INPUT CONDITIONS FOR CONTINUOUS-TIMEADAPTIVE SYSTEM PROBLEMS

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Abstract

The exponential convergence of certain algorithms used in *continuous-time adaptive identification* and control is investigated by deriving persistence of excitation conditions which depend on the system inputs exclusively. Stable plants and unstable plants with external stabilizing control are considered in the identification study. For the control algorithm considered, it is shown that in the absence of foreknowledge of a gain parameter, exponential stability is impossible to claim.

1. Introduction

This paper seeks to establish the exponential convergence *of a class of continuous-time* algorithms used in adaptive identification and control. At the outset we stress the need for obtaining exponential convergence as opposed to simple asymptotic convergence for a variety of reasons. To begin with, exponential convergence imparts a measure of robustness by making algorithms substantially tolerant of noise, errors in modelling and various other inadequacies of realization. Secondly, in situations where adaptive algorithms are invoked for tracking *slowly varying plant parameters*, a convergence rate less than exponential in the time-invariant case can scarcely be expected to translate into satisfactory performance in the time-varying case. Furthermore, proofs of exponential convergence frequently simultaneously yield additional information regarding actual rates of convergence; such information generally does not come as a byproduct of proofs of simple convergence.

The existing results for continuous-time plants derived variously by authors such as Morgan and Narendra [1,2], Kreisselmeier [3], Sondhi and Mitra [4] and Anderson [5,6] involve conditions for exponential convergence which depend on both inputs and outputs. In practice, however, their dependence on the system outputs renders them ill-suited to the task of input design; and to our knowledge no criterion for exponential convergence, explicitly independent of system outputs, has as yet been derived. Indeed, the closest attempt to this end had been made by Yuan and Wonham [7] who developed techniques for input design based, nevertheless, on *conditions similar to those established in* [1-6].

The first contribution (Section 2) of this paper is to use the criteria given in [6] as the basis for deriving a convergence condition on input signals alone for the identification of a stable plant. This is similar to a condition derived in [8] for discrete-time plants, and for inputs which are linear combinations of sinusoids; the result is directly in accord with intuition. In Section 3, we extend the ideas to consider plants which are not necessarily stable but which are embedded in a larger system which has *an overall stability property*. (Such an analysis has not been done in discrete time.) The results of Section 2 require adjustment: plants with poles on $Re[s] = 0$ can cause a problem. In Sections 4 and 5, *we consider* also the exponential stability of a model reference adaptive control algorithm, studied by Morse [9], by examining in turn the two separate situations where (a) the constant gain of the process transfer function is known and (b) where no such information is available a priori. In the former case exponential stability *is shown to be conditional* on the satisfaction of a persistence of excitation condition on the reference input while in the latter such stability is shown as impossible to achieve, even with a persistence of excitation condition.

Finally we remark, that owing to space constraints we have stated the main results only, and have left out their proofs and all lemmata required for those proofs.

2. Identification of Stable Plants

Consider an n-dimensional stable, single-input, single-output system

$$y^{(n)} + \sum_{i=0}^{n-1} a_i y^{(i)} = \sum_{j=0}^{m} b_j u^{(j)} \qquad (m<n) \qquad (2.1)$$

with all *derivatives of the input and output* being filtered through state variable filters of the form $\frac{1}{(s+\alpha)^n}$.

Define $V = [y_{n-1}, y_{n-2}, \ldots, y_0, u_m, u_{m-1}, \ldots, u_0]^T$ with $y_i = s^i y/(s+\alpha)^n$ and $u_i = s^i u/(s+\alpha)^n$, where the abuse of notation has been introduced for the sake of notational convenience.

In the sequel, we shall often require vector time functions such as V to belong *to a special set* $\Omega_\Delta[0,\infty)$, defined below. (The definition is for scalar functions; a vector or matrix function belongs to the set if each entry belongs to the set.) The importance of this set in identification problems was first illustrated in [7].

Definition 2.1:

C_Δ is a set $\{t_i\}$ of points in $[0,\infty)$ for which there exists a Δ such that for any $t_i, t_j \in C_\Delta$ with $t_j \neq t_i$, one has $|t_i - t_j| > \Delta$. (i.e. C_Δ comprises points spaced at least Δ apart).

Definition 2.2:

A function v(.) belongs to Ω_Δ if there corresponds *some* Δ *and some* C_Δ such that

(1) $v(t)$ and $\dot{v}(t)$ are continuous and bounded on $\{[0,\infty) - C_\Delta\}$

(2) $v(t)$ and $\dot{v}(t)$ have finite limits as $t \downarrow t_i$ and $t \uparrow t_i$, $t_i \epsilon C_\Delta$.

In other words functions in Ω_Δ are smooth enough to have bounded continuous derivatives, save that at a countable number of points, finite-step switchings are allowed, as long as such switchings do not occur too frequently.

For $V(t) \epsilon \Omega_\Delta[0,\infty)$ it has been shown in [6], that with minor modifications, virtually all known equation error identification schemes are exponentially convergent if and only if for some positive δ, α_1 and α_2 and for all $S \epsilon R_+$

$$\alpha_1 I < \int_s^{s+\delta} V(t)\, V^T(t)dt < \alpha_2 I \qquad (2.2)$$

The main result of this section, which we now state, stipulates conditions on the input which imply and are implied by the inequalities in (2.2). The proof of this and other results in this paper exploits a possibly little known but potentially very useful inequality relating derivatives of functions of one variable [see [10]].

Theorem 2.1: Consider an asymptotically stable n-dimensional single-input, single-output system with

$$y^{(n)} + \sum_{i=0}^{n-1} a_i y^{(i)} = \sum_{j=0}^{m} b_j u^{(j)} \qquad (2.3)$$

$m < n$, the polynomials $A(s) = s^n + \sum_{i=0}^{n-1} a_i s^i$ and $B(s) = \sum_{j=0}^{m} b_j s^j$

coprime, $u(t) \epsilon \Omega_\Delta[0,\infty)$ and $V(t)$ and $W(t)$ defined as

$$V \triangleq [y_{n-1}, \ldots y_0, u_m, \ldots u_0]^T \qquad (2.4)$$

$$W \triangleq [u, \frac{u}{s+\beta}, \frac{u}{(s+\beta)^2}, \ldots, \frac{u}{(s+\beta)^{n+m}}]^T$$

$$(2.5)$$

where $y_i = s^i y/(s+\alpha)^n$ and $u_i = s^i u/(s+\beta)^n, \beta, \alpha > 0$. If there exist some positive α_1, α_2 and δ' independent of σ, such that

$$\alpha_1 I < \int_\sigma^{\sigma+\delta'} W(t)W^T(t)dt < \alpha_2 I \qquad (2.6)$$

for all $\sigma \epsilon R_+$, then there exist positive α_3, α_4 and a suitably large $\delta > \delta'$ also independent of σ such that

$$\alpha_3 I < \int_\sigma^{\sigma+\delta} V(t)V^T(t)dt < \alpha_4 I \qquad (2.7)$$

Conversely, if there exist some positive α_3, α_4 and δ'' independent of σ, such that

$$\alpha_5 I < \int_\sigma^{\sigma+\delta''} V(t)V^T(t)dt < \alpha_6 I \qquad (2.8)$$

for all $\sigma \epsilon R_+$, then there exist positive α_7, α_8 and a suitably large $\delta > \delta''$ independent, as before, of σ such that

$$\alpha_7 I < \int_\sigma^{\sigma+\delta} W(t)W^T(t)dt < \alpha_8 I \qquad (2.9)$$

Remarks:

(2.1) It is worth noting that the satisfaction of (2.2) for some $\delta = \delta_0$ implies the same for all $\delta > \delta_0$, indeed this applies to (2.6-2.9) and all similar inequalities.

(2.2) Theorem 2.1 demonstrates that the input-only conditions (2.6) and (2.8) are necessary and sufficient to assure the exponential convergence of most equation error identification schemes, at least for stable plants.

(2.3) The reduction in intervals of (2.6) and (2.8) as compared with (2.7) and (2.9) is essentially due to the need for transients associated with nonzero boundary conditions at $t=\sigma$ to die away.

(2.4) The system (2.1) is described by a total of m+n+1 parameters, (once the integers m,n have been specified). Now application of a single sinusoid to a stable linear system allows identification of the real and imaginary parts of the transfer function at one frequency. More generally, application of

$$u(t) = \sum_{i=1}^{q} u_i e^{j\omega_i t} \qquad (2.10)$$

where the ω_i are real, $\omega_i \neq \omega_j$ for $i \neq j$, $u_i \neq 0$ and $u(t)$ is real allows identification of q pieces of information about the transfer function.

Thus for the identification problem being studied q must exceed m+n. Indeed it is not difficult to show that theorem 2.1 encompasses this result.

3. Identification of Unstable Plants Inside an Overall Stable System

Not only do the arguments of the preceding section use the boundedness of signals in stable plants, but the stability of those plants is also used. Throughout this section, we shall consider the plant still described by (2.1), but such that the zeros of $s^n + \sum_{i=0}^{n-1} a_i s^i$ are not necessarily in $Re[s] < 0$.

As a standing assumption, we take the plant to be part of an overall system which is stable, and in which all signals are bounded. We shall find that while the first part of theorem 2.1 (2.6)=>(2.7)) remains unaltered the second part ((2.8)=>(2.9)) needs adjustment.

Note that whereas the main result of the last section paralleled, in its statement though not its proof, a similar result for discrete-time systems [8], no discrete-time result parallel to the result of this section has been stated.

Theorem 3.1: Assume the plant is described by (2.1), with $A(s) = s^n + \sum_{i=0}^{n-1} a_i s^i$ and $B(s) = \sum_{j=0}^{m} b_j s^j$ coprime, $u(t)\epsilon\Omega_\Delta[0,\infty)$. Let $A(s)$ have no more than p zeros with zero real part, and thus at least n-p zeros with non-zero real part. Define

$$V(t) \triangleq [y_{n-1} \cdots y_0 \quad u_m \cdots u_0]^T \qquad (3.1)$$

$$W(t) \triangleq [u \ \frac{u}{(s+\beta)} \cdots \frac{u}{(s+\beta)^{n+m}}]^T \qquad (3.2)$$

$$\text{and } \bar{W}(t) \triangleq [u, \frac{u}{(s+\beta)} \cdots \frac{u}{(s+\beta)^{n+m-p}}]^T \qquad (3.3)$$

where

$$y_i = s^i y/(s+\alpha)^n, \quad u_i = s^i u/(s+\alpha)^n, \quad \beta,\alpha > 0.$$

If there exist positive α_1 and δ' such that

$$\alpha_1 I < \int_\sigma^{\sigma+\delta'} W(t)W^T(t)dt \qquad (3.4)$$

for all $\sigma\epsilon R_+$, then there exist a positive α_2 and a suitably large $\delta>\delta'$, also independent of σ, such that

$$\alpha_2 I < \int_\sigma^{\sigma+\delta} V(t)V^T(t)dt \qquad (3.5)$$

Conversely if there exist a positive α_3 and δ' such that

$$\alpha_3 I < \int_\sigma^{\sigma+\delta'} V(t)V^T(t)dt \qquad (3.6)$$

for all σ, then there exist a positive α_4, a σ_0 and suitably large $\delta>\delta'$, also independent of σ, such that

$$\alpha_4 I < \int_\sigma^{\sigma+\delta} \bar{W}(t)\bar{W}^T(t)dt \qquad (3.7)$$

for all $\sigma > \sigma_0$.

Remarks: (3.1) If nothing is known about the zeros of $A(s)$, one must assume p=n in applying this theorem.

(3.2) Theorem 3.1 shows that if $u(.)$ is a real linear combination of imaginary exponentials, m+n+1 different exponents are sufficient, but only m+n+1-p may be necessary, to identify the system. Nondecaying, nongrowing components of the system output stemming from nonzero initial conditions may make up the remaining information required to give the spanning condition on $V(.)$. Given that the unknown system is part of an overall stable system, such special output components can only be present if the external input to the overall stable system contains a sinusoidal component at each of the relevant frequencies. We return to the question of conditions on the external input below.

Theorems 3.1 relates $V(t)$, the vector which is crucial in the adaptive parameter adjustment law, to the input u of a subsystem of a larger system, itself possessing an external input, r. It is relevant to ask what conditions on r imply condition (3.4) on W (which in turn implies condition (3.5) on V, guaranteeing exponential convergence of an equation error identifier).

The standing assumption for the section implies that

$$u = \frac{b(s)}{a(s)} r \qquad (3.8)$$

for some $a(s)$ with all zeros in $Re[s] < 0$ and with b/a proper. Note that it may be the case that b,a depend on the parameters of the subsystem, and so may not be known. We can get insight into the result (Theorem 3.2 below) by considering the effect of

$$r(t) = \sum_{i=1}^{N} \bar{r}_i e^{j\omega_i t}.$$ This ensures that $$u(t) = \sum_{k=1}^{M} \bar{u}_k e^{j\omega_k t}$$ where $M < N$ is possible if $b(s)$

has purely imaginary zeros, and if the ω_i are such that $j\omega_i$ is a zero of $b(s)$. Notice that if $A(s)$, the denominator of the plant or unknown subsystem transfer function, has $j\omega$-axis zeros, $b(s)$ necessarily has such zeros when the overall setup is stable. Now in order that a condition like (3.4) hold for $u(t)$ above it is necessary and sufficient that $M = m + n + 1$. In the absence of any information about the zeros of $b(s)$, one needs

$$N = m + n + 1 + \deg b(s). \qquad (3.9)$$

as up to $\deg(b(s))$ of the ω_i's may coincide with the zeros of $b(s)$.
The above argument is developed for sinusoids. The content of the next theorem shows that it is in fact more general.

Theorem 3.2: Assume that the overall system input $r\epsilon\Omega_\Delta[0,\infty)$ is related to the input of the plant being identified by (3.8), with b/a proper and $a(s)$ possessing all zeros in $Re[s] < 0$. Suppose that the number of $j\omega$-axis zeros of $b(s)$ is no greater than ρ ($\rho = \deg b(s)$ is a possibility). Define

$$R(t) = [r \ \frac{r}{(s+\gamma)} \cdots \frac{r}{(s+\gamma)^{n+m+\rho}}]^T \qquad (3.10)$$

with $\gamma>0$ and suppose that for some positive α_1,α_2 and δ, and for all σ

$$\alpha_1 I < \int_{\sigma}^{\sigma+\delta} R(t)R^T(t)dt < \alpha_2 I \qquad (3.11)$$

Then there exist positive α_3, α_4 such that for some $\delta' > \delta$, independent of σ, with $W(t)$ defined as in (3.2),

$$\alpha_3 I < \int_{\sigma}^{\sigma+\delta'} W(t)W^T(t)dt < \alpha_4 I \qquad (3.12)$$

for all σ.

Up to now we have been concerned with relating inputs to vectors which mix input and output quantities and which are relevant in identification.

For completeness, we remark that their exists a result, directly in accord with intuition, which ties together outputs, a vector mixing input and output quantities, and the inputs. Note that such a tie has proved useful in considering a trajectory-following adaptive control problem in discrete time [8]. Rather than stating it in a theorem form, we illustrate it through the self explanatory figure 3 where

$$Y(t) = [\frac{y}{(s+\beta)^{n+m}} \cdots y]^T$$

$$\bar{Y}(t) = [\frac{y}{(s+\beta)^{n+m-z}} \cdots y]^T,$$

and $W, \bar{W}$ and V as defined in theorem 3.1. (N.B. In the figure X=>U means that the boundedness of a gramian with X and a given δ^1 implies the same for a gramian involving U and a $\delta > \delta^1$.

4. Model Reference Control: Known Gain

In the remainder of this paper we investigate the exponential stability of a model reference adaptive control scheme, proposed by Morse [9]. At the outset we briefly outline the philosophy and nature of the scheme, adhering closely to the terminology employed in [9], but omitting details which do not bear direct relevance to the course of our development.

Consider the task of controlling a single-input single-output process, modelled by a strictly proper transfer function

$$T_p = g_p \frac{\alpha_p(s)}{\beta_p(s)} \qquad (4.1)$$

having degree and relative degree of n and n^* respectively; g_p is a nonzero constant and α_p and β_p are monic, strictly stable, coprime polynomials but are otherwise unknown. It is assumed that the output $y(t)$ of this system is required to follow a reference trajectory $y_r(t)$, generated in turn as the output of a system defined by equation (4.2) below, $r(t)$ being a bounded, piecewise continuous reference input and T_r a known stable transfer function having a relative degree no smaller than n^*. Then following Morse, it can be shown that the signal e in Figure 4.1 represents, within an exponentially decaying term, the measurable error $y-y_r$. Among the other quantities in this diagram $\theta^T(t) \equiv [\theta_u^T, \theta_y^T, \theta_r]$ is a known signal generated indirectly by $r(t)$, through the equations (4.2) to (4.6), $c(sI-A)^{-1}b$ is a realization of an arbitrary known stable all-pole transfer function $(1/\beta_r)$ having degree n^*, and $k = \hat{k}-k_p$ is the estimation error in $k_p = [k_u^T, k_y^T, k_r]^T$, the latter being related to the fixed unknown parameters of the system with k_r in particular being equal to $1/g_p$.

$$y_r = T_r(s)r \qquad (4.2)$$

$$\dot{\theta}_u = A_0\theta_u + b_0 u \qquad (4.3)$$

$$\dot{\theta}_y = A_0\theta_y + b_0 y \qquad (4.4)$$

$$u = \hat{k}^T\theta \qquad (4.5)$$

$$\theta_r = \beta_r(s)T_r(s)r \qquad (4.6)$$

In the above u is the scalar control input to the plant; $[A_0, b_0]$ define a completely controllable n dimensional stable system and $\beta_r T_r$ and T_r are proper and strictly proper transfer functions respectively.

Morse's algorithm then involves in essence the addition of a signal to the actual error e to generate an augmented error e', illustrated in Figure 4.2, the use of which, instead of e, facilitates considerably the task of implementing the updating algorithm for $\hat{k}$. In our exposition we deal only with what Morse terms his second parameter adjustment law, defined by the following equations and represented schematically in Figure 4.3. (Similar conditions can be expected to apply in respect of his more complicated first parameter law.)

$$\dot{\hat{k}} = -Q\phi_2 e_2 \qquad (4.7a)$$

$$e_2 = \frac{1}{\lambda_0+\phi_2^T Q\phi_2}(\hat{g}_p\psi_2+e) \qquad (4.7b)$$

$$\psi_2 = \hat{k}^T\phi_2 - cz_2 \qquad (4.7c)$$

$$\dot{z}_2 = Az_2 + b\hat{k}^T\theta \qquad (4.7d)$$

$$\phi_2^T = cH_2$$

$$\dot{H}_2 = AH_2 + b\theta^T \qquad (4.7e)$$

$$\dot{\hat{g}}_p = -q\psi_2 e_2 \qquad (4.7f)$$

Here Q and q are a preselected positive definite matrix and a positive scalar constant respectively. λ_0 is also positive and obeys the relation $\beta_r = (s+\lambda_0)\beta_1$.

It is easily seen that as $k_p (=\hat{k}-k)$ is time invariant, the signal $(\lambda_0 + \phi_2^T Q\phi_2)e_2$ in Figure 4.3

is identical to e' in Figure 4.2 whenever $\hat{g}_p$ equals g_p. And indeed in this section we consider the case where g_p is known a priori so that $\hat{g}_p$ can be set directly equal to g_p and $\hat{k}_r$ to $1/g_p$. Thus under the assumptions of a known g_p, a fixed $\hat{g}_p = g_p$ and a block diagonal Q of the form

$$Q = \begin{bmatrix} Q_1 & 0 \\ 0 & q_1 \end{bmatrix}$$

with q_1 a scalar and Q_1 an $(n-1) \times (n-1)$ matrix, one can show, specializing equations in [9] (where unknown g_p was assumed) that the resultant error system reduces to

$$\dot{x} = \bar{A}x + \bar{b}(\bar{k}^T\theta + r) \quad (4.8a)$$

$$\theta = \bar{c}x + \bar{d}r + \bar{\varepsilon}(t) \quad (4.8b)$$

$$\dot{H} = AH + b\theta^T \quad (4.8c)$$

$$\phi^T = cH \quad (4.8d)$$

$$\dot{z} = Az + b\bar{k}^T\theta \quad (4.8e)$$

$$\dot{\bar{k}} = -Q_1\bar{\phi}\bar{e} \quad (4.8f)$$

$$\bar{e} = (\lambda_0 + \phi^TQ\phi)^{-1}(g_p\bar{k}^T\bar{\phi} + \varepsilon(t)) \quad (4.8g)$$

Here $\bar{\varepsilon}(t)$ and $\varepsilon(t)$ are exponentially decaying quantities, $(\bar{A},\bar{b},\bar{c},\bar{d})$ is an unknown but strictly stable system and $\bar{k}$ and $\bar{\phi}$ are given by

$$\bar{k} = \begin{bmatrix} I_{2n} & 0 \\ 0 & 0 \end{bmatrix} k \qquad \bar{\phi} = \begin{bmatrix} I_{2n} & 0 \\ 0 & 0 \end{bmatrix} \phi \quad (4.9)$$

Note that the replacement of k and ϕ by $\bar{k}$ and $\bar{\phi}$ respectively becomes possible since, with known g_p, the last element of k becomes zero. The stability results of [9], applicable for unknown g_p, may in this case be trivially modified to yield the Proposition below. Following the Proposition, we indicate in Theorem 4.1 a development of this stability result to reflect exponential stability under a persistence of excitation condition.

Proposition 4.1: For any time $t > 0$ and bounded, piecewise continuous input r(t), the state response of the adaptive control system defined by (4.8) and (4.9) is uniformly bounded and the quantities $\bar{e}$ and $\dot{\bar{k}}$ decay asymptotically to zero.

Theorem 4.1: For a reference input $r(t) \in \Omega_\Delta[0,\infty)$ and the adaptive control system with known g_p defined by (4.8) and (4.9), the quantities k and $\bar{e}$ approach zero exponentially fast, provided that for some α_{11}, α_{12} and $\Delta > 0$ and all $\sigma \in R^+$ the following relation holds:

$$\alpha_{11}I < \int_\sigma^{\sigma+\Delta} R(t)R^T(t)dt < \alpha_{12}I \quad (4.10)$$

where

$$R = \left[r, \frac{r}{s+\bar{\beta}}, \ldots, \frac{r}{(s+\bar{\beta})^{2n+n_r-1}} \right]^T$$

$\bar{\beta}$ is any positive number, and n_r is the number of imaginary axis zeros of $T_r(s)$.

Remark: 4.1) It is evident from figure 4.1, that the exponential decay of k leads to the exponential decay of the tracking error e. Thus the condition in (4.10) also guarantees the exponential convergence of e.

5. Model Reference Control: Unknown Gain

In this section we consider Morse's algorithm for unknown gain parameter g_p. Using an implication of exponential convergence we show that such a scheme cannot be exponentially convergent.

Let us define $x_0(t)$, $\theta_0(t)$, H_0, $\phi_0(t)$ as the values obtained for $x(.)$, $\theta(.)$, $H(.)$ and $\phi(.)$ if $k \equiv 0$ and $g = \hat{g}-g_p \equiv 0$. Then x_0, θ_0 etc can be regarded as prescribed functions of time and the error system is the same as in (4.8) with the following modifications.

(i) (4.8g) should read

$$\bar{e} = \frac{1}{\lambda_0+\phi^TQ\phi}[g_pk^T\phi + g\psi + \varepsilon(t)]$$

(ii) The following equations should be added

$$\psi = k^T\phi - cz$$

$$\dot{g} = -q\psi\bar{e}$$

Thus defining $\tilde{x} = x-x_0$, $\tilde{\theta} = \theta-\theta_0$, $\tilde{H} = H-H_0$, and $\tilde{\phi} = \phi-\phi_0$, we obtain the following modified error equations

$$\dot{\tilde{x}} = \bar{A}\tilde{x} + \bar{b}(k^T\tilde{\theta} + k^T\theta_0)$$

$$\tilde{\theta} = \bar{c}\tilde{x} + \bar{\varepsilon}(t)$$

$$\dot{\tilde{H}} = A\tilde{H} + b\tilde{\theta}^T$$

$$\tilde{\phi}' = c\tilde{H}$$

$$\dot{z} = Az + bk^T\tilde{\theta} + bk^T\theta_0 \quad (5.1)$$

$$\psi = k^T\tilde{\phi} + k^T\phi_0 - cz$$

$$\dot{k} = -Q(\tilde{\phi} + \phi_0)e$$

$$\dot{g} = -q\psi\tilde{e}$$

$$\tilde{e} = \frac{1}{\lambda_0(\phi_0+\tilde{\phi})^T Q(\phi_0+\tilde{\phi})}\left[g_p k^T(\phi_0+\tilde{\phi}) + g\psi + \varepsilon(t)\right]$$

The state variable set now becomes

$$w \triangleq \begin{bmatrix} \tilde{x} \\ \tilde{\tilde{H}} \\ z \\ k \\ g \end{bmatrix}$$

where $\tilde{\tilde{H}}$ is $\tilde{H}$ rearranged as a big column vector. It is easily seen by studying (5.1) that if k and g approach zero exponentially fast, so must $\tilde{x}$, $\tilde{H}$ and z.

Thus if we regard $\dot{w} = f(w,t)$ as representing (5.1) where the time dependence comes about through the influence ultimately of r and more directly through the immediate inputs of the equations corresponding to the variables concerned then we can show using a result in [11, p86] that the linearization of (5.1) around the zero trajectory will likewise be exponentially stable.

We note, however, that the equation involving g associated with this linearization is [because $|\dot{\psi}|$ and $|\tilde{e}|$ are both $O(\|w\|)$] $\dot{g} = 0$, which clearly is not exponentially stable. It is impossible therefore for the quantities k and g to be exponentially stable.

6. Conclusion

We have established persistence of excitation conditions on the inputs for exponential convergence of adaptive identification algorithms and of a particular adaptive control algorithm with known gain parameter, under the assumption that the inputs satisfy certain criteria for smoothness. We have also demonstrated that for systems with unknown gain parameters exponential stability is impossible to claim, even when the reference input satisfies this persistence of excitation condition. Whether or not the adaptive control results apply to other adaptive control algorithms is in principle an open question. However, in view of the broad similarity of the algorithms of [9,11], for example, we would expect the conclusion to be general.

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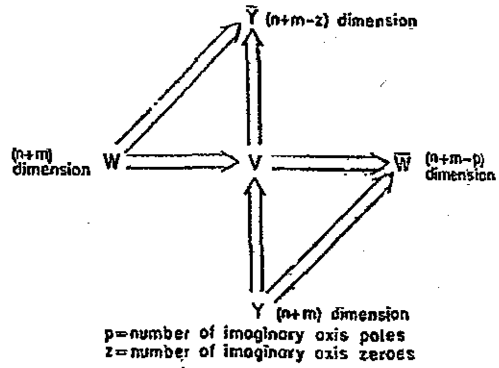

Fig.3: Results in Section 3.

Fig.4.1: Representation of the tracking error e.

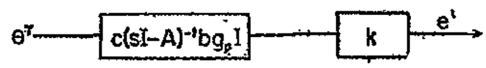

Fig.4.2: Representation of augmented error e'

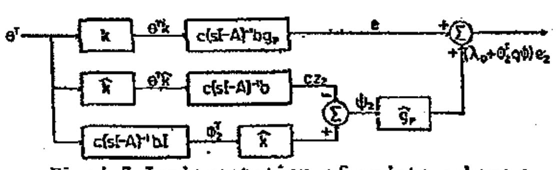

Fig:4.3 Implementation of update scheme