}$, $\varpi_3 = 3e^{-0.8t}$, $\varpi_4 = 2e^{-0.7t}$ in simulations [1]. Note that all these functions $\varpi_i(t)$ satisfy the requirements in Theorem 1.

Fig. 3 shows the trajectories of $\{Lp(t)\}_i$ of the four agents. Note that $\{Lp(t)\}_i$ for each agent crosses zeros several times, but Zeno triggering does not occur for any agent, thanks to the term $\varpi_i(t)$. This is shown in Fig. 4, where there are no dense events or Zeno triggering behaviors for any agent. By numerical calculation, the minimum inter-event time interval is 0.1003, which is almost 20000 times larger than that with the original event function (10) shown in Fig. 2. Fig. 5 depicts the state evolutions of the four agents which reach the consensus as desired.

Table I illustrates the event-triggering performance of the consensus dynamics with both event functions (10) and (12). Note: with the event function (10), agent 1 and agent 2 actually contain accumulation triggering points around the zero-crossing time and the number of triggered events approaches infinity from a theoretical viewpoint. For comparison and demonstration reasons in the table, the count of triggered events is performed by choosing a threshold minimum time (which is 5×10^{-6} in this case) that can be observed in the simulation with the specific numerical accuracy setting in Matlab.

TABLE I
COMPARISIONS OF EVENT TRIGGERING PERFORMANCE BETWEEN THE EVENT FUNCTION (10) AND (12)

event counts	event function (10)	event function (12)
agent 1	82+	13
agent 2	83+	22
agent 3	26	24
agent 4	48	11
total	239+	70
minimum inter-event time	5×10^{-6}	0.1003
maximum inter-event time	1.6727	8.2542

E. Comparisons with other approaches

This paper builds on the results in [22], [15], while we have improved the triggering behavior in two major aspects: the exclusion of Zeno triggering for all the agents, and less frequent control updates via a general and flexible event-triggered control framework. This is achieved from a modified event function by including a bounded and convergent function in the event detector function (12). To summarize, we highlight the advantages of the event-triggered consensus algorithm of this paper compared with previous works:

- The consensus controller and event function are totally distributed with only local information required, and in measuring relative states (i.e. relative positions) only local coordinate frames are involved;
- Each agent updates its control input at its own triggering instants, and the inter-event intervals can be enlarged

[1]The magnitudes and exponential rates in these functions are chosen just for the purpose of a clear simulation demonstration. Other bounded and convergent functions, such as $\frac{1}{t^2+1}$, are also good candidates.

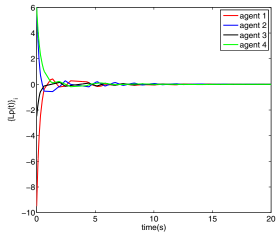

Fig. 3. Trajectories of $\{Lp(t)\}_i$ under the event-triggered function (12).

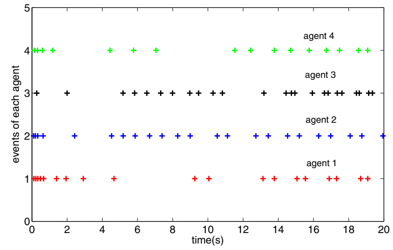

Fig. 4. Event time instants under the event-triggered function (12).

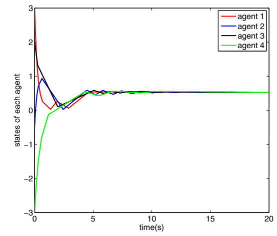

Fig. 5. States trajectories of the four agents under the event-triggered function (12).

or adjusted by choosing different $\varpi_i(t)$. This results in less frequent triggering compared to the event-triggered schemes proposed in e.g., [15], [14], [16], [17];

- In contrast to the algorithms in [16], [14], in the current event-triggered scheme each individual agent is not required to have knowledge of some global information (such as graph algebraic connectivity) or common parameters;
- It guarantees a truly Zeno-free triggering behavior for all the agents, with much simpler (and more flexible) event functions and triggering algorithms compared to the complex Zeno-free consensus mechanisms proposed in e.g. [25], [17].

We note that the converged consensus value with the event-triggered algorithm in this paper is not necessarily the average value, which would be a disadvantage if the average of initial states should be preserved for the multi-agent group.

IV. Conclusions

In this paper we propose a novel event-triggered algorithm for achieving consensus of a multi-agent group. We first analyze a basic event-triggered scheme, which is similar to the one in [15], and point out its defects relating to Zeno-triggering issues. We then propose a modified event-based consensus controller, which guarantees the exclusion of Zeno behavior for all the agents. It also possesses other advantageous properties, such as the independence of global coordinate frame and enlarged inter-event triggering intervals (thus enabling fewer triggering events).

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