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Model-Independent Rendezvous of Euler-Lagrange Agents on Directed Networks

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***Abstract*—This paper proposes a distributed, model-independent algorithm to achieve rendezvous to a stationary leader for a directed network where each fully-actuated agent has Euler-Lagrange self-dynamics. We show that if the directed graph contains a directed spanning tree, with the leader as the root node and with no incoming edges, then a model-independent algorithm semi-globally achieves the rendezvous objective exponentially fast. By model-independent we mean that each agent can execute the algorithm with no knowledge of the parameters of the self-dynamics of any agent in the network. For stability, a pair of control gain terms for each agent are required to meet several inequalities and so design of the algorithm requires some limited knowledge of global information. Numerical simulations are provided to illustrate the algorithm's effectiveness.**

***Index Terms*—model-independent, euler-lagrange agent, semi-global, directed graph, distributed network, rendezvous**

I. Introduction

Coordination and cooperative control of autonomous multiagent systems has received sustained attention over the last decade. A system comprised of multiple agents, possibly heterogeneous, acting together to achieve a common objective offers numerous advantages over a single complex agent. The term "agent" can be applied to any number of controllable subsystems connected together via a sensing or communication network. An overview of multiagent systems coordination and control is provided in [1].

The consensus problem is an important and widely studied area of multiagent coordination. Rendezvous to a leader is variation of the consensus problem where, given a common state variable defined for every agent, all follower agents are controlled to converge to the state value of the stationary leader(s) by communication and negotiation between neighbors. A moving leader results in a trajectory tracking problem. In general it is desirable to develop distributed control laws; by distributed we mean that each agent can execute the control law without requiring information about the network as a whole [2]. The topological constraints of a network are linked to the self-dynamics of the agents when studying control laws which guarantee achievement of the coordination objective; reducing the topology constraints allows greater flexibility during the design phase. Rendezvous for single and double-integrator agents are covered thoroughly in [3].

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The Euler-Lagrange equation of motion can be used to model a class of nonlinear dynamic systems including mechanical, electrical and electromechanical systems. As such, multiagent systems problems where the agent dynamics are described by Euler-Lagrange equations are well motivated. In most previous works, the control laws require knowledge of the agent dynamical model or require a linearized parametrization which can be used in an adaptive algorithm. Contraction analysis is used in [4] to study trajectory tracking while assuming the exact agent dynamics are known. Adaptive algorithms for tracking are studied in [5], [6]. Convergence to a bounded region around a stationary leader with guarantee of no collisions using an adaptive algorithm is studied in [7]. Containment control, where the group of followers converge to a convex hull spanned by a number of leaders using adaptive algorithms is studied in [8], [9], [10].

In the area of *model-independent* algorithms for Euler-Lagrange networks, there have been relatively few developments. By model-independent, we mean that execution of the control law does not require knowledge of the individual agent dynamics. The pioneering work in [11] considered leaderless position consensus and relied on an undirected, connected graph. In [12], flocking is achieved on an undirected graph. The passivity analysis in [13] achieved model-independent synchronization of the velocities (but not the positions) on balanced and strongly connected graphs. Tracking of a leader with non-constant velocity is studied in [14] where the subgraph of followers is undirected.

Further development of model-independent algorithms is desirable for several reasons. Firstly, the design stage is greatly simplified. For a unique Euler-Lagrange equation, the linearized parametrization required for adaptive algorithms is not unique and determining the minimum number of parameters is difficult in general [15]. The algorithm itself is also greatly simplified and therefore requires less computation during execution. As mentioned previously, the constraints on network topology are an important aspect. In general, directed graphs representing unilateral communication flow are more desirable than undirected graphs (which requires bilateral communication flow). In a practical context, the number of neighbors that an agent must communicate with is reduced when there is only a unilateral communication requirement.

The key contribution of this paper is to show that a network of Euler-Lagrange follower agents can achieve rendezvous to a single stationary leader if the directed graph representing the communication topology contains a directed spanning tree, with the leader as the root vertex. The algorithm proposed in this paper allows for heterogeneous agents; for example several UAVs in a formation may have different payload weights and motion characteristics. The algorithm studied in this paper requires each agent to have a control gain scalar and gain matrix satisfy a set of lower bounding inequalities. These inequalities require limited knowledge of the bounds on the agent model parameters, knowledge of the fixed network topology and the set of all possible initial conditions (which may be arbitrarily large). The last requirement means the algorithm is semi-globally stable. We therefore say that the algorithm is model-independent in execution but the design stage is, to a limited degree, centralized.

The rest of the paper is structured as follows. Section II provides background information on matrix mathematics, graph theory, the Euler-Lagrange equations, and we formally define the rendezvous objective. The main result and proof is presented in Section III. Simulations are provided in Section IV and the paper is concluded in Section V.

II. Background and Problem Statement

A. Notation and Matrix Theory

Here, we provide definitions of notation and several results which will be used later. We denote the Kronecker product as $\otimes$, the $p \times p$ identity matrix by $\boldsymbol{I}_p$, the column vector of all ones by $\mathbf{1}$, and the Euclidean norm of a vector by $|\cdot|$. We denote a matrix $\boldsymbol{A}$ as positive definite (respectively positive semidefinite) by $\boldsymbol{A} > 0$ (respectively $\boldsymbol{A} \geq 0$). The minimum and maximum eigenvalues of a square symmetric matrix $\boldsymbol{A}$ are $\lambda_{\min}(\boldsymbol{A})$ and $\lambda_{\max}(\boldsymbol{A})$ respectively. For two symmetric matrices $\boldsymbol{A}, \boldsymbol{B}$, we say $\boldsymbol{A} > \boldsymbol{B}$ if $\boldsymbol{A} - \boldsymbol{B} > 0$. Furthermore, the following expressions hold

$$\lambda_{\min}(\boldsymbol{A}) > \lambda_{\max}(\boldsymbol{B}) \Rightarrow \boldsymbol{A} > \boldsymbol{B} \tag{1}$$
$$\lambda_{\max}(\boldsymbol{A}+\boldsymbol{B}) \leq \lambda_{\max}(\boldsymbol{A}) + \lambda_{\max}(\boldsymbol{B}) \tag{2}$$
$$\lambda_{\min}(\boldsymbol{A}+\boldsymbol{B}) \geq \lambda_{\min}(\boldsymbol{A}) + \lambda_{\min}(\boldsymbol{B}) \tag{3}$$
$$\lambda_{\min}(\boldsymbol{A})\boldsymbol{x}^\top\boldsymbol{x} \leq \boldsymbol{x}^\top\boldsymbol{A}\boldsymbol{x} \leq \lambda_{\max}(\boldsymbol{A})\boldsymbol{x}^\top\boldsymbol{x} \tag{4}$$

Lemma 1 ([16]). *Let $\boldsymbol{A} \in \mathbb{R}^{n\times n}$, $\boldsymbol{B} \in \mathbb{R}^{m\times m}$, $\boldsymbol{C} \in \mathbb{R}^{n\times n}$ and $\boldsymbol{D} \in \mathbb{R}^{m\times m}$ be matrices. Then the following identities hold:*

1) $(\boldsymbol{A}\otimes\boldsymbol{B})(\boldsymbol{C}\otimes\boldsymbol{D}) = \boldsymbol{AC}\otimes\boldsymbol{BD}$
2) $(\boldsymbol{A}\otimes\boldsymbol{B}) + (\boldsymbol{C}\otimes\boldsymbol{B}) = (\boldsymbol{A}+\boldsymbol{C})\otimes\boldsymbol{B}$
3) $(\boldsymbol{A}\otimes\boldsymbol{B})^\top = \boldsymbol{A}^\top\otimes\boldsymbol{B}^\top$
4) *If $\boldsymbol{A}$ and $\boldsymbol{B}$ are symmetric positive definite, then $\boldsymbol{A}\otimes\boldsymbol{B}$ is symmetric positive definite.*

Theorem 1 (The Schur Complement [16]). *Consider a symmetric matrix*

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{B} & \boldsymbol{C} \\ \boldsymbol{C}^\top & \boldsymbol{D} \end{bmatrix} \tag{5}$$

where the submatrices $\boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ are appropriately sized. Then $\boldsymbol{A} > 0$ if and only if $\boldsymbol{B} > 0$ and $\boldsymbol{D} - \boldsymbol{C}^\top\boldsymbol{B}^{-1}\boldsymbol{C} > 0$

Definition 1 ([17]). *Let $\boldsymbol{Z}_n \subset \mathbb{R}^{n\times n}$ be the set of all square matrices with nonpositive off-diagonal entries. A matrix $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ is said to be a nonsingular M-matrix if $\boldsymbol{A} \in \boldsymbol{Z}_n$ and all eigenvalues of $\boldsymbol{A}$ have positive real part.*

Lemma 2 ([17]). *If $\boldsymbol{A} \in \boldsymbol{Z}_n$ is a nonsingular M-matrix, there exists a positive definite diagonal matrix $\boldsymbol{\Gamma}$ such that $\boldsymbol{\Gamma A} + \boldsymbol{A}^\top\boldsymbol{\Gamma} > 0$*

Definition 2 (See [18] for details). *The spectral norm for a matrix $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ is defined as*

$$\|\boldsymbol{A}\|_2 = \sigma_{\max}(\boldsymbol{A}), \text{ the largest singular value of } \boldsymbol{A} \tag{6}$$

The spectral norm has the following properties

1) $\|\boldsymbol{A}\|_2 \geq 0$ and $\|\boldsymbol{A}\|_2 = 0 \Leftrightarrow \boldsymbol{A} = \mathbf{0}$
2) $\|c\boldsymbol{A}\|_2 = |c|\|\boldsymbol{A}\|_2 \ \forall c \in \mathbb{C}$
3) $\|\boldsymbol{A}+\boldsymbol{B}\|_2 \leq \|\boldsymbol{A}\|_2 + \|\boldsymbol{B}\|_2$
4) $\|\boldsymbol{AB}\|_2 \leq \|\boldsymbol{A}\|_2\|\boldsymbol{B}\|_2$
5) $\|\boldsymbol{A}\|_2 = \|\boldsymbol{A}^\top\|_2 = \|-\boldsymbol{A}\|_2$
6) $|\lambda_i(\boldsymbol{A})| \leq \|\boldsymbol{A}\|_2, \forall i$

Lemma 3. *Let*

$$W = \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix}^\top \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{B}^\top & \boldsymbol{C} \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{bmatrix}, \text{ with } \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{B}^\top & \boldsymbol{C} \end{bmatrix} > 0 \tag{7}$$

Then there holds

$$\boldsymbol{y}^\top[\boldsymbol{C} - \boldsymbol{B}^\top\boldsymbol{A}^{-1}\boldsymbol{B}]\boldsymbol{y} \leq W \tag{8}$$

Proof. See that

$$\begin{aligned} W = &\ \boldsymbol{x}^\top[\boldsymbol{A} - \boldsymbol{B}\boldsymbol{C}^{-1}\boldsymbol{B}^\top]\boldsymbol{x} \\ &+ [\boldsymbol{x}^\top\boldsymbol{B}\boldsymbol{C}^{-1} + \boldsymbol{y}]\boldsymbol{C}[\boldsymbol{C}^{-1}\boldsymbol{B}^\top\boldsymbol{x} + \boldsymbol{y}^\top] \end{aligned}$$

Using the conditions for positive definiteness given in Theorem 1, the inequality in (8) is immediately proven. □

B. Graph Theory

Let a weighted directed graph modelling interacting agents be $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, with the set of nodes $\mathcal{V} = \{v_1, \ldots, v_n\}$, and with a corresponding set of edges $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$. An ordered edge of $\mathcal{G}$ is $e_{ij} = (v_i, v_j)$ and the assumption $e_{ij} = e_{ji}$ does not hold. The $n \times n$ weighted adjacency matrix $\mathcal{A}$ of $\mathcal{G}$ has nonnegative adjacency elements a_{ij}. The finite index set of node indices is $\mathcal{I} = \{1, 2, ..., n\}$. The elements of $\mathcal{A}$ are defined such that $a_{ij} > 0 \Leftrightarrow e_{ji} \in \mathcal{E}$ while $a_{ij} = 0$ if $e_{ji} \notin \mathcal{E}$ and it is assumed $a_{ii} = 0, \forall i$. An edge $e_{ij} = (v_i, v_j)$ is outgoing with respect to v_i and incoming with respect to v_j, i.e. the edge (v_i, v_j) indicates that v_i is passing information to v_j. The neighbors of v_i are the set $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_i, v_j) \in \mathcal{E}\}$. The $n \times n$ Laplacian matrix, $\mathcal{L} = \{l_{ij}\}$, of the associated digraph $\mathcal{G}$ is defined as

$$l_{ij} = \begin{cases} \sum_{k=1,k\neq i}^{n} a_{ik} & \text{for } j = i \\ -a_{ij} & \text{for } j \neq i \end{cases}$$

A directed spanning tree is a directed graph formed by directed edges of the graph that connects all the nodes, and where every vertex apart from the root has exactly one parent [19]. A graph is said to contain a directed spanning tree if a subset of the edges forms a spanning tree. We now provide some basic results on algebraic graph theory which will be relevant to this paper.

Theorem 2 (From [19]). *The Laplacian matrix $\mathcal{L}$ associated with a graph $\mathcal{G}$ which has a directed spanning tree has a single eigenvalue of zero with associated eigenvector* $\mathbf{1}$. *All other eigenvalues have positive real part.*

Corollary 1. *Let the graph $\mathcal{G}$ contain a directed spanning tree, and there are not edges of $\mathcal{G}$ which are incoming to the root vertex of the tree. Number the root vertex as v_1. The Laplacian associated with $\mathcal{G}$ is given as*

$$\mathcal{L} = \begin{bmatrix} 0 & \mathbf{0} \\ \mathcal{L}_{21} & \mathcal{L}_{22} \end{bmatrix} \tag{9}$$

Then there exists a positive definite diagonal matrix $\mathbf{\Gamma}$ such that $\boldsymbol{Q} \triangleq \mathbf{\Gamma}\mathcal{L}_{22} + \mathcal{L}_{22}^\top \mathbf{\Gamma} > 0$.

Proof. Theorem 2 establishes that $\mathcal{L}$ has precisely one zero eigenvalue and all others have positive real part. Then all eigenvalues of $\mathcal{L}_{22}$ have positive real part. By the definition of the Laplacian, $\mathcal{L}_{22} \in \boldsymbol{Z}_n$. Definition 1 shows that $\mathcal{L}_{22}$ is a nonsingular M-matrix and Lemma 2 completes the proof. □

C. Euler-Lagrange Systems

The dynamics of a class of systems can be described using the Euler-Lagrange equations, and the general form for the i^{th} agent equation of motion is:

$$\boldsymbol{M}_i(\boldsymbol{q}_i)\ddot{\boldsymbol{q}}_i + \boldsymbol{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\dot{\boldsymbol{q}}_i + \boldsymbol{g}_i(\boldsymbol{q}_i) = \boldsymbol{\tau}_i \tag{10}$$

where $\boldsymbol{q}_i \in \mathbb{R}^p$ is a vector of the generalized coordinates, $\boldsymbol{M}_i(\boldsymbol{q}_i) \in \mathbb{R}^{p\times p}$ is the inertia matrix, $\boldsymbol{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i) \in \mathbb{R}^{p\times p}$ is the Coriolis and centrifugal force matrix, $\boldsymbol{g}_i \in \mathbb{R}^p$ is the vector of gravitational forces and $\boldsymbol{\tau}_i \in \mathbb{R}^p$ is the control input vector, with the assumption that the agent is fully-actuated. For each agent, we use superscript (j) to denote the j^{th} component of a vector. For agent i, we have $\boldsymbol{q}_i = [q_i^{(1)}, ..., q_i^{(p)}]^\top$. Several useful properties universal to all systems described using (10) are given below, with details found in [15] and [20]:

1) The matrix $\boldsymbol{M}_i(\boldsymbol{q}_i)$ is symmetric positive definite.
2) There exists scalar constants $k_{\underline{m}i}, k_{\overline{M}i} > 0$ such that $k_{\underline{m}i}\boldsymbol{I}_p \le \boldsymbol{M}_i(\boldsymbol{q}_i) \le k_{\overline{M}i}\boldsymbol{I}_p, \forall i$. We assume $\sup_{\boldsymbol{q}_i} \|\boldsymbol{M}_i\|_2 \le k_{\overline{M}i}$ and $k_{\underline{m}i} \le \inf_{\boldsymbol{q}_i} \|\boldsymbol{M}_i^{-1}\|_2$ and denote $k_{\underline{m}} = \min_i\{k_{\underline{m}i}\}$, $k_{\overline{M}} = \max_i\{k_{\overline{M}i}\}$.
3) The Coriolis and centrifugal matrix $\boldsymbol{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)$ can always be defined such that the matrix $\dot{\boldsymbol{M}}_i - 2\boldsymbol{C}_i$ is skew-symmetric. It follows that $\dot{\boldsymbol{M}}_i = \boldsymbol{C}_i + \boldsymbol{C}_i^\top$.
4) There exists a scalar constant $k_{Ci} > 0$ such that $\|\boldsymbol{C}_i\|_2 \le k_{Ci}|\boldsymbol{q}_i|, \forall i$. We denote $k_C = \max_i\{k_{Ci}\}$.

In this work, the gravitational forces are assumed to be zero to simplify the problem; that is $\boldsymbol{g}_i = \mathbf{0}, \forall i$. This case can occur in a wide range of mechanical systems, for example, when mobile robots, UAVs or aircraft move in a two-dimensional plane parallel to the Earth's surface. If there is a nonzero gravitational force then there will be an error in the final rendezvous dependent on $\boldsymbol{g}_i$.

D. Problem Statement

Denote the leader as agent 1 with $\boldsymbol{q}_1$ and $\dot{\boldsymbol{q}}_1$ being the generalized coordinates and generalized velocity of the leader respectively. The aim is to develop a model-independent, distributed algorithm which allows a directed network of Euler-Lagrange agents to rendezvous to a stationary leader. That is, $\lim_{t\to\infty} |\boldsymbol{q}_i - \boldsymbol{q}_1| = 0, \forall i$ and $\lim_{t\to\infty} \dot{\boldsymbol{q}}_i(t) = \dot{\boldsymbol{q}}_1 = \mathbf{0}, \forall i$. By model-independent, we mean that the algorithm does not contain $\boldsymbol{M}_i, \boldsymbol{C}_i, \forall i$. By distributed, we mean that each agent is able to execute the algorithm with no knowledge of the network topology or the multiagent system as a whole; it only needs to receive information about its neighbors.

We assume that each agent can measure the relative generalized coordinates of its neighbors and this is depicted by the fixed, weighted, directed graph $\mathcal{G}$. That is, if for agent i, $a_{ij} > 0$ then $\boldsymbol{q}_i - \boldsymbol{q}_j$ and a_{ij} are separately available. The method by which $\boldsymbol{q}_i - \boldsymbol{q}_j$ is obtained is left open to interpretation and adaptation to the specific scenario. One can imagine ground vehicles equipped with GPS communicating global coordinates or UAVs measuring relative positions. It is also assumed that agent i can measure its own generalized velocity, $\dot{\boldsymbol{q}}_i$.

Finally, we assume that all possible initial conditions lie in some fixed and arbitrarily large set, Ω, which is known a priori. This is not unreasonable, as many systems will have an expected operating range for $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$.

Notice that $\boldsymbol{M}_1 \ne \boldsymbol{M}_2 \ne ... \ne \boldsymbol{M}_n$ and $\boldsymbol{C}_1 \ne \boldsymbol{C}_2 \ne ... \ne \boldsymbol{C}_n$ is a possibility but $\boldsymbol{q}_i \in \mathbb{R}^p, \forall i$. In other words, the inertia matrices and Coriolis and centrifugal force matrices of the agents can differ but the generalized coordinates must be defined using the same quantities for all agents. This work thus treats for Euler-Lagrange agents of the same type but which have heterogeneous parameters.

III. Main Result

Consider a model-independent algorithm for the i^{th} agent, excluding the leader agent 1, of the form

$$\boldsymbol{\tau}_i = -\sum_{j\in\mathcal{N}_i} a_{ij}\gamma_i(\boldsymbol{q}_i - \boldsymbol{q}_j) - \mu\boldsymbol{K}_i\dot{\boldsymbol{q}}_i \tag{11}$$

where a_{ij} is the weighted (i, j) entry of the adjacency matrix $\mathcal{A}$ associated with the weighted directed graph $\mathcal{G}$. The constant γ_i is defined such that $\mathbf{\Gamma} = diag[\gamma_2, ..., \gamma_n]$ satisfies $\mathbf{\Gamma}\mathcal{L}_{22} + \mathcal{L}_{22}^\top\mathbf{\Gamma} > 0$ as in Corollary 1. The control gain matrix $\boldsymbol{K}_i \in \mathbb{R}^{p\times p}$ is symmetric positive definite and can vary between different agents. The control gain scalar $\mu > 0$ is universal to all agents. To ensure the control objective is achieved, the control pair $\mu, \boldsymbol{K}_i$ must be designed to satisfy several inequalities, which will be detailed below. It will become apparent that keeping $\mu, \boldsymbol{K}_i$ as separate terms will make the proof intuitively easier.

We define the new state variables $\boldsymbol{u}_i = \boldsymbol{q}_{i+1} - \boldsymbol{q}_1$, $\boldsymbol{v}_i = \dot{\boldsymbol{q}}_{i+1}$. Let $\boldsymbol{u} = [\boldsymbol{u}_1^\top, ..., \boldsymbol{u}_{n-1}^\top]^\top$ be the stacked column vector of all $\boldsymbol{u}_i$. Similarly, let $\boldsymbol{v} = [\boldsymbol{v}_1^\top, ..., \boldsymbol{v}_{n-1}^\top]^\top$ and $\boldsymbol{q} = [\boldsymbol{q}_2^\top, ..., \boldsymbol{q}_n^\top]^\top$. The rendezvous objective is achieved when $\boldsymbol{u} = \boldsymbol{v} = \mathbf{0}$. Let us define the following block diagonal matrices $\boldsymbol{M}(\boldsymbol{q}) = diag[\boldsymbol{M}_2(\boldsymbol{q}_2), ..., \boldsymbol{M}_n(\boldsymbol{q}_n)]$,

$\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}}) = diag[\boldsymbol{C}_2(\boldsymbol{q}_2,\dot{\boldsymbol{q}}_2),...,\boldsymbol{C}_n(\boldsymbol{q}_n,\dot{\boldsymbol{q}}_n)]$ and $\boldsymbol{K} = diag[\boldsymbol{K}_2,...,\boldsymbol{K}_n]$. Since $\boldsymbol{M}_i$ and $\boldsymbol{K}_i$ are symmetric positive definite for all i, then $\boldsymbol{M}$ and $\boldsymbol{K}$ are also symmetric positive definite. With this notation and applying control law (11) to each agent we can express the networked system using the new variables $\boldsymbol{u},\boldsymbol{v}$ as below

$$\boldsymbol{M}(\boldsymbol{q})\dot{\boldsymbol{v}} + \boldsymbol{C}(\boldsymbol{q},\boldsymbol{v})\boldsymbol{v} + (\boldsymbol{\Gamma}\mathcal{L}_{22}\otimes \boldsymbol{I}_p)\boldsymbol{u} + \mu\boldsymbol{K}\boldsymbol{v} = \boldsymbol{0} \quad (12)$$

and expressed as the non-autonomous system

$$\dot{\boldsymbol{u}} = \boldsymbol{v} \quad (13a)$$
$$\dot{\boldsymbol{v}} = -\boldsymbol{M}(\boldsymbol{q})^{-1}\left[\boldsymbol{C}(\boldsymbol{q},\boldsymbol{v})\boldsymbol{v} + (\boldsymbol{\Gamma}\mathcal{L}_{22}\otimes \boldsymbol{I}_p)\boldsymbol{u} + \mu\boldsymbol{K}\boldsymbol{v}\right] \quad (13b)$$

Although the system (13) is not a self-contained system (since the arguments $\boldsymbol{M}$ and $\boldsymbol{C}$ are dependent on $\boldsymbol{q}$) it turns out using arguments like those of usual Lyapunov theory, we will be able to prove the stability of (13). Alternatively, we could proceed by formally expanding (13) to become the autonomous system

$$\dot{\boldsymbol{q}} = \boldsymbol{v} \quad (14a)$$
$$\dot{\boldsymbol{u}} = \boldsymbol{v} \quad (14b)$$
$$\dot{\boldsymbol{v}} = -\boldsymbol{M}(\boldsymbol{q})^{-1}\left[\boldsymbol{C}(\boldsymbol{q},\boldsymbol{v})\boldsymbol{v} + (\boldsymbol{\Gamma}\mathcal{L}_{22}\otimes \boldsymbol{I}_p)\boldsymbol{u} + \boldsymbol{K}\boldsymbol{v}\right] \quad (14c)$$

It is easily verified that the set of equilibrium points for (14) are defined by $\boldsymbol{u} = 0, \boldsymbol{v} = 0$ and $\boldsymbol{q}_{i+1} = \boldsymbol{q}_1, \forall\, i$. For convenience, we will work with (13) but (14) delivers the same stability result. We are now ready to show the main result of the paper.

Theorem 3. *The equilibrium of system* (13) *is semi-globally stable at an exponentially fast rate if 1) the networked system contains a directed spanning tree with the leader as the root vertex (and thus with no incoming edges) and 2) the control terms μ and $\boldsymbol{K}_i, \forall\, i$ exceed lower bounds computable from bounds on the matrices of the agent models, the graph topology, and Ω. For a given graph topology satisfying 1), there always exists $\mu, \boldsymbol{K}_i$ which satisfy the lower bounds.*

Proof. Consider the Lyapunov-like candidate function

$$V = \frac{1}{2}\boldsymbol{u}^\top\boldsymbol{R}\boldsymbol{u} + \frac{1}{2}\boldsymbol{v}^\top\boldsymbol{M}\boldsymbol{v} + \mu^{-1}\boldsymbol{u}^\top\boldsymbol{M}\boldsymbol{v} \quad (15)$$

where $\boldsymbol{R} = \boldsymbol{K} + \frac{1}{2}(\boldsymbol{\Gamma}\mathcal{L}_{22} + \mathcal{L}_{22}^\top\boldsymbol{\Gamma})\otimes\boldsymbol{I}_p$. It may also expressed as a quadratic in the variables $\boldsymbol{u}$ and $\boldsymbol{v}$

$$V = \begin{bmatrix}\boldsymbol{u}\\ \boldsymbol{v}\end{bmatrix}^\top \begin{bmatrix}\frac{1}{2}\boldsymbol{R} & \frac{1}{2}\mu^{-1}\boldsymbol{M}\\ \frac{1}{2}\mu^{-1}\boldsymbol{M} & \frac{1}{2}\boldsymbol{M}\end{bmatrix}\begin{bmatrix}\boldsymbol{u}\\ \boldsymbol{v}\end{bmatrix} \quad (16)$$

Both summands of $\boldsymbol{R}$ are symmetric positive definite, which implies that $\boldsymbol{R}$ is positive definite. Then by Theorem 1, V is positive definite if

$$\frac{1}{2}\boldsymbol{R} - \frac{1}{2}\mu^{-2}\boldsymbol{M} > 0 \quad (17)$$

which is implied by

$$\lambda_{\min}(\boldsymbol{R}) - \mu^{-2}\lambda_{\max}(\boldsymbol{M}) > 0 \quad (18)$$

Observe that (18) is implied by

$$\mu > \sqrt{\frac{k_{\overline{M}}}{\lambda_{\min}(\boldsymbol{K})}} \quad (19)$$

because expression (3) implies $\lambda_{\min}(\boldsymbol{R}) \geq \lambda_{\min}(\boldsymbol{K})$, and it also holds that $\lambda_{\max}(\boldsymbol{M}) \leq k_{\overline{M}}$. Furthermore, once $\boldsymbol{K}$ is known, and since $\lambda_{\min}(\boldsymbol{K}) > 0$ then there can always be found a $\mu > 0$ which satisfies (19), and henceforth we assume μ satisfies this condition (we discuss below how $\boldsymbol{K}$ should be determined). Therefore V is positive definite in $\boldsymbol{u}$ and $\boldsymbol{v}$.

Taking the derivative of V with respect to time, we have

$$\dot{V} = \boldsymbol{u}^\top\boldsymbol{R}\boldsymbol{v} + \boldsymbol{v}^\top\boldsymbol{M}\dot{\boldsymbol{v}} + \frac{1}{2}\boldsymbol{v}^\top\dot{\boldsymbol{M}}\boldsymbol{v} + \mu^{-1}\boldsymbol{v}^\top\boldsymbol{M}\boldsymbol{v} + \mu^{-1}\boldsymbol{u}^\top\dot{\boldsymbol{M}}\boldsymbol{v} + \mu^{-1}\boldsymbol{u}^\top\boldsymbol{M}\dot{\boldsymbol{v}} \quad (20)$$

Substitute $\boldsymbol{M}\dot{\boldsymbol{v}}$ from (12) into (20) and note that 1) $\dot{\boldsymbol{M}} - 2\boldsymbol{C}$ is skew-symmetric, and 2) $\dot{\boldsymbol{M}} = \boldsymbol{C} + \boldsymbol{C}^\top$. It follows that

$$\dot{V} = -\mu\boldsymbol{v}^\top\boldsymbol{K}\boldsymbol{v} + \mu^{-1}\boldsymbol{v}^\top\boldsymbol{M}\boldsymbol{v} - \mu^{-1}\boldsymbol{u}^\top(\boldsymbol{\Gamma}\mathcal{L}_{22}\otimes\boldsymbol{I}_p)\boldsymbol{u} - \frac{1}{2}\boldsymbol{u}^\top\left((\mathcal{L}_{22}^\top\boldsymbol{\Gamma} - \boldsymbol{\Gamma}\mathcal{L}_{22}^\top)\otimes\boldsymbol{I}_p\right)\boldsymbol{u} + \mu^{-1}\boldsymbol{u}^\top\boldsymbol{C}^\top\boldsymbol{v} \quad (21)$$

Let $\boldsymbol{X} = (\boldsymbol{\Gamma}\mathcal{L}_{22} + \mathcal{L}_{22}^\top\boldsymbol{\Gamma})\otimes\boldsymbol{I}_p$ and $\boldsymbol{Y} = \boldsymbol{Y}_0 - \mu^{-1}\boldsymbol{C}^\top$. Here, $\boldsymbol{Y}_0 = (\mathcal{L}_{22}^\top\boldsymbol{\Gamma} - \boldsymbol{\Gamma}\mathcal{L}_{22}^\top)\otimes\boldsymbol{I}_p$. We can then express $\dot{V}$ in quadratic form as below:

$$\dot{V} = -\begin{bmatrix}\boldsymbol{u}\\ \boldsymbol{v}\end{bmatrix}^\top\begin{bmatrix}\frac{1}{2}\mu^{-1}\boldsymbol{X} & \frac{1}{2}\boldsymbol{Y}\\ \frac{1}{2}\boldsymbol{Y}^\top & \mu\boldsymbol{K} - \mu^{-1}\boldsymbol{M}\end{bmatrix}\begin{bmatrix}\boldsymbol{u}\\ \boldsymbol{v}\end{bmatrix} \quad (22)$$

From Corollary 1 and Lemma 1 we conclude that $\boldsymbol{X}$ is positive definite. The term $\mu\boldsymbol{K} - \mu^{-1}\boldsymbol{M}$ is positive definite if the inequality (19) is satisfied. Then, by Theorem 1, the matrix in (22) is positive definite if

$$\mu\boldsymbol{K} - \mu^{-1}\boldsymbol{M} - \frac{1}{2}\mu\boldsymbol{Y}^\top\boldsymbol{X}^{-1}\boldsymbol{Y} > 0 \quad (23)$$

which is implied by

$$\mu\lambda_{\min}(\boldsymbol{K}) - \lambda_{\max}(\mu^{-1}\boldsymbol{M} + \frac{1}{2}\mu\boldsymbol{Y}^\top\boldsymbol{X}^{-1}\boldsymbol{Y}) > 0 \quad (24)$$

Using the properties of the spectral norms set out below Definition 2, and expression (2), straightforward calculations (which are omitted due to space limitations) show that

$$\lambda_{\max}(\mu^{-1}\boldsymbol{M} + \frac{1}{2}\mu\boldsymbol{Y}^\top\boldsymbol{X}^{-1}\boldsymbol{Y}) \leq \frac{1}{2}\mu\psi + \beta + \mu^{-1}(\frac{1}{2}\epsilon + k_{\overline{M}}) \quad (25)$$

where $\psi = \|\boldsymbol{X}^{-1}\|_2\|\boldsymbol{Y}_0\|_2{}^2$, $\beta = \|\boldsymbol{X}^{-1}\|_2\|\boldsymbol{Y}_0\|_2\|\boldsymbol{C}\|_2$ and $\epsilon = \|\boldsymbol{X}^{-1}\|_2\|\boldsymbol{C}\|_2{}^2$. It then follows that, using (25), the inequality in (24) is implied by

$$\mu^2\left[2\lambda_{\min}(\boldsymbol{K}) - \psi\right] - 2\mu\beta - (\epsilon + 2k_{\overline{M}}) > 0 \quad (26)$$

Think of this as another constraint on μ, which once again depends on $\boldsymbol{K}$. By designing each $\boldsymbol{K}_i$ such that

$$\lambda_{\min}(\boldsymbol{K}_i) > \frac{1}{2}\psi, \forall\, i \quad (27)$$

then the coefficient of μ^2 is always positive; this guarantees the existence of μ satisfying (26) and for which $\mu > 0$. In turn, this ensures (24) and then (23) holds, which ensures that $\dot{V}$ is negative definite. Because (23) holds, so does (19) and V is positive definite. Since $\|\boldsymbol{C}\|_2$ is a function of $|\boldsymbol{v}|$, in the following section we explicitly derive an upper bound on $|\boldsymbol{v}|$, and thus $\|\boldsymbol{C}\|_2$, using Ω. In summary, if (27) and (26) are satisfied then the matrix in (22) is positive definite. We then conclude that $\dot{V}$ and V are, respectively, negative definite and positive definite in the variables $\boldsymbol{u}$ and $\boldsymbol{v}$.

From the equations for V and $\dot{V}$, we conclude that $\dot{V} \leq -\alpha V$ for some strictly positive scalar α. From this, it follows that V decays exponentially fast to zero, at least as fast as $e^{-\alpha t}$. From the fact that V is positive definite in $\boldsymbol{u}$ and $\boldsymbol{v}$, we then conclude that $\boldsymbol{u}$ and $\boldsymbol{v}$ decay exponentially fast to zero and thus the rendezvous objective is achieved. □

Remark 1 (The scalar α)**.** *Let us call the matrices in* (15) *and* (22)*,* $\boldsymbol{G}$ *and* $\boldsymbol{H}$ *respectively. From the definiteness of* V *and* $\dot{V}$*, we have* $V \leq \lambda_{\max}(\boldsymbol{G}) \parallel \boldsymbol{u}^\top \boldsymbol{v}^\top \parallel^2$ *and* $\dot{V} \leq -\lambda_{\min}(\boldsymbol{H}) \parallel \boldsymbol{u}^\top \boldsymbol{v}^\top \parallel^2$*. Then* $\dot{V} \leq -\alpha V$ *if* $\alpha = \lambda_{\min}(\boldsymbol{H})/\lambda_{\max}(\boldsymbol{G})$*. Note that the nonsingularity of* $\boldsymbol{H}$ *guarantees* α *is positive.*

A. Semi-Global Stability

The explicit upper bound on $|\boldsymbol{v}| = |\dot{\boldsymbol{q}}|$ is derived as follows. Suppose $\boldsymbol{K}_i^*$ is such that it satisfies (27) and notice that any $\boldsymbol{K}_i \geq \boldsymbol{K}_i^*$ also satisfies the inequality. Let $\boldsymbol{R}^* = \boldsymbol{R}$ when $\boldsymbol{K}_i^* = \boldsymbol{K}_i$. Let μ^* be such that it satisfies (19) and note then that any $\mu \geq \mu^*$ will also satisfy the inequality. Since we do not have knowledge of $\boldsymbol{M}_i, \forall i$, we introduce a function $\bar{V}$ which overbounds V, and use this for bounding calculations. Let

$$\bar{V} = \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{v} \end{bmatrix}^\top \begin{bmatrix} \boldsymbol{R} & \frac{1}{2}\mu^{-1}k_M\boldsymbol{I}_{np} \\ \frac{1}{2}\mu^{-1}k_M\boldsymbol{I}_{np} & \frac{1}{2}k_M\boldsymbol{I}_{np} \end{bmatrix} \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{v} \end{bmatrix} \tag{28}$$

Verify using Theorem 1 that for all nonzero values of $[\boldsymbol{u}^\top, \boldsymbol{v}^\top]^\top$ there holds $V < \bar{V}, \forall t$. From Lemma 3, expression (4) and the definition of $\bar{V}$, the following holds

$$|\boldsymbol{v}(0)|^2 \leq \frac{\bar{V}(0)}{\rho} \tag{29}$$

where $\rho = \lambda_{\min}\left(k_M\boldsymbol{I}_{np} - \frac{1}{2}(\mu^*)^{-2}{k_M}^2(\boldsymbol{R}^*)^{-1}\right)$. It continues to hold for all $\mu \geq \mu^*$ and $\boldsymbol{K}_p \geq \boldsymbol{K}_p^*$.

From Ω, there are possible values for $\bar{V}(0)$ that lie in some bounded set. Now suppose that the supremum of the values of $\bar{V}(0)$ over all of Ω is found. Now calculate the maximal set of $\boldsymbol{v}$ corresponding to this supremum $\bar{V}(0)$. For this maximal set, for some arbitrarily small but positive δ, there holds

$$|\boldsymbol{v}|^2 \leq \frac{\sup \bar{V}(0)}{\rho} + \delta \tag{30}$$

This set includes the initial conditions, but in general will be bigger. Using this bound on $\boldsymbol{v}$, which we shall argue is satisfied for all $\boldsymbol{v}$ on the system trajectory, compute $\|\boldsymbol{C}\|_2$. Using this, modify μ^* if required to satisfy (26). The two earlier conditions (19) and (27) will continue to hold for the modified μ^*. We now assert

Lemma 4. *With notation as above, there holds* $\dot{V} < 0$ *for all t, and the bound on* $\|\boldsymbol{C}\|_2$ *applies on the whole trajectory.*

Proof. It is evident that there will be an interval with left point 0 on which $\dot{V} < 0$. Suppose the maximal such interval is $[0, T]$ where $T < \infty$. We shall establish a contradiction. There will be points arbitrarily close to and on the right of T for which $\dot{V} \geq 0$. Up to time T however, there must hold $V(t) < \bar{V}(0)$ and in particular $V(T) < \bar{V}(0)$. The argument leading to (29) will then imply that, for all $t \leq T$, there holds

$$|\boldsymbol{v}(t)|^2 \leq \frac{V(t)}{\rho} < \frac{\bar{V}(0)}{\rho} \tag{31}$$

In the light of our (30), this will imply that the bound on $\|\boldsymbol{C}\|_2$ will apply on $[0, T]$ and by continuity for at least some small interval to the right. By that fact, $\dot{V} < 0$ will continue to apply, i.e. T is not maximal, or a contradiction is obtained. □

Remark 2 (Knowledge of agent dynamics)**.** *Notice that* $\boldsymbol{X}$ *and* $\boldsymbol{Y}_0$ *depend only on the graph topology. From our assumption that the graph topology is fixed and known,* $\boldsymbol{K}_i$ *can be designed first using* (27)*, since it relies only on the graph topology. We then design* μ *to satisfy* (19) *and lastly modify* μ*, if required, to satisfy* (26) *using the bound on* $\|\boldsymbol{C}\|_2$*. Note that the inequalities are conservative; there may be a pair* $\mu, \boldsymbol{K}_i$ *which stabilizes the system but does not fully satisfy the inequalities. The inequalities give control gains which guarantee stability. However, arbitrarily increasing* $\mu\boldsymbol{K}_i$ *adversely affects convergence performance, as we show in the following section.*

Remark 3 (Centralized design)**.** *Since design of the control pair* $\mu, \boldsymbol{K}_i$ *requires knowledge of global information, this algorithm is distributed and model-independent in execution but centralized in design. Previous model-independent algorithms also have similar requirements for global information, see [14]. Existing literature uses adaptive algorithms based on a linear parametrization of the Euler-Lagrange equation to estimate uncertain system parameters. However, this requires precise knowledge of the structure of the Euler-Lagrange equations. The algorithm presented in this paper requires limited knowledge of some bounds on the agent dynamics, the graph topology, and a set of possible initial conditions. The proposed algorithm therefore presents an option with different benefits to existing adaptive algorithms and a trade-off can be made based on application.*

IV. Application to Industrial Robotics

A simulation is provided to demonstrate the distributed algorithm (11) for application to industrial robotic manipulators. In a number of manufacturing processes, industrial robotic arms are required to pick up objects and move them to a machine to undergo a certain process, e.g. friction welding. A smart, programmable leader arm can first move into a position to pick up the object at its workstation. Other robotic arms run the algorithm in (11) to move into position at their respective workstations. A similar procedure is used for placing the object in the machine. An advantage in this setup is the need to only program a single leader, even when changing the motion for a new process.

Each agent is a two-link robotic arm and five agents rendezvous to the arm angles of the leader agent. A model is shown in Fig. 1 and the equations of motion are given in [15], pp. 259-262. Recall that we have assumed $\boldsymbol{g}_i = \boldsymbol{0}$, $\forall i$. The generalized coordinates for agent i are $\boldsymbol{q}_i = [q_i^{(1)}, q_1^{(2)}]^\top$, which are the angles of each link in radians. The parameters are shown in Table I with SI units of kg for m_1, m_2, units of m for l_1, l_2, l_{c1}, l_{c2}, and $\mathrm{kg\,m^2}$ for I_1, I_2. The initial conditions of the system are also provided in Table I. The directed graph has the following Laplacian

$$\mathcal{L} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & -1 & 0 & 2 \end{bmatrix} \quad (32)$$

and see that it contains a spanning tree rooted at agent 1 with no incoming edges to agent 1. The control gain pair for agents 2,...,5 is $\mu\boldsymbol{K}_i = \boldsymbol{I}_2$. For the above Laplacian, $\boldsymbol{\Gamma} = \boldsymbol{I}_{n-1}$ satisfies Corollary 1.

Figure 2 shows rendezvous of the generalized coordinates $q^{(1)}$ and $q^{(2)}$ to Agent 1. The generalized velocities, $\dot{q}^{(1)}$ and $\dot{q}^{(2)}$, all tend to zero in Fig. 3. The control torques are shown in Fig. 4. Recall from Theorem 3 that we require a sufficiently large $\mu\boldsymbol{K}$ for stability. Figure 5 shows results of a simulation where the only change is that $\mu\boldsymbol{K}_i = 0.01\boldsymbol{I}_2$, $\forall i$. We see the divergence of the generalized coordinates due to $\mu\lambda_{\min}(\boldsymbol{K}_i)$ being insufficiently large. Conversely, increasing $\mu\boldsymbol{K}_i$ to $\mu\boldsymbol{K}_i = 5\boldsymbol{I}_2$, $\forall i$ degrades performance by increasing the time required for rendezvous, as shown in Fig. 6. This is intuitive by seeing $\mu\boldsymbol{K}$ as a damping term in the nonlinear system (12); as the damping increases then the transient response is slowed. Multiplying the term $\boldsymbol{\Gamma}$ by a positive scalar increases the system oscillations. This is intuitive if we consider the term in (12) involving $\boldsymbol{\Gamma}\mathcal{L}_{22}$ to be the spring in a nonlinear spring-mass-damper system.

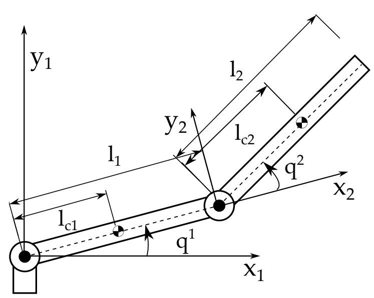

Figure 1. Two-link arm with generalized coordinates $\boldsymbol{q} = [q^1, q^2]^\top$

V. Conclusion

In this paper, we propose a model-independent algorithm to achieve rendezvous to a stationary leader for a directed network of agents with Euler-Lagrange self-dynamics. We prove that exponentially fast convergence to the rendezvous objective, requires the directed network to contain a directed

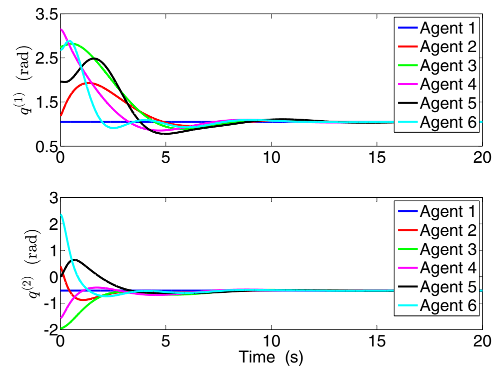

Figure 2. Plot of generalized coordinates vs. time

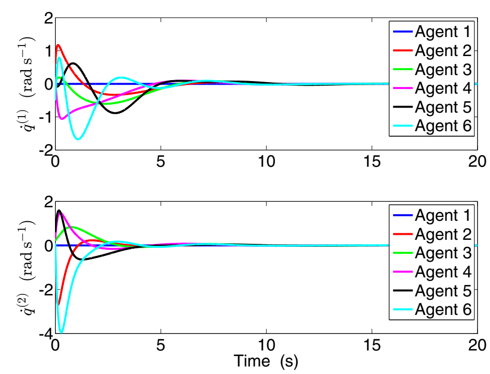

Figure 3. Plot of generalized velocities vs. time

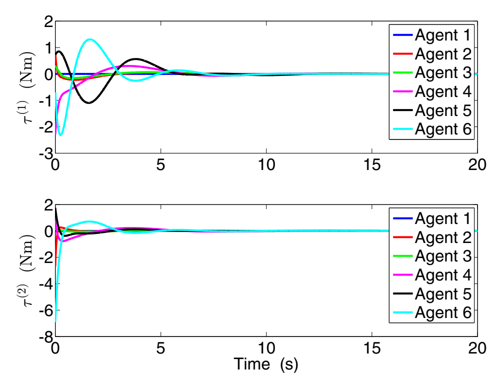

Figure 4. Plot of control input torque vs. time

spanning tree with the leader as the root node. Furthermore, stability requires a control gain matrix and control gain scalar to satisfy a set of inequalities; design of the algorithm therefore requires some limited knowledge on the bounds of the parameters describing the agent self-dynamics, a fixed topology and a fixed but arbitrarily large set of initial conditions. Numerical simulations are provided to show the algorithm effectiveness. We also show the instability and divergence of the agents from the leader when the control

Table I
AGENT PARAMETERS USED IN SIMULATION

	m_1	m_2	l_1	l_2	l_{c1}	l_{c2}	I_1	I_2	$q_i^{(1)}(0)$	$q_i^{(2)}(0)$	$\dot{q}_i^{(1)}(0)$	$\dot{q}_i^{(2)}(0)$
Agent 1	0.5	0.4	0.4	0.3	0.2	0.15	0.1	0.05	$\pi/3$	$-\pi/6$	0	0
Agent 2	0.2	0.4	0.6	0.1	0.35	0.08	0.15	0.08	$3\pi/8$	$\pi/8$	0.6	-0.6
Agent 3	0.5	0.4	0.4	0.3	0.2	0.15	0.1	0.05	$7\pi/8$	$-5\pi/8$	0.1	0.2
Agent 4	1	0.6	0.45	0.8	0.2	0.4	0.15	0.5	π	$-\pi/2$	0.2	0.3
Agent 5	0.25	0.4	0.8	0.5	0.3	0.1	0.45	0.15	$5\pi/8$	0	0	0.6
Agent 6	0.5	0.4	0.6	0.25	0.3	0.1	0.15	0.3	$6\pi/7$	$-\pi/8$	-0.5	0.1

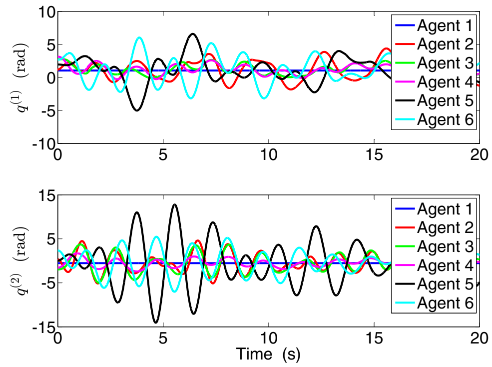

Figure 5. Plot of generalized coordinates vs. time, $\mu \boldsymbol{K}_i = 0.01\boldsymbol{I}_2$

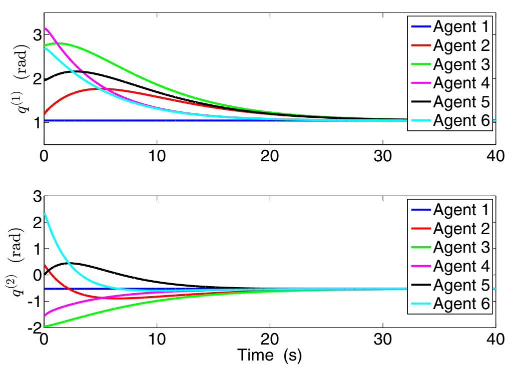

Figure 6. Plot of generalized coordinates vs. time, $\mu \boldsymbol{K}_i = 5\boldsymbol{I}_2$

gain pair does not satisfy the lower bounding inequalities. A number of directions exist for future work. A major focus will be on introducing the gravitational term and finding a model-independent method to account for its effect. Other future works include considering effects such as time-delay in the network or study of switching topologies.

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