A Variable Gain Model-Independent Algorithm for Rendezvous of Euler-Lagrange Agents on Directed Networks

Mengbin Ye[1], Brian D.O. Anderson[1,2,3] and Changbin Yu[1]

Abstract— **A variable-gain model-independent control law is proposed to solve the problem of rendezvous to a leader for a directed network of Euler-Lagrange agents. A sufficiency condition for stability is developed, requiring centralised design of the two control gains. Compared to existing results which use constant-gain model-independent controllers for directed networks, our work has two key differences. Firstly, the damping term may begin at zero, and increases only if rendezvous has not been achieved. Constant-gain controllers use conservative gain values which negatively impact convergence speed. Secondly, we introduce a novel method of analysing the Lyapunov derivative, which provides useful and unique insight into analysis of variable-gain controllers for multi-agent systems. Simulations are provided to show the effectiveness of the controller.**

I. Introduction

A key area in the research field of multiagent systems is to coordinate cooperating agents to achieve a common global objective, whereby the term agent may be applied to any type of controllable system, e.g. Unmanned Aerial Vehicle, marine vessel or mobile sensor), and a survey of recent multiagent coordination problems is available in [1]. Several key issues are present in any problem studying coordination of multiagent systems. The first is that the control laws are desired to be distributed (as opposed to being implemented by a centralised controller) due to scalability [2].

Secondly, each agent interacts only with its neighbours, i.e. other agents in some way close by, as opposed to every agent in the multiagent system. Such interactions are commonly captured using graphs and thus the phrase "networked systems" may sometimes be used. The specifics of the "interactions" are typically specified for each individual problem and may include e.g. sensing or information transmission. Directed graphs representing unilateral interactions are seen as more desirable than undirected graphs (bilateral interaction) from the following two points of view. Firstly, a directed network allows for agents with heterogeneous sensing and/or communicating capabilities (e.g. different sensing radius). Secondly, a directed graph allows each agent to reduce its sensing/communicating burden by reducing the number of neighbouring agents.

Lastly, the agent dynamics must be considered. There has been extensive study of networks where agents have single and double-integrator dynamics [2], [3], as well as linear and general nonlinear dynamics [4]. There is motivation to study agents with dynamics described by the nonlinear Euler-Lagrange equations as the equations can model a large class of mechanical, electrical and electromechanical systems [5]. Exact knowledge of the Euler-Lagrange equations is used for feedback linearisation in e.g. [6], [7]. A popular technique is to utilise the linear parametrisation of Euler-Lagrange equations [8] to construct adaptive algorithms which handle uncertain agent parameters. In this framework, containment control is studied in e.g. [9], [10]. Tracking of a moving leader is studied in e.g. [11], [12].

The work in [13] studied leaderless consensus and pioneered the idea of *model-independent* algorithms whereby the controller can be executed without knowledge of the agent model. The majority of existing work studies undirected Euler-Lagrange networks. Consensus to the intersection of individual agent targets is studied in [14]. Leaderless consensus in the presence of time-delays [15] and flocking [16] are studied on undirected graphs. Tracking of a leader with nonconstant velocity is studied in [17] [18] while containment control is considered in [19]. Rendezvous to a stationary leader is studied in [20].

Finding model-independent algorithms on directed graphs remains an open problem due to the difficulty in obtaining a stability result when combining the nonlinear Euler-Lagrange equations and the nonsymmetric matrices typically used to describe the graph topology. Synchronisation of the velocities (but not of the positions) is studied on strongly connected graphs using passivity analysis in [21]. Rendezvous to a stationary leader is studied on a directed spanning tree in [22]. Model-independent control laws are desirable for several reasons. By definition, they may be applied to different agents with only minor modifications. While the previously mentioned adaptive algorithms allow for uncertain agent parameters (e.g. the mass of a robotic arm), the linear parametrisation requires precise knowledge of the equation structures, including the functions appearing in them. In contrast, model-independent algorithms are reminiscent of robust controllers. Limited knowledge of upper bounds on parameters of the multiagent system guarantees stability [15], [17], [22]. This requires control gains to be designed centrally at the start; the controllers are model-independent in execution. In addition, model-independent controllers may be semi-globally stable; knowledge of the initial conditions is required to obtain a control gain which guarantees stability [17], [22].

The key contribution of this paper is to improve our work in [22] by incorporating a variable-gain controller (different to the adaptive algorithms mentioned previously). We are motivated to study a variable-gain controller because the gain associated with the damping term was designed in [23] according to a necessarily conserva-

[1]M. Ye, B. D.O. Anderson and C. Yu are with the Research School of Engineering, Australian National University. [2]B. D.O. Anderson and C. Yu are also with Hangzhou Dianzi University, Hangzhou, China. [3]B. D.O. Anderson is also with Data61-CSIRO (formerly NICTA Ltd.) in Canberra, A.C.T., Australia. This work was supported by the Australian Research Council (ARC) under grants DP-130103610 and DP-160104500, by the National Natural Science Foundation of China (grant 61375072), and by Data61-CSIRO.

{Mengbin.Ye, Brian.Anderson, Brad.Yu}@anu.edu.au

tive lower bound. This negatively impacted the convergence speed. In this paper, the gain associated with the damping term in the controller may begin at zero and increases monotonically, stopping when rendezvous has been achieved. There is however a trade-off. In [22], we show that the algorithm results in an exponentially stable system with a minimum computable rate of convergence. The exponential stability property offers better rejection of noise and disturbances when compared to systems which are asymptotically stable, but not exponentially so. By incorporating a variable gain, and similar to the adaptive algorithms above, we only reach the conclusion (at this stage of our work) of asymptotic stability, although simulations suggest exponential stability behaviour is present. Adaptive controllers will yield exponential stability under certain conditions, e.g. persistency of excitation. However, most existing adaptive controllers in Euler-Lagrange networks have not been verified as being exponentially stable. As with other model-independent algorithms, centralised design of parameters in the controller is required, involving bounds on matrices describing the agent dynamics, bounds on the graph topology and the initial conditions (i.e. the algorithm is semi-globally stable). The analysis method for the Lyapunov-like candidate function is also novel when compared to existing variable gain controllers in multi-agent systems, providing added insight.

The rest of the paper is presented in the following structure. Section II provides mathematical background and formally defines the rendezvous problem. The controller is proposed in Section III and its stability analysed. Simulations are provided in Section IV and Section V concludes the paper.

II. Background and Problem Statement

A. Mathematical Notation and Matrix Theory

To begin we provide definitions of notation, a lemma and a theorem for later use. We use $\otimes$ to denote the Kronecker product, the properties of which are in [24]. The $p \times p$ identity matrix is $\boldsymbol{I}_p$. The Euclidean vector norm is denoted by $\|\cdot\|_2$. For a square matrix $\boldsymbol{A}$, the Euclidean norm induces the spectral matrix norm, which is denoted as $\|\boldsymbol{A}\|_2$. See [24] for details on the spectral norm, which is used frequently in this paper. We use $\lambda_{\min}(\boldsymbol{A})$ and $\lambda_{\max}(\boldsymbol{A})$ to denote respectively the smallest and largest eigenvalues of a square matrix $\boldsymbol{A}$. A symmetric matrix $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ which is positive definite (respectively nonnegative definite) is denoted by $\boldsymbol{A} > 0$ (respectively $\boldsymbol{A} \geq 0$). For two symmetric matrices $\boldsymbol{A}, \boldsymbol{B}$, the expression $\boldsymbol{A} > \boldsymbol{B}$ is equivalent to $\boldsymbol{A} - \boldsymbol{B} > 0$. The following eigenvalue inequalities will be frequently used.

$$\lambda_{\min}(\boldsymbol{A}) > \lambda_{\max}(\boldsymbol{B}) \Rightarrow \boldsymbol{A} > \boldsymbol{B} \tag{1}$$
$$\lambda_{\max}(\boldsymbol{A}+\boldsymbol{B}) \leq \lambda_{\max}(\boldsymbol{A}) + \lambda_{\max}(\boldsymbol{B}) \tag{2}$$
$$\lambda_{\min}(\boldsymbol{A}+\boldsymbol{B}) \geq \lambda_{\min}(\boldsymbol{A}) + \lambda_{\min}(\boldsymbol{B}) \tag{3}$$
$$\lambda_{\min}(\boldsymbol{A})\boldsymbol{x}^\top\boldsymbol{x} \leq \boldsymbol{x}^\top\boldsymbol{A}\boldsymbol{x} \leq \lambda_{\max}(\boldsymbol{A})\boldsymbol{x}^\top\boldsymbol{x} \tag{4}$$

Theorem 1 ([24]). *Consider a symmetric block matrix, partitioned as*

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{B} & \boldsymbol{C} \\ \boldsymbol{C}^\top & \boldsymbol{D} \end{bmatrix}$$

Then $\boldsymbol{A} > 0$ if and only if $\boldsymbol{D} > 0$ and $\boldsymbol{B} - \boldsymbol{C}\boldsymbol{D}^{-1}\boldsymbol{C}^\top > 0$.

Lemma 1. *Let $g(x, y)$ be a function given as*

$$g(x, y) = ax^2 + by^2 - cxy^2 - dx^2y^2 \tag{5}$$

for real positive scalars $a, c, d > 0$. Then for a given $\mathcal{X} > 0$, there holds $g(x, y)$ positive definite for all $x \in [0, \mathcal{X}]$ and $y \in [0, \infty)$ if

$$b > c\mathcal{X} + d\mathcal{X}^2 \tag{6}$$

Proof. By expressing $b = c\mathcal{X} + d\mathcal{X}^2 + \delta$, where δ is strictly positive, observe that

$$\begin{aligned} g(x, y) &= ax^2 + (b - cx - dx^2)y^2 \\ &\geq ax^2 + (cX + dX^2 + \delta - cx - dx^2) > 0 \end{aligned} \tag{7}$$

because $X \geq x$ and $c, d > 0$ Thus $g(x, y)$ is positive in the region $x \in [0, \mathcal{X}]$ and $y \in [0, \infty)$, and $g(x, y) = 0$ if and only if $x = y = 0$. □

B. Graph Theory

A weighted directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is used to model the information flow between agents (in the sequel we define the precise nature of this information). The finite, nonempty set of nodes is $\mathcal{V} = \{v_0, v_1, \ldots, v_n\}$ with node indices $\mathcal{I} = \{0, 1, 2, ..., n\}$, and with $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ being the corresponding set of ordered edges. We denote an ordered edge of $\mathcal{G}$ as $e_{ij} = (v_i, v_j)$ and the assumption $e_{ij} = e_{ji}$ does not hold in general because $\mathcal{G}$ is directed. An edge $e_{ij} = (v_i, v_j)$ is outgoing with respect to v_i and incoming with respect to v_j, i.e. the edge (v_i, v_j) indicates that information flows from v_i to v_j. The weighted adjacency matrix $\mathcal{A} \in \mathbb{R}^{(n+1)\times(n+1)}$ of $\mathcal{G}$ has nonnegative elements a_{ij}. The elements of $\mathcal{A}$ are such that $a_{ij} > 0 \Leftrightarrow e_{ji} \in \mathcal{E}$ while $a_{ij} = 0$ if $e_{ji} \notin \mathcal{E}$. We assume $a_{ii} = 0, \forall i$. The neighbour set of v_i is denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_i, v_j) \in \mathcal{E}\}$. The $(n+1) \times (n+1)$ Laplacian matrix, $\mathcal{L}$, of the associated digraph $\mathcal{G}$, has elements

$$l_{ij} = \begin{cases} \sum_{k=1,k\neq i}^{n} a_{ik} & \text{for } j = i \\ -a_{ij} & \text{for } j \neq i \end{cases}$$

A directed spanning tree is a directed graph formed by directed edges of the graph that connects all the nodes, and where every vertex apart from the root has exactly one parent. A graph is said to contain a spanning tree if a subset of the edges forms a spanning tree. We provide a lemma to be used in the sequel.

Lemma 2 ([22]). *Let the graph $\mathcal{G}$ contain a directed spanning tree, and without loss of generality number the root vertex as v_0. Suppose there are no edges incoming to v_0. Then $\mathcal{L}$ associated with $\mathcal{G}$ can be partitioned as*

$$\mathcal{L} = \begin{bmatrix} 0 & \mathbf{0}_{1\times n} \\ \mathcal{L}_{11} & \mathcal{L}_{22} \end{bmatrix} \tag{8}$$

and there exists a positive definite diagonal matrix $\boldsymbol{\Gamma} = diag[\gamma_1, ..., \gamma_n]$ such that $\boldsymbol{Q} \triangleq \boldsymbol{\Gamma}\mathcal{L}_{22} + \mathcal{L}_{22}^\top\boldsymbol{\Gamma} > 0$.

C. Euler-Lagrange Systems

The dynamics of each fully-actuated follower agent are described using the nonlinear Euler-Lagrange equations. The general form for the i^{th} agent equation of motion is:

$$\boldsymbol{M}_i(\boldsymbol{q}_i)\ddot{\boldsymbol{q}}_i + \boldsymbol{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\dot{\boldsymbol{q}}_i = \boldsymbol{\tau}_i \tag{9}$$

where $\boldsymbol{q}_i \in \mathbb{R}^p$ is a vector of the generalised coordinates, $\boldsymbol{M}_i(\boldsymbol{q}_i) \in \mathbb{R}^{p\times p}$ is the inertia matrix, $\boldsymbol{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i) \in \mathbb{R}^{p\times p}$ is the Coriolis and centrifugal force matrix and $\boldsymbol{\tau}_i \in \mathbb{R}^p$ is the control input vector. Note that (9) is within a subclass of Euler-Lagrange equations as it does not contain the vector of potential forces, commonly denoted as $\boldsymbol{g}_i(\boldsymbol{q}_i)$. We assume that the systems described using (9) have the following properties (typical for this class of problems) [25], [8]:

A1 The matrix $\boldsymbol{M}_i(\boldsymbol{q}_i)$ is symmetric positive definite.

A2 There exist constants $k_{\underline{m}}, k_{\overline{M}} > 0$ such that $k_{\underline{m}}\boldsymbol{I}_p \leq \boldsymbol{M}_i(\boldsymbol{q}_i) \leq k_{\overline{M}}\boldsymbol{I}_p \; \forall i, \boldsymbol{q}_i$. It follows that $\sup_{\boldsymbol{q}_i} \|\boldsymbol{M}_i\|_2 \leq k_{\overline{M}}$ and $k_{\underline{m}} \leq \inf_{\boldsymbol{q}_i} \|\boldsymbol{M}_i^{-1}\|_2{}^{-1}$ for all i.

A3 The matrix $\boldsymbol{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)$ is defined such that $\dot{\boldsymbol{M}}_i - 2\boldsymbol{C}_i$ is skew-symmetric. It follows that $\dot{\boldsymbol{M}}_i = \boldsymbol{C}_i + \boldsymbol{C}_i^\top$.

A4 There exists a scalar constant $k_C > 0$ such that $\|\boldsymbol{C}_i\|_2 \leq k_C\|\dot{\boldsymbol{q}}_i\|_2, \forall i$.

D. Problem Statement

Denote the leader as agent 0, corresponding to the node v_0 on $\mathcal{G}$, with $\boldsymbol{q}_0$ and $\dot{\boldsymbol{q}}_0$ being the leader's generalised coordinates and generalised velocity respectively. The aim is to develop a model-independent, distributed algorithm which allows a directed network of Euler-Lagrange agents to rendezvous to a leader, *which we assume is stationary*. The rendezvous objective is satisfied if

$$\begin{aligned} \lim\nolimits_{t\to\infty} \|\boldsymbol{q}_i(t) - \boldsymbol{q}_0\|_2 = 0, \; \forall\, i \\ \lim\nolimits_{t\to\infty} \dot{\boldsymbol{q}}_i(t) = \dot{\boldsymbol{q}}_0 = \boldsymbol{0}, \; \forall\, i \end{aligned} \tag{10}$$

The interaction between the agents is captured by the fixed, weighted, directed graph $\mathcal{G}$ with the associated Laplacian $\mathcal{L}$. Specifically, if $a_{ij} > 0$, then agent v_i knows a_{ij} and knows the quantity $\boldsymbol{q}_i - \boldsymbol{q}_j$, i.e. the relative generalised coordinates, and $\dot{\boldsymbol{q}}_i - \dot{\boldsymbol{q}}_j$, i.e. the relative generalised velocities. The method by which these quantities are obtained depends on the specific multiagent scenario. One can imagine robotic manipulators transmitting measured coordinates or vehicles measuring relative positions. We also assume that agent i can measure its own velocity $\dot{\boldsymbol{q}}_i$.

By model-independent, we mean that the algorithm does not contain $\boldsymbol{M}_i, \boldsymbol{C}_i \,\forall\, i$ and does not contain the linear parametrisation. An agent's algorithm is distributed if, during execution, the agent only needs to receive information about its neighbours.

Notice that it is possible for $\boldsymbol{M}_i \neq \boldsymbol{M}_j$ and $\boldsymbol{C}_i \neq \boldsymbol{C}_j$ for any $i, j \in \mathcal{I}$ but $\boldsymbol{q}_i \in \mathbb{R}^p, \forall i$. In other words this work considers Euler-Lagrange agents which have heterogeneous parameters but with generalised coordinates which are defined such that $\boldsymbol{q}_i - \boldsymbol{q}_j, \forall i, j$ is meaningful.

III. Main Result

For the i^{th} follower agent, consider the following model-independent algorithm

$$\boldsymbol{\tau}_i = -\eta \sum_{j\in\mathcal{N}_i} a_{ij}(\boldsymbol{q}_i - \boldsymbol{q}_j) - \frac{1}{\eta}\sum_{j\in\mathcal{N}_i} a_{ij}(\dot{\boldsymbol{q}}_i - \dot{\boldsymbol{q}}_j) - \mu_i\dot{\boldsymbol{q}}_i \tag{11a}$$

$$\dot{\mu}_i = \alpha\dot{\boldsymbol{q}}_i^\top\dot{\boldsymbol{q}}_i \tag{11b}$$

where a_{ij} is an element of the adjacency matrix $\mathcal{A}$ associated with $\mathcal{G}$. The constant, strictly positive control gain scalars $\eta < 1$ and α will be designed in the sequel, and are universal to all agents. The variable-gain control scalar $\mu_i(t)$ is initialised at $t = 0$ with an arbitrary nonnegative value, but for simplicity of analysis we assume in this paper that $\mu_i(0) = 0, \forall\, i$. This implies that $\mu_i(t) \geq 0 \,\forall\, t > 0$. It is also easily verified that $\mu_i(t)$ is a monotonically increasing function.

Define new state variables $\boldsymbol{u}_i = \boldsymbol{q}_i - \boldsymbol{q}_0$ and $\boldsymbol{v}_i = \dot{\boldsymbol{q}}_i$. Define the stacked column vectors of all $\boldsymbol{u}_i, \boldsymbol{v}_i, \boldsymbol{q}_i$ as $\boldsymbol{u} = [\boldsymbol{u}_1^\top, ..., \boldsymbol{u}_n^\top]^\top$, $\boldsymbol{v} = [\boldsymbol{v}_1^\top, ..., \boldsymbol{v}_n^\top]^\top$ and $\boldsymbol{q} = [\boldsymbol{q}_1^\top, ..., \boldsymbol{q}_n^\top]^\top$ respectively. The rendezvous objective is achieved when there holds $\boldsymbol{u} = \boldsymbol{v} = \boldsymbol{0}$. Next, define the following block diagonal matrices $\boldsymbol{M}(\boldsymbol{q}) = diag[\boldsymbol{M}_1(\boldsymbol{q}_1), ..., \boldsymbol{M}_n(\boldsymbol{q}_n)]$, $\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = diag[\boldsymbol{C}_1(\boldsymbol{q}_1, \dot{\boldsymbol{q}}_1), ..., \boldsymbol{C}_n(\boldsymbol{q}_n, \dot{\boldsymbol{q}}_n)]$ and $\boldsymbol{K} = diag[\mu_1\boldsymbol{I}_p, ..., \mu_n\boldsymbol{I}_p]$. Since $\boldsymbol{M}_i > 0, \forall\, i$ then $\boldsymbol{M}$ is also symmetric positive definite. With this notation and applying control law (11) to each follower agent, the closed-loop networked system expressed in the new variables $\boldsymbol{u}, \boldsymbol{v}$, is

$$\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{v}} + \boldsymbol{C}(\boldsymbol{q}, \boldsymbol{v})\boldsymbol{v} + (\mathcal{L}_{22} \otimes \boldsymbol{I}_p)(\eta\boldsymbol{u} + \frac{1}{\eta}\boldsymbol{v}) + \boldsymbol{K}\boldsymbol{v} = \boldsymbol{0} \tag{12}$$

Here, $\mathcal{L}_{22}$ is defined as in Lemma 2 with the root node v_0 being the leader, agent 0. The closed-loop network can also be expressed as the non-autonomous system

$$\dot{\boldsymbol{u}} = \boldsymbol{v} \tag{13a}$$

$$\dot{\boldsymbol{v}} = -\boldsymbol{M}(\boldsymbol{q})^{-1}\Big[\boldsymbol{C}(\boldsymbol{q}, \boldsymbol{v})\boldsymbol{v} + (\mathcal{L}_{22} \otimes \boldsymbol{I}_p)(\eta\boldsymbol{u} + \frac{1}{\eta}\boldsymbol{v}) + \boldsymbol{K}\boldsymbol{v}\Big] \tag{13b}$$

$$\dot{\boldsymbol{K}} = (\boldsymbol{\Xi} \otimes \boldsymbol{I}_p) \tag{13c}$$

where $\boldsymbol{\Xi} = diag[\alpha\|\boldsymbol{v}_1\|_2{}^2, \alpha\|\boldsymbol{v}_2\|_2{}^2, ..., \alpha\|\boldsymbol{v}_n\|_2{}^2]$, and with the set of equilibrium points $\mathcal{M} = \{\boldsymbol{u}, \boldsymbol{v}, \mu_i : \boldsymbol{u} = \boldsymbol{0}, \boldsymbol{v} = \boldsymbol{0}, \mu_i \in \mathbb{R}_+ \forall\, i\}$. From (11b), observe that for some $T^* \geq 0$, there holds

$$\dot{\mu}_i(t) = 0, \forall\, i, \forall t \geq T^* \tag{14}$$

if and only if (13) has converged a single point in $\mathcal{M}$ by time T^*. (equivalently $\mu_i(t)$ is constant, $\forall\, i, \forall\, t \geq T^*$). The if direction is trivial. For the only if direction, suppose that (14) holds. This implies that $\boldsymbol{v}(t) = \boldsymbol{0}, \forall\, t \geq T^*$, which in turn implies that $\dot{\boldsymbol{v}}(t) = \boldsymbol{0}, \forall\, t \geq T^*$. Because $\boldsymbol{M}$ and $\mathcal{L}_{22}$ are nonsingular, (13b) will deliver the conclusion that $\dot{\boldsymbol{v}}(t) = \boldsymbol{v}(t) = \boldsymbol{0}, \forall\, t \geq T^*$ only if $\boldsymbol{u} = \boldsymbol{0}, \forall\, t \geq T^*$ holds. Note that convergence must be to a single point in $\mathcal{M}$ as opposed to a nonconstant trajectory because in $\mathcal{M}$ there holds $\boldsymbol{v} = 0$. One can also verify that any point in $\mathcal{M}$ implies that the rendezvous objective has been achieved.

It is obvious that solutions around $t = 0$ exist for the system (13). We will show in the main theorem below that the system does not have a finite escape time for times well away from $t = 0$. Although the system (13) is not a self-contained system (since the arguments $\boldsymbol{M}$ and $\boldsymbol{C}$ are dependent on $\boldsymbol{q}$) it will turn out that, using arguments like those of usual Lyapunov theory, we will be able to prove the stability of (13). Alternatively, we could proceed by formally expanding (13) to become an autonomous system with the supplementary differential equation

$$\dot{\boldsymbol{q}} = \boldsymbol{v} \tag{15}$$

For convenience, we will work with (13) but analysis of the formally expanded autonomous system delivers the same stability result. We are now ready to show the main result of the paper.

Theorem 2. *The equilibrium of system* (13) *is globally asymptotically stable if the directed graph $\mathcal{G}$ contains a directed spanning tree and η is sufficiently small.*

Proof. Before we introduce the Lyapunov-like candidate function, we introduce the constant $\bar{\mu}$, which is used only in the proof. Its precise value will be given in the sequel.

For arbitrary initial conditions $\boldsymbol{u}(0)$, $\boldsymbol{v}(0)$, consider the following Lyapunov-like candidate function

$$\begin{aligned} V &= \frac{1}{2}\boldsymbol{u}^\top \boldsymbol{R}\boldsymbol{u} + \eta\boldsymbol{u}^\top \boldsymbol{\Gamma}_p\boldsymbol{M}\boldsymbol{v} + \frac{1}{2}\eta^{-1}\boldsymbol{v}^\top\boldsymbol{\Gamma}_p\boldsymbol{M}\boldsymbol{v} \\ &+ \sum_{i=1}^n \frac{\eta}{2\alpha\gamma_i}s_i^2 = V_1 + V_2 + V_3 + V_4 \end{aligned} \tag{16}$$

where $\boldsymbol{R} = \eta\boldsymbol{\Gamma}_p\boldsymbol{K} + \boldsymbol{Q}\otimes\boldsymbol{I}_p$ and $\boldsymbol{\Gamma}_p = \boldsymbol{\Gamma}\otimes\boldsymbol{I}_p$, with $\boldsymbol{\Gamma}$, γ_i and $\boldsymbol{Q}$ defined in Lemma 2. Here, $s_i = \gamma_i\mu_i/\eta - \bar{\mu}$ and denote $\boldsymbol{s} = [s_1, ..., s_n]^\top$. Note that $\dot{s}_i = \frac{\gamma_i\alpha}{\eta}\boldsymbol{v}_i^\top\boldsymbol{v}_i$. Denote $\boldsymbol{w} = [\boldsymbol{u}^\top, \boldsymbol{v}^\top]^\top$, $\boldsymbol{x} = [\boldsymbol{w}^\top, \boldsymbol{s}^\top]^\top$ and $\boldsymbol{w}(0) = [\boldsymbol{u}(0)^\top, \boldsymbol{v}(0)^\top]^\top$. Note that $\boldsymbol{\Gamma}_p\boldsymbol{M}$ is symmetric positive definite. While V is a function with argument $\boldsymbol{x}(t)$, for convenience we use $V(t)$ to denote $V(\boldsymbol{x}(t))$.

Define $\bar{\gamma}$ and $\underline{\gamma}$ to be the largest and smallest element of the diagonal matrix $\boldsymbol{\Gamma}$ respectively. We may also express $V_1 + V_2 + V_3$ as a quadratic function, $\widetilde{V}$, in the variables $\boldsymbol{u}, \boldsymbol{v}$

$$\widetilde{V} = \begin{bmatrix}\boldsymbol{u}\\ \boldsymbol{v}\end{bmatrix}^\top \begin{bmatrix}\frac{1}{2}\boldsymbol{R} & \frac{1}{2}\eta\boldsymbol{\Gamma}_p\boldsymbol{M}\\ \frac{1}{2}\eta\boldsymbol{\Gamma}_p\boldsymbol{M} & \frac{1}{2}\eta^{-1}\boldsymbol{\Gamma}_p\boldsymbol{M}\end{bmatrix}\begin{bmatrix}\boldsymbol{u}\\ \boldsymbol{v}\end{bmatrix} \tag{17}$$

and define the matrix on the right of (17) as $\boldsymbol{H}(\eta)$. Both summands of $\boldsymbol{R}$ are symmetric positive definite, which implies that $\boldsymbol{R}$ is positive definite. We conclude using the Schur complement argument in Theorem 1 that $\boldsymbol{H}(\eta)$ is positive definite if $\frac{1}{2}\boldsymbol{R} - \frac{1}{2}\eta^3\boldsymbol{\Gamma}_p\boldsymbol{M} > 0$,

$$\lambda_{\min}(\boldsymbol{Q}) - \eta^3\lambda_{\max}(\boldsymbol{\Gamma}_p\boldsymbol{M}) > 0 \tag{18}$$

because expression (3) implies $\lambda_{\min}(\boldsymbol{R}) \geq \lambda_{\min}(\boldsymbol{Q})$. Observe that (18) is satisfied for any positive $\eta \leq \hat{\eta}$ such that

$$\hat{\eta} < \sqrt[3]{\frac{\lambda_{\min}(\boldsymbol{Q})}{2\bar{\gamma}k_{\overline{M}}}} \tag{19}$$

because $\lambda_{\max}(\boldsymbol{\Gamma}_p\boldsymbol{M}) \leq \bar{\gamma}k_{\overline{M}}$. Furthermore, there always exists an $\eta > 0$ which satisfies (19) because $\lambda_{\min}(\boldsymbol{Q}), \bar{\gamma}k_{\overline{M}} > 0$. It will turn out that there is another condition on η, which we provide later, but it does not conflict with (19). We thus conclude that V is positive definite in $\boldsymbol{u}, \boldsymbol{v}$ and $\boldsymbol{s}$, and is radially unbounded. It is also obvious that $\|\boldsymbol{x}(t)\|_2 \to 0$ implies $V(\boldsymbol{x}(t)) \to 0$. It can be shown that there exist two constants ϕ, ζ such that $\lambda_{\max}(\boldsymbol{H}(\eta)) < \phi$ and $\lambda_{\min}(\boldsymbol{H}(\eta)) > \zeta$ for all values of $\boldsymbol{u}, \boldsymbol{v}$ and for any $\eta < \eta^*$. The values of ϕ, ζ are computable from the quantities $\underline{\gamma}, \bar{\gamma}, \lambda_{\min}(\boldsymbol{Q}), \lambda_{\max}(\boldsymbol{Q}), k_{\overline{M}}$ and $k_{\underline{m}}$. We omit the calculations due to spatial limitations.

Taking the derivative of V with respect to time, along the trajectory of the system (13), we have $\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4$. Evaluating $\dot{V}_1$ yields

$$\begin{aligned}\dot{V}_1 &= \boldsymbol{u}^\top\boldsymbol{R}\boldsymbol{v} + \frac{1}{2}\eta\boldsymbol{u}^\top\boldsymbol{\Gamma}_p\dot{\boldsymbol{K}}\boldsymbol{u}\\ &= \eta\boldsymbol{u}^\top\boldsymbol{\Gamma}_p\boldsymbol{K}\boldsymbol{v} + \boldsymbol{u}^\top(\boldsymbol{Q}\otimes\boldsymbol{I}_p)\boldsymbol{v}\\ &\quad + \frac{1}{2}\eta\alpha\sum_{i=1}^n\gamma_i(\|\boldsymbol{u}_i\|_2\|\boldsymbol{v}_i\|_2)^2\end{aligned}$$

Next, the derivative $\dot{V}_2$ is evaluated to be

$$\dot{V}_2 = \eta\boldsymbol{v}^\top\boldsymbol{\Gamma}_p\boldsymbol{M}\boldsymbol{v} + \eta\boldsymbol{u}^\top\boldsymbol{\Gamma}_p\dot{\boldsymbol{M}}\boldsymbol{v} + \eta\boldsymbol{u}^\top\boldsymbol{\Gamma}_p\boldsymbol{M}\dot{\boldsymbol{v}}$$

By substituting $\dot{\boldsymbol{v}}$ from (13b) and recalling assumption A3, we obtain

$$\begin{aligned}\dot{V}_2 &= \eta\boldsymbol{u}^\top\boldsymbol{\Gamma}_p\boldsymbol{C}^\top\boldsymbol{v} - \eta^2\boldsymbol{u}^\top(\boldsymbol{\Gamma}\mathcal{L}_{22}\otimes\boldsymbol{I}_p)\boldsymbol{u} - \eta\boldsymbol{u}^\top\boldsymbol{\Gamma}_p\boldsymbol{K}\boldsymbol{v}\\ &\quad + \eta\boldsymbol{v}^\top\boldsymbol{\Gamma}_p\boldsymbol{M}\boldsymbol{v} - \boldsymbol{u}^\top(\boldsymbol{\Gamma}\mathcal{L}_{22}\otimes\boldsymbol{I}_p)\boldsymbol{v}\end{aligned}$$

With analysis similar to $\dot{V}_2$, and by verifying that there holds $\boldsymbol{v}^\top\boldsymbol{\Gamma}_p\boldsymbol{K}\boldsymbol{v} = \sum_{i=1}^n\mu_i\gamma_i\|\boldsymbol{v}_i\|_2{}^2$, we easily obtain

$$\begin{aligned}\dot{V}_3 &= \eta^{-1}\Big(\boldsymbol{v}^\top\boldsymbol{\Gamma}_p\boldsymbol{M}\dot{\boldsymbol{v}} + \frac{1}{2}\boldsymbol{v}^\top\boldsymbol{\Gamma}_p\dot{\boldsymbol{M}}\boldsymbol{v}\Big)\\ &= -\boldsymbol{v}^\top(\boldsymbol{\Gamma}\mathcal{L}_{22}\otimes\boldsymbol{I}_p)\boldsymbol{u} - \eta^{-2}\boldsymbol{v}^\top(\boldsymbol{\Gamma}\mathcal{L}_{22}\otimes\boldsymbol{I}_p)\boldsymbol{v}\\ &\quad - \eta^{-1}\sum_{i=1}^n\mu_i\gamma_i\|\boldsymbol{v}_i\|_2{}^2\end{aligned}$$

Lastly, $\dot{V}_4$ evaluates to

$$\begin{aligned}\dot{V}_4 &= \sum_{i=1}^n\Big(\eta^{-1}\mu_i\gamma_i\|\boldsymbol{v}_i\|_2{}^2 - \bar{\mu}\|\boldsymbol{v}_i\|_2{}^2\Big)\\ &= \sum_{i=1}^n\Big(\eta^{-1}\mu_i\gamma_i\|\boldsymbol{v}_i\|_2{}^2\Big) - \bar{\mu}\boldsymbol{v}^\top\boldsymbol{v}\end{aligned}$$

Summing the above yields

$$\begin{aligned}\dot{V} = -\Bigg[&\bar{\mu}\boldsymbol{v}^\top\boldsymbol{v} + \frac{1}{2}\eta^2\boldsymbol{u}^\top(\boldsymbol{Q}\otimes\boldsymbol{I}_p)\boldsymbol{u} - \eta\boldsymbol{v}^\top\boldsymbol{\Gamma}_p\boldsymbol{M}\boldsymbol{v}\\ &- \eta\boldsymbol{u}^\top\boldsymbol{\Gamma}_p\boldsymbol{C}^\top\boldsymbol{v} - \frac{1}{2}\eta^{-2}\boldsymbol{v}^\top(\boldsymbol{Q}\otimes\boldsymbol{I}_p)\boldsymbol{v}\\ &- \frac{1}{2}\eta\alpha\sum_{i=1}^n\gamma_i(\|\boldsymbol{u}_i\|_2\|\boldsymbol{v}_i\|_2)^2\Bigg] := -f(\boldsymbol{u},\boldsymbol{v})\end{aligned} \tag{20}$$

Note that the second summand of $\dot{V}_1$, the last summand of the second expression of $\dot{V}_2$ and the first summand, second equality of $\dot{V}_3$ sum to zero. The term $(\eta^{-2}/2)\boldsymbol{v}^\top(\boldsymbol{Q}\otimes\boldsymbol{I}_p)\boldsymbol{v}$ arises from the fact that $\boldsymbol{Q}/2$ is the symmetric part of $\boldsymbol{\Gamma}\mathcal{L}_{22}$. Note that any η satisfying (19) implies that $\frac{1}{2}\eta^{-2}\lambda_{\min}(\boldsymbol{Q}) - \eta\bar{\gamma}k_{\overline{M}} > 0$. It is then straightforward to compute that

$$\begin{aligned}f(\boldsymbol{u},\boldsymbol{v}) &\geq \bar{\mu}\|\boldsymbol{v}\|_2{}^2 + \frac{1}{2}\eta^2\lambda_{\min}(\boldsymbol{Q})\|\boldsymbol{u}\|_2{}^2\\ &\quad - \eta\bar{\gamma}k_C\|\boldsymbol{u}\|_2\|\boldsymbol{v}\|_2{}^2 - \frac{1}{2}\eta\alpha\bar{\gamma}(\|\boldsymbol{u}\|_2\|\boldsymbol{v}\|_2)^2\\ &:= g(\|\boldsymbol{u}\|_2, \|\boldsymbol{v}\|_2)\end{aligned} \tag{21}$$

Notice that $g(\|\boldsymbol{u}\|_2, \|\boldsymbol{v}\|_2)$ is of the form of $g(x,y)$ in Lemma 1, with $x = \|\boldsymbol{u}\|_2$, $y = \|\boldsymbol{v}\|_2, a = \eta\lambda_{\min}(\boldsymbol{Q})/2$, $b = \bar{\mu}$, $c = \epsilon\bar{\gamma}k_C$, and $d = \eta\alpha\bar{\gamma}/2$.

For some given value $\mathcal{X}$, define the region $\mathcal{D}_{\mathcal{X}}$ as $\|\boldsymbol{u}\|_2 \in [0, \mathcal{X}]$ and $\|\boldsymbol{v}\|_2 \in [0, \infty)$. The precise value of

$\mathcal{X}$ will be specified below. It follows from Lemma 1 that $g(\|\boldsymbol{u}\|_2, \|\boldsymbol{v}\|_2) > 0$ in the region $\mathcal{D}_{\mathcal{X}}$ if $\bar{\mu}$ satisfies

$$\bar{\mu} = \eta\bar{\gamma}k_C\mathcal{X} + \frac{\eta\alpha\bar{\gamma}}{2}\mathcal{X}^2 + \delta \tag{22}$$

where the strictly positive constant δ may be arbitrarily small. Consider now a ball region, defined as $\mathcal{B}$, centred about $\boldsymbol{w} = \boldsymbol{0}$ with radius $\mathcal{X}$. We observe that $\mathcal{B} \subset \mathcal{D}_{\mathcal{X}}$. Denote the set of points $\|\boldsymbol{w}\|_2 = \mathcal{X}$ as $\bar{\mathcal{D}}$. On the outer surface of $\mathcal{B}$ (i.e. the set of points $\bar{\mathcal{D}}$), we use expression (4) to conclude that

$$\begin{aligned}\inf_{\bar{\mathcal{D}}} \widetilde{V}(\boldsymbol{w}) &= \inf_{\|\boldsymbol{w}\|_2 = \mathcal{X}} \boldsymbol{w}^\top \boldsymbol{H}(\eta)\boldsymbol{w} \\ &\geq \lambda_{\min}(\boldsymbol{H}(\eta))\mathcal{X}^2 > \zeta\mathcal{X}^2 \end{aligned}\tag{23}$$

In addition, observe that $s_i(t)^2 \geq 0$ for all t. It follows that

$$\inf_{\bar{\mathcal{D}}\times\mathbb{R}^n} V(\boldsymbol{w}, \boldsymbol{s}) = \inf_{\bar{\mathcal{D}}} \widetilde{V}(\boldsymbol{w}) > \zeta\mathcal{X}^2 \tag{24}$$

Defining

$$W(\eta, \alpha, \mathcal{X}) := \inf_{\bar{\mathcal{D}}\times\mathbb{R}^n} V(\boldsymbol{w}, \boldsymbol{s}) - V(\boldsymbol{w}(0), \boldsymbol{s}(0)) \tag{25}$$

and recalling that we have assumed $\mu_i(0) = 0, \forall\, i$ yields

$$W(\eta, \alpha, \mathcal{X}) > \zeta\mathcal{X}^2 - \phi\|\boldsymbol{w}(0)\|_2{}^2 - \frac{\beta}{\alpha}\bar{\mu}^2 \tag{26}$$

where $\beta = n/2\underline{\gamma}$. Substituting $\bar{\mu}$ from (22) into (26) yields

$$\begin{aligned}W(\eta, \alpha, \mathcal{X}) \geq &\Big(\zeta - A(\eta, \alpha)\Big)\mathcal{X}^2 - B(\eta)\mathcal{X}^3 \\ &- C(\eta, \alpha)\mathcal{X}^4 - \phi\|\boldsymbol{w}(0)\|_2{}^2 \end{aligned}\tag{27}$$

where $A(\eta, \alpha) = \alpha^{-1}\beta\eta^3\bar{\gamma}^2{k_C}^2$, $B(\eta) = \beta\eta^3\bar{\gamma}^2 k_C$, $C(\eta, \alpha) = \frac{1}{4}\beta\eta^3\alpha\bar{\gamma}^2$ (we will use below the cubic dependence of these expressions on η). Note that we discard the terms of $\bar{\mu}^2$ involving δ as δ is arbitrarily small.

It is possible to show that there exists some positive value of $\mathcal{X}^*$ which ensures that the right hand side of (27) is positive. This is achievable by adjusting α and making η sufficiently small (which ensures that $A(\eta, \alpha), B(\eta), C(\eta)$ decrease at a rate of η^3 while ζ is unaffected). This requires a centralised design process, and we leave out the details, which are straightforward. Let η^*, α^* be such that there exists a value $\mathcal{X}^*$ which ensures that $W(\eta^*, \alpha^*, \mathcal{X}^*) > 0$ (note that $\eta^* \leq \hat{\eta}$ to ensure that (19) continues to hold).

This implies that $\dot{V} \leq -g(\|\boldsymbol{u}\|_2, \|\boldsymbol{v}\|_2) < 0$ in the region $\mathcal{D}_{\mathcal{X}^*}$. Specifically, by recalling that $\mu_i(t) \geq 0, \forall i, t$, we conclude that $\dot{V}$ is negative for all nonzero values of $\boldsymbol{u}, \boldsymbol{v} \in \mathcal{D}_{\mathcal{X}^*}$ and arbitrary values of $\boldsymbol{s}$, i.e. $\dot{V}$ is negative semidefinite. In addition, $\dot{V}$ is zero if and only if $\boldsymbol{u} = \boldsymbol{v} = 0$ (i.e. $\dot{V} = 0$ for $\mathcal{M}$, the set of equilibrium points of (13)). Let $\mathcal{Q}$ be defined as a ball of radius R, centred around $\boldsymbol{x} = \boldsymbol{0}$. From our conclusions on V below (19), it follows that there exists an R such that, for all $\|\boldsymbol{x}\|_2 \leq R$, $V(\boldsymbol{x})$ has a value less than $\zeta\mathcal{X}^2$. The definition of $W(\eta, \alpha, \mathcal{X})$ in (25), and our conclusion that $W(\eta^*, \alpha^*, \mathcal{X}^*) > 0$ implies the trajectory of the system with initial conditions $\boldsymbol{w}(0), \boldsymbol{s}(0)$ starts inside the region $\mathcal{Q}$. We thus establish that 1) $V(\boldsymbol{x}(t))$ in $\mathcal{Q}$ is less than $\inf_{\bar{\mathcal{D}}\times\mathbb{R}^n} V(\boldsymbol{x})$, 2) $\dot{V} \leq 0$ in $\mathcal{B} \subset \mathcal{D}_{\mathcal{X}^*}$, and 3) the trajectory begins in $\mathcal{Q}$. It follows that the trajectory of the system (13) remains in $\mathcal{B}\times\mathbb{R}^n$ for all time. Thus $\dot{V}(t) \leq 0$ for all time, which implies that $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{s}$ are bounded. We thus conclude that system (13) does not have a finite escape time. We also conclude that $\dot{\boldsymbol{v}}$ is bounded because each term on the right hand side of (13b) is bounded. It is straightforward to conclude that $\dot{f}(\boldsymbol{u}, \boldsymbol{v})$ is bounded because $\boldsymbol{u}, \boldsymbol{v}$ and $\dot{\boldsymbol{v}}$ are all bounded. As is easily checked, this implies that $\ddot{V}$ is bounded. Barbalat's Lemma [26] will then allow us to conclude that $\lim_{t\to\infty} \dot{V}(t) = 0$. From our knowledge of $\dot{V}$, it follows that system (13) converges to the set $\mathcal{M}$. In fact, convergence must be to a single point in $\mathcal{M}$, as opposed to a nonconstant trajectory, because in $\mathcal{M}$ there holds $\boldsymbol{v} = 0$. It is straightforward to verify that any point in $\mathcal{M}$ achieves the rendezvous objective.

$\square$

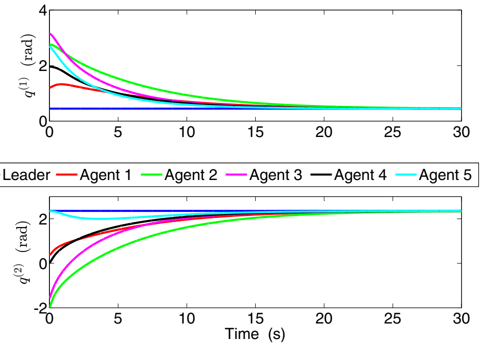

Fig. 1. Plot of generalised coordinates vs. time

IV. Simulations

A simulation is now provided to demonstrate the effectiveness of the algorithm. As a direct comparison, we use the simulation example in [22]. Each of five follower agents is modelled as a two-link robotic manipulator and they must rendezvous to the leader robotic arm. A figure and the equations of motion may be found in [25], pp.259-262. The generalised coordinates are the two arm angles, given for agent i as $\boldsymbol{q}_i = [q_i^{(1)}, q_i^{(2)}]^\top$. The agent parameters are given in Table I of [22] and the directed graph topology is found in the same reference. We initialise $\mu_i(0) = 0, \forall\, i$ and set $\alpha = 0.5, \forall\, i$ and $\eta = 0.5$. Figure 1 shows that the generalised coordinates of the follower agents rendezvous to the leader's generalised coordinates. Figures 2 and 3 show the generalised velocity and control input respectively. The variable control gain $\mu_i(t)$ is shown in Fig. 4.

V. Conclusion

In this paper we present a variable gain model-independent algorithm which ensures that a network of Euler-Lagrange follower agents will rendezvous to a stationary leader. Using a variable-gain controller, we provide a sufficiency condition for stability. The results of this paper is different to the work of [22] in two key aspects. Firstly, the variable-gain controller removes the need for a highly conservative constant gain for the damping term in the controller. Secondly, novel analysis is applied to the Lyapunov-like candidate function. Simulations are provided to show the algorithm's effectiveness. Future work will include attempt to relax the design conditions on

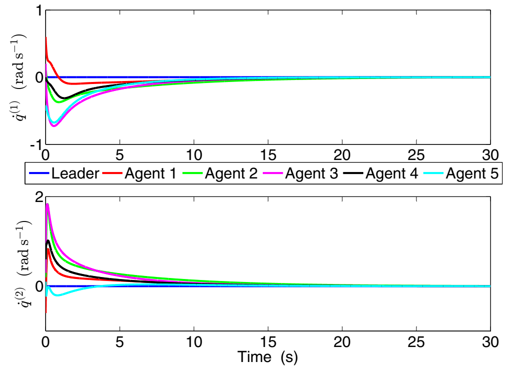

Fig. 2. Plot of generalised velocities vs. time

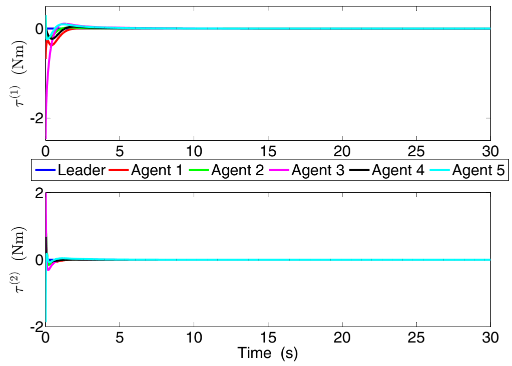

Fig. 3. Plot of control input vs. time

the control parameters and study of related coordination problems such as leaderless consensus.

References

[1] Y. Cao, W. Yu, W. Ren, and G. Chen, “An Overview of Recent Progress in the Study of Distributed Multi-Agent Coordination,” *IEEE Transactions on Industrial Informatics*, vol. 9, no. 1, pp. 427–438, Feb 2013.

[2] W. Ren and R. Beard, *Distributed Consensus in Multi-vehicle Cooperative Control: Theory and Applications*. Springer, 2007.

[3] W. Ren and Y. Cao, *Distributed Coordination of Multi-agent Networks*. Springer, 2011.

[4] Z. Li and Z. Duan, *Cooperative Control of Multi-Agent Systems: A Consensus Region Approach*. CRC Press, 2014.

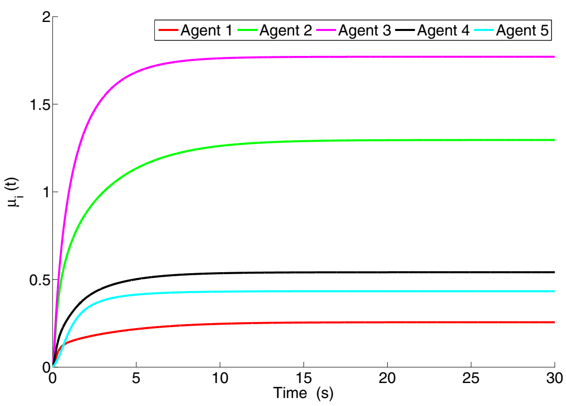

Fig. 4. Plot of $\mu_i(t)$ vs. time

[5] R. Ortega, J. A. L. Perez, P. J. Nicklasson, and H. Sira-Ramirez, *Passivity-based Control of Euler-Lagrange systems: Mechanical, Electrical and Electromechanical Applications*. Springer Science & Business Media, 2013.

[6] Z. Meng and Z. Lin, “Distributed Finite-time Cooperative Tracking of Networked Lagrange Systems via Local Interactions,” in *American Control Conference (ACC), 2012*. IEEE, 2012, pp. 4951–4956.

[7] G. Chen, “Cooperative controller design for synchronization of networked uncertain Euler–Lagrange systems,” *International Journal of Robust and Nonlinear Control*, vol. 25, no. 11, pp. 1721–1738, 2015.

[8] R. Kelly, V. S. Davila, and J. A. L. Perez, *Control of Robot Manipulators in Joint Space*. Springer Science & Business Media, 2006.

[9] J. Mei, W. Ren, J. Chen, and G. Ma, “Distributed adaptive coordination for multiple lagrangian systems under a directed graph without using neighbors velocity information,” *Automatica*, vol. 49, no. 6, pp. 1723–1731, 2013.

[10] Z. Meng, Z. Lin, and W. Ren, “Leader–Follower Swarm Tracking for Networked Lagrange Systems,” *Systems & Control Letters*, vol. 61, no. 1, pp. 117–126, 2012.

[11] Z. Meng, D. V. Dimarogonas, and K. H. Johansson, “Leader–Follower Coordinated Tracking of Multiple Heterogeneous Lagrange Systems Using Continuous Control,” *IEEE Transactions on Robotics*, vol. 30, no. 3, pp. 739–745, 2014.

[12] E. Nuno, R. Ortega, L. Basanez, and D. Hill, “Synchronization of Networks of Nonidentical Euler-Lagrange Systems with Uncertain Parameters and Communication Delays,” *IEEE Transactions on Automatic Control*, vol. 56, no. 4, pp. 935–941, 2011.

[13] W. Ren, “Distributed Leaderless Consensus Algorithms for Networked Euler–Lagrange Systems,” *International Journal of Control*, vol. 82, no. 11, pp. 2137–2149, 2009.

[14] Z. Meng, T. Yang, G. Shi, D. V. Dimarogonas, Y. Hong, and K. H. Johansson, “Set Target Aggregation of Multiple Mechanical Systems,” in *2014 IEEE 53rd Annual Conference on Decision and Control*. IEEE, 2014, pp. 6830–6835.

[15] E. Nuno, I. Sarras, and L. Basanez, “Consensus in Networks of Nonidentical Euler-Lagrange Systems using P+ d Controllers,” *IEEE Transactions on Robotics*, vol. 29, no. 6, pp. 1503–1508, 2013.

[16] A. Tavasoli, S. Taghvaee, and M. Eghtesad, “Flocking of a Team of Lagrangian Agents,” in *IEEE International Conference on Robotics and Biomimetics (ROBIO)*. IEEE, 2009, pp. 1433–1438.

[17] J. Mei, W. Ren, and G. Ma, “Distributed Coordinated Tracking With a Dynamic Leader for Multiple Euler-Lagrange Systems,” *IEEE Transactions on Automatic Control*, vol. 56, no. 6, pp. 1415–1421, 2011.

[18] Y. Zhao, Z. Duan, and G. Wen, “Distributed finite-time tracking of multiple Euler–Lagrange systems without velocity measurements,” *International Journal of Robust and Nonlinear Control*, vol. 25, no. 11, pp. 1688–1703, 2015.

[19] J. R. Klotz, T.-H. Cheng, and W. E. Dixon, “Robust containment control in a leader–follower network of uncertain Euler–Lagrange systems,” *International Journal of Robust and Nonlinear Control*, 2016.

[20] P. F. Hokayem, D. M. Stipanović, and M. W. Spong, “Coordination and Collision Avoidance for Lagrangian Systems With Disturbances,” *Applied Mathematics and Computation*, vol. 217, no. 3, pp. 1085–1094, 2010.

[21] N. Chopra, “Output Synchronization on Strongly Connected Graphs,” *IEEE Transactions on Automatic Control*, vol. 57, no. 11, pp. 2896–2901, Nov 2012.

[22] M. Ye, C. Yu, and B. D. O. Anderson, “Model-Independent Rendezvous of Euler-Lagrange Agents on Directed Networks,” in *Proceedings of IEEE 54th Annual Conference on Decision and Control*, 2015, pp. 3499–3505.

[23] ——, “Distributed model-independent consensus of euler-lagrange agents on directed networks,” *Submitted for publication*.

[24] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge university press, 2012.

[25] M. W. Spong, S. Hutchinson, and M. Vidyasagar, *Robot Modeling and Control*. Wiley New York, 2006, vol. 3.

[26] J.-J. E. Slotine, W. Li *et al.*, *Applied Nonlinear Control*. Prentice-Hall Englewood Cliffs, NJ, 1991, vol. 199, no. 1.