Cooperative Information Forwarding in Vehicular Networks Subject to Channel Randomness

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Abstract—**This paper investigates the information dissemination process in wireless communication networks formed by vehicles. As vehicles are moving constantly, a vehicular ad-hoc network exhibits a highly-dynamic network topology and a fast-changing radio environment. These distinguishing characteristics result in random and unreliable wireless connections between vehicles. Consequently, vehicles need to work cooperatively to disseminate a piece of information to the destination. This paper analyses cooperative information forwarding schemes where each vehicle determines whether or not to forward a received packet in a decentralized manner, without the costly or even impractical demand for the knowledge of network topology. Considering a generic wireless connection model incorporating wireless channel randomness, analytical results are derived for the probability of successful delivery and the expected number of packet forwardings. Moreover, analysis is conducted on the optimal information forwarding scheme that meets a pre-designated probability of successful delivery objective using the minimum number of packet forwardings.**

Index Terms—**vehicular ad-hoc network; cooperative forwarding; channel randomness; random connection model**

I. Introduction

With recent developments of telecommunication technologies, vehicle-to-vehicle communication has been taking an increasingly important role in improving safety, productivity and environmental performance of our transport system [1]. This paper considers vehicular ad-hoc networks (VANETs) formed by vehicles travelling on a highway and investigates the information dissemination process using cooperative information forwarding schemes. Information dissemination in VANETs is a challenging task mainly due to the highly dynamic wireless connections between vehicles, which are attributable to two key factors: dynamic network topology and wireless channel randomness.

The network topology of a VANET exhibits high dynamics as vehicles are constantly moving. In particular, the distances between vehicles are changing from time to time, where a wireless connection between two vehicles only appears when vehicles move close to each other and disappears when two vehicles move far apart. Fast changing network topology makes traditional routing mechanisms (e.g. AODV [2] or DSR [3]) ineffective, because they require time to learn the network topology to determine all relays connecting source and destination before any actual data transmission.

In addition to the fast-changing network topology, wireless channel randomness is another key factor affecting the wireless connections between vehicles. When a vehicle moves along the road, its surrounding radio environment varies as it passes by buildings and trees. Therefore, the condition of the wireless channel between two vehicles can vary from time to time even when their distance remains unchanged. Moreover, it is shown by Boban et al. [4] that nearby vehicles moving on the road can also significantly change the condition of wireless channel between two vehicles. Consequently, the information dissemination schemes for VANETs need to take channel randomness into account.

Though wireless communication brings a number of challenges, it also comes with potential benefits: a wireless transmission is naturally a broadcast transmission. When a vehicle transmits a packet, there can be more than one vehicle who receive the packet successfully. These vehicles can then forward the packet again, resulting in multiple information forwarding paths as illustrated in Fig. 1. Multiple paths of information forwarding can significantly improve the reliability of information dissemination compared with that using a single path, when the cost (in terms of the number of forwardings) is carefully controlled.

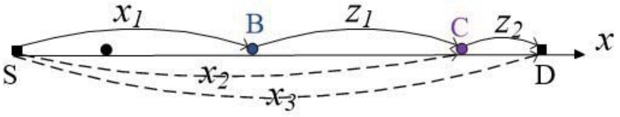

Fig. 1. An illustration of an information dissemination process from a source S to a destination D. A curve between two nodes represents a successful transmission of a packet. For example, Nodes B and C, at distance x_1 and x_2 from node S respectively, receive the packet from node S at the same time. There are three information forwarding paths: $SBCD$, SCD and SD.

This paper investigates a cooperative information forwarding scheme for VANETs, where vehicles work cooperatively to forward a piece of information to its destination. Further, each vehicle determines whether or not to forward a received packet in a decentralized manner without the knowledge of network topology, which can be costly or even impractical to obtain in VANETs taking into account channel randomness and highly dynamic network topology. The main contributions of this paper include: 1) a branching process model for the multiple-path information dissemination process using a

This research is supported by ARC Discovery projects DP110100538 and DP120102030. National ICT Australia (NICTA) is funded by the Australian Government as represented by the Department of Broadband, Communications and the Digital Economy and the Australian Research Council through the ICT Centre of Excellence program. Dr Tao Han would like to acknowledge the support from the National Natural Science Foundation of China (NSFC) under the grants 61271224, NFSC Major International Joint Research Project under the grant 61210002, and the Ministry of Science and Technology (MOST) of China under the grants 0903, the Hubei Provincial Science and Technology Department under the grant 2011BFA004, the Fundamental Research Funds for the Central Universities under the grant 2013QN136. His research is partially supported by EU FP7- PEOPLE-IRSES, Contract/Grant No. 247083, 318992 and 610524.

978-1-4799-2003-7/14/$31.00 ©2014 IEEE

distributed cooperative forwarding scheme is established for VANETs under a generic wireless connection model; 2) an analytical result on the expected number of packet forwardings required to disseminate a piece of information to a destination at a given distance from the source is derived; 3) an upper bound on the probability of successful delivery is derived; 4) guidelines are provided on the design of the optimal forwarding scheme to meet a pre-designated performance objective on the probability of successful delivery using the minimum number of packet forwardings.

The rest of this paper is organized as follows: Section II reviews related work. Section III introduces the system model, including the wireless connection model and the cooperative forwarding scheme. Performance analysis on the information dissemination process is presented in Section IV. Section V validates the analysis using simulations. Section VI concludes this paper and discusses future research directions.

II. Related work

Motivated by the great potential in improving safety and productivity of road transport, there are an increasing number of studies on wireless communication between vehicles [1]. Due to the highly-dynamic network topology that distinguishes VANETs from traditional static networks where nodes do not move, research in this area has mainly focused on distributed information forwarding schemes [5]–[7].

A popular category of distributed information forwarding schemes, for either unicast, multicast or broadcast, is the store-carry-forward scheme [8], using which *every* vehicle carries its received packets while moving, and forwards them to other vehicles coming into its vicinity. The store-carry-forward approach is particularly suitable for delay-tolerant information dissemination in sparse VANETs. However in a dense network, this approach becomes less efficient and can even cause a broadcast storm problem [9]. In view of this, a number of probabilistic information forwarding schemes are proposed, where two main categories are the delay-based forwarding and the probability-based forwarding schemes.

Using a delay-based forwarding approach, after a packet is transmitted by a vehicle, the vehicles receiving this packet wait different amounts of time before relaying the packet. A well-known scheme in this category is proposed by Korkmaz et al. [10], where the key idea is to maximise the packet forwarding progress (viz. the distance between adjacent relays) by letting each vehicle waits an amount of time that is a decreasing function of the source-relay distance (viz. the distance between the relay and the source). The studies in this category [5], [10] usually assumed a simple wireless channel model where once a vehicle (i.e. the one with the shortest waiting time) transmits a packet, all its neighbours can hear the transmission and stop forwarding the same packet, so that the broadcast storm problem can be avoid.

Using probability-based forwarding approaches, each vehicle is assigned with a certain probability to forward its received packet. While the simplest protocol uses a fixed value for the forwarding probability [7], usually the forwarding probability of each relay is assigned as an increasing function of the transmitter-relay distance (viz. the distance between a relay and its corresponding transmitter from which the packet is received) [5], [9], [11]. In particular, Wisitpongphan et al. [9] considered a probability-based cooperative information broadcast scheme where the forwarding probability of each relay is assigned as $p(z) = z/r_0$ where r_0 is the transmission range and z is the transmitter-relay distance. Through simulations, they showed that their broadcast scheme can guarantee 100% reachability (viz. the probability that the information is received by all the vehicles within a given geographical area) while achieving up to 70% reduction in the number of packet transmissions compared with the traditional broadcast approach where each vehicle keeps broadcasting until all its neighbours receive the packet.

A recent work of Panichpapiboon and Cheng [12] assigned the forwarding probability of each relay as $p(z) = (1 - F(r_0 - z))^{1/c}$, where c is a shaping parameter, r_0 is the transmission range, z is the *transmitter-relay distance* (viz. the distance between the relay and its corresponding transmitter from which the packet is received) and $1 - F(r_0 - z)$ is the probability that there is no vehicle to the right of the relay within the transmission range of the transmitter. Using simulations driven by real traffic traces, they showed that their scheme, taking into account not only the distance z but also the vehicular density (in the calculation of $F(r_0-z)$), can guarantee 100% reachability while further reducing the number of forwardings compared with the schemes (e.g. [9]) where the forwarding probability is a linear function of transmitter-receiver distance z. Yet there is limited understanding on the optimal design of the forwarding probability that achieves a pre-designated performance objective with the minimum number of forwardings.

Further, many existing work (e.g. [8]–[12]) assumed the unit disk connection model (UDM), under which two vehicles can directly communicate with each other if their distance is not larger than the transmission range r_0. As introduced in Section I, the wireless connection between vehicles can be affected by a number of random factors, which are taking into account in this paper using a generic connection model to be described in the next section, hence the analysis in this paper can be more relevant to real world scenarios.

III. System model

A. Network model

Consider a 1D VANET where vehicles follow a homogeneous Poisson distribution with intensity ρ [7], [12]. As commonly done is this field, movements of vehicles are not considered because packet forwarding time is in milliseconds, during which the movements of vehicles are negligible [7].

A generic wireless connection model is considered, where the probability that a receiver separated by distance z from the transmitter receives a transmission successfully is given by a connection function $g(z)$, independent of other transmissions. Taking different forms, the connection function $g(z)$ can include a number of widely-used wireless connection models as special cases. For example, under the UDM, there is

$$g(z) = \begin{cases} 1, & \text{for } 0 < z \le r_0 \\ 0, & \text{for } z > r_0 \end{cases}, \tag{1}$$

where r_0 is the transmission range. More specifically, under the UDM, the received signal strength (RSS) at a receiver separated by distance z from the transmitter is $P_u(z) = CP_t z^{-\eta}$, where C is a constant, P_t is the transmission power common to all vehicles and η is the path loss exponent [13]. A transmission is successful if the RSS exceeds a given threshold $P_{\min}$. Consequently, the transmission range (r_0) under UDM is the solution to $P_{\min} = CP_t r_0^{-\eta}$. Moreover, under the log-normal shadowing model (LSM) [13], the RSS attenuation (in dB) follows a Gaussian distribution: $10\log_{10}(P_l(z)/CP_t z^{-\eta}) \sim G$, where $P_l(z)$ is the RSS under LSM and G is a zero-mean Gaussian random variable with standard deviation σ. When $\sigma = 0$, the LSM reduces to the UDM. It has been shown in [14] that under LSM,

$$g(z) = \Pr(P_l(z) \geq P_{\min}) = \frac{1}{2}\left(1 - \mathrm{erf}\left(\frac{10\eta\log_{10}(\frac{z}{r_0})}{\sqrt{2\sigma^2}}\right)\right). \quad (2)$$

Further, taking other forms, channel fading can also be incorporated into the connection function as shown in [14].

Though we consider a generic connection function $g(z)$, there are two reasonable constraints on $g(z)$: 1) it is a monotone non-increasing function of z and 2) $\lim_{z\to\infty} g(z) = 0$.

B. Cooperative forwarding scheme

As illustrated in Fig. 1, consider a source S located at the origin of the x axis. The source transmits a *packet* containing a piece of *information* (e.g. an advertisement or a traffic update [12]) intended to a destination located at $x = D$.

If a vehicle at position x_2 receives a packet from a transmitter (either the source or a relay vehicle) at position x_1 for $x_1 < x_2 < D$, which happens with probability $g(x_2 - x_1)$, then the vehicle at position x_2 forwards the packet with probability $p(x_2 - x_1)$, independent of other vehicles. Hereafter, the vehicles that receive the information *and* choose to forward the information are referred to as the *relays*. We consider that the forwarding probability $p(z)$ is a function of the transmitter-relay distance z because previous research (e.g. [5], [9], [10], [12]) showed that the transmitter-relay distance is a key metric determining the performance of the information dissemination process. Further, note that our analysis on the optimal design of $p(z)$, shown in the next section, takes into account not only the transmitter-relay distance but also the connection function $g(z)$ and density ρ.

Further, it is assumed that a proper MAC mechanism (e.g. CSMA) is employed to avoid collision, so that we can focus on the design of the forwarding probability $p(z)$. Moreover, the transmitter-relay distance z can be calculated by each relay using its own position, provided by GPS [9] or other localization techniques [15], and the position of transmitter carried in the packet header.

As we allow multiple information forwarding paths, there can be more than one copy of the same information propagating in the network. Initially, the source node transmits a single *packet* containing the information, where the time-to-live (TTL) [12] of the packet is set to a pre-designated positive integer k_m. The TTL value of a packet decreases by one each time the packet is forwarded. In the example shown in Fig. 1, node D receives three packets containing the information from node S forwarded via three paths $SBCD$, SCD and SD, where the TTL values of these packets are $k_m - 2$, $k_m - 1$ and k_m respectively.

The dissemination process of an information stops naturally when there is no relay in the network (recall that the aforementioned constraint $x_1 < x_2 < D$ means that only the vehicles located between a transmitter and the destination can be a relay) *or* when the TTL values of all packets containing the information decrease to 0. Note that the TTL only limits the maximum path length (viz. the number of forwardings along a single path), while the total number of forwardings along all paths needs to be analysed separately.

IV. Analysis of the information dissemination process

In this section, a branching process model is established for the information dissemination process. Then the analytical results on the probability of successful delivery and the total number of forwardings are derived. Based on these results, Section IV-B investigates the optimal forwarding probability $p(z)$ that meets a pre-designated probability of successful delivery objective using the minimum number of forwardings.

A. Analysis under generic $g(z)$ and $p(z)$

The main results of this subsection are summarised in the following two theorems, whose proofs are provided later.

Theorem 1. *Denote by $\mathcal{M}(D)$ the expected total number of forwardings of an information along all paths before the information dissemination process stops. There holds*

$$\mathcal{M}(D) = \sum_{k=1}^{k_m} \int_0^D \overbrace{(\zeta * \zeta * \ldots * \zeta)}^{k\text{-fold convolution}}(x)dx, \quad (3)$$

where $\zeta(z) \triangleq \rho p(z)g(z)$ and k_m is the maximum path length (set by the TTL).

Theorem 2. *Denote by $\phi(D)$ the probability that the information sent from a source node is successfully received by the destination at distance D from the source. There holds*

$$\phi(D) \leq 1 - (1 - g(D))\exp\left(-\sum_{k=1}^{k_m}(\overbrace{\zeta * \zeta * \ldots * \zeta}^{k\text{-fold convolution}} * g)(D)\right), \quad (4)$$

where $\zeta(z) \triangleq \rho p(z)g(z)$.

Proof of Theorem 1. We first construct a branching process model for the relays in the information dissemination process. The root, i.e. the 0^{th} generation, of the branching process is the source node. The children of a node (say node B) are the nodes that receive the packet directly from node B *and* choose to forward the packet. Note that as we allow multiple forwarding paths, multiple nodes in the branching process can represent the same vehicle. In the example shown in Fig. 1, the children of both vehicles B and S include the vehicle C. Hereafter, the packets transmitted by the k^{th} generation nodes are called the k^{th} generation packets.

Denote by $\psi_k(x)$ the expected number of the k^{th} generation packets forwarded by a given k^{th} generation node at distance $x \in (0, D)$ from the source. Then it is straightforward that

$$\psi_1(x) = g(x)p(x). \quad (5)$$

Next we consider the 2^{nd} generation. Due to the independence of the packet transmissions between every pair of vehicles and the fact that each vehicle chooses its forwarding probability independently of other vehicles, the 1^{st} generation nodes follow an inhomogeneous Poisson point process with intensity $\zeta(x) \triangleq \rho p(x)g(x)$. Given that there is a 1^{st} generation node at distance h from the source, a node at x transmits a 2^{nd} generation packet if it receives a packet from the 1^{st}

generation node (with probability $g(x-h)$) *and* it chooses to forward (with probability $p(x-h)$). Then there holds

$$\psi_2(x) = \int_0^x p(x-h)g(x-h)\zeta(h)dh = \frac{1}{\rho}(\zeta * \zeta)(x). \quad (6)$$

Similarly, consider that the 1^{st} and the 2^{nd} generation nodes are at distance h and z from the source respectively. Then the expected number of the 3^{rd} generation packets forwarded by a given node at $x \in (0, D)$ is

$$\begin{aligned}\psi_3(x) &= \int_0^x \int_h^x p(x-z)g(x-z)\zeta(z-h)\zeta(h)dzdh \\ &= \frac{1}{\rho}(\zeta * \zeta * \zeta)(x). \quad (7)\end{aligned}$$

Consequently, it can be shown that

$$\psi_k(x) = \frac{1}{\rho}\overbrace{(\zeta * \zeta * ... * \zeta)}^{\text{k-fold convolution}}(x). \quad (8)$$

Finally, the expected total number of forwardings made by all relays between the source and the destination is

$$\mathcal{M}(D) = \sum_{k=1}^{k_m}\int_0^D \psi_k(x)\rho dx = \sum_{k=1}^{k_m}\int_0^D \overbrace{(\zeta * \zeta * ... * \zeta)}^{\text{k-fold convolution}}(x)dx. \quad (9)$$

□

Before going into the next proof, let Fig. 2 illustrate the forwarding process, which shows the simulation (whose setting is described in Section V) and analytical results (given by Eq. 8) of the expected number of forwardings in each generation, under UDM and a constant forwarding probability $p(z) = p_c$. It can be seen that the analytical result matches well with the simulation result.

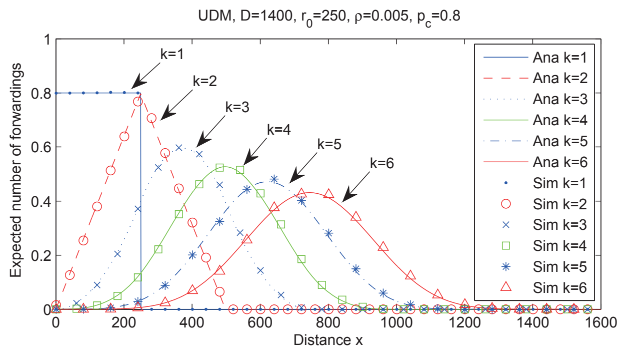

Fig. 2. Simulation (Sim) and analytical (Ana) results on the expected number of the k^{th} generation packets forwarded by vehicles located at distance x from the source node.

Next we study the probability of successful delivery.

Proof of Theorem 2. Construct a branching process for the relays in the same way as that in the proof of Theorem 1. Denote by $\phi_k(D)$ the probability that the destination at distance D from the source receives at least one k^{th} generation packet. It is clear that when $k = 0$, there is $\phi_0(D) = g(D)$.

Consider a differential interval Δx_1 at distance x_1 from the source. Due to Poisson distribution of vehicles, the probability that there is a 1^{st} generation node in the interval Δx_1 is $\rho p(x_1)g(x_1)\Delta x_1 = \zeta(x_1)\Delta x_1$. Consider another differential interval Δx_2 at distance x_2 from the source. Let Ξ_1 (resp. Ξ_2) be the event that the destination at D does *not* receive the 1^{st} generation packet transmitted by the node at Δx_1 (resp. Δx_2). When Δx_1 and Δx_2 are geometrically close to each other, the events Ξ_1 and Ξ_2 can be positively correlated, i.e. conditioned on the occurrence of the event Ξ_1, the event Ξ_2 is more likely to occur and vice versa. Hence there is $\Pr(\Xi_1 \cap \Xi_2) \geq (1 - g(D-x_1)\zeta(x_1)\Delta x_1)(1 - g(D-x_2)\zeta(x_2)\Delta x_2)$. This is known as the spatial correlation problem [14], which may occur in some cases depending on the property of $g(z)$. The impact of the spatial correlation problem on the analysis is discussed further in Section V.

Considering all differential intervals between the source and the destination at D, there is

$$\begin{aligned}\phi_1(D) &\leq 1 - \lim_{\Delta x_1 \to 0}\prod_{\Delta x_1 \in (0,D)}(1 - g(D-x_1)\zeta(x_1)\Delta x_1) \quad (10)\\ &= 1 - \lim_{\Delta x_1 \to 0}\exp\left(\sum_{\Delta x_1 \in (0,D)} -g(D-x_1)\zeta(x_1)\Delta x_1\right)\\ &= 1 - \exp\left(\int_0^D -g(D-x_1)\zeta(x_1)dx_1\right)\\ &= 1 - \exp\left(-(\zeta * g)(D)\right).\end{aligned}$$

Through a similar analysis, it can be shown that for $k \geq 1$,

$$\begin{aligned}\phi_k(D) \leq\ & 1 - \lim_{\Delta x_1 \to 0} ... \lim_{\Delta x_k \to 0}\prod_{\Delta x_1 \in (0,D)} ... \prod_{\Delta x_k \in (x_{k-1},D)}(1 - g(D-x_k)\\ & \zeta(x_k - x_{k-1})...\zeta(x_1 - 0)\Delta x_k...\Delta x_1) \quad (11)\\ =\ & 1 - \exp\left(\int_0^D\int_{x_1}^D ... \int_{x_{k-1}}^D -g(D-x_k)\right.\\ & \left.\zeta(x_k - x_{k-1})...\zeta(x_2 - x_1)\zeta(x_1 - 0)dx_k...dx_2dx_1\right)\\ =\ & 1 - \exp\left(-(\overbrace{\zeta * \zeta * ... * \zeta}^{\text{k-fold convolution}} * g)(D)\right).\end{aligned}$$

Finally, the probability of successful delivery $\phi(D)$ is equal to the probability that the destination receives at least one of the k^{th} generation packets for $k = \{0, 1, ..., k_m\}$. Then there is

$$\begin{aligned}\phi(D) &\leq 1 - \prod_{k=0}^{k_m}(1 - \phi_k(D)) \quad (12)\\ &\leq 1 - (1 - g(D))\prod_{k=1}^{k_m}\exp\left(-(\overbrace{\zeta * \zeta * ... * \zeta}^{\text{k-fold convolution}} * g)(D)\right)\\ &= 1 - (1 - g(D))\exp\left(-\sum_{k=1}^{k_m}(\overbrace{\zeta * \zeta * ... * \zeta}^{\text{k-fold convolution}} * g)(D)\right).\end{aligned}$$

where the first inequality is again due to the spatial correlation problem, i.e. due to a finite transmission range determined by certain forms of $g(z)$, the event that a node does not receive the information in the k^{th} generation is positively correlated to the event that the node does not receive the information in the i^{th} generation for $i \neq k$. □

B. Optimization

Based on the above analysis, this subsection presents a conjecture on the optimal design of the forwarding probability that meets a pre-designated probability of successful delivery objective using the minimum number of forwardings, which is formulated as an optimization problem as follows

$$\begin{aligned}&\underset{p(z)}{\text{Minimise}} \quad \mathcal{M}(D)\\ &\text{Subject to} \quad \phi(D) = P_t,\end{aligned}$$

where $P_t \in [0, 1]$ is the pre-designated probability of successful delivery objective. Note that it can be seen later that the optimization result using the constraint $\phi(D) \geq P_t$ is the same as that using $\phi(D) = P_t$.

The following analysis is based on the upper bound given in Theorem 2. Note that it is an upper bound only due to the spatial correlation problem arisen in some cases depending on $g(z)$. Further, it has been shown in [14], and can be seen later in Section V, that the spatial correlation problem becomes less notable in the presence of channel randomness.

Seeing that the k-fold convolution is the common term in both $\phi(D)$ and $\mathcal{M}(D)$, we first define

$$\eta_k(x) \triangleq \overbrace{(\zeta * \zeta * \ldots * \zeta)}^{\text{k-fold convolution}}(x). \tag{13}$$

Then perform functional derivative [16] on $\mathcal{M}(D)$ given by Theorem 1. Using the delta function $\delta(.)$ as a test function, for $0 < y < D$ there holds

$$\begin{aligned}\frac{\partial \mathcal{M}(D)}{\partial \eta_k(y)} &= \frac{\partial}{\partial \eta_k(y)} \sum_{k=1}^{k_m} \int_0^D \eta_k(x)dx \qquad (14)\\ &= \lim_{\epsilon \to 0} \frac{\sum_{k=1}^{k_m} \int_0^D \eta_k(x) + \epsilon\delta(x-y)dx - \sum_{k=1}^{k_m} \int_0^D \eta_k(x)dx}{\epsilon}\\ &= \lim_{\epsilon \to 0} \frac{\sum_{k=1}^{k_m} \int_0^D \epsilon\delta(x-y)dx}{\epsilon} = \lim_{\epsilon \to 0} \frac{k_m \epsilon}{\epsilon} = k_m.\end{aligned}$$

Denote by $\phi_u(D)$ an upper bound on the probability of successful delivery, Then using Theorem 2, there is

$$\phi_u(D) = 1 - (1 - g(D)) \exp\left(-\sum_{k=1}^{k_m} \int_0^D \eta_k(x)g(D-x)dx\right). \tag{15}$$

Then perform functional derivative on $\phi_u(D)$. There is

$$\begin{aligned}\frac{\partial \phi_u(D)}{\partial \eta_k(y)} &= (1 - g(D)) \qquad (16)\\ &\lim_{\epsilon \to 0}\left(\frac{-\exp\left(-\sum_{k=1}^{k_m} \int_0^D (\eta_k(x) + \epsilon\delta(x-y))g(D-x)dx\right)}{\epsilon} + \frac{\exp\left(-\sum_{k=1}^{k_m} \int_0^D \eta_k(x)g(D-x)dx\right)}{\epsilon}\right)\\ &= (1 - g(D)) \exp\left(-\sum_{k=1}^{k_m} \int_0^D \eta_k(x)g(D-x)dx\right)\\ &\quad \lim_{\epsilon \to 0} \frac{-\exp\left(-\sum_{k=1}^{k_m} \int_0^D \epsilon\delta(x-y)g(D-x)dx\right) + 1}{\epsilon}\\ &= (1 - g(D)) \exp\left(-\sum_{k=1}^{k_m} \int_0^D \eta_k(x)g(D-x)dx\right)\\ &\quad \lim_{\epsilon \to 0} \frac{-\exp(-k_m \epsilon g(D-y)) + 1}{\epsilon}\\ &= (1 - g(D)) \exp\left(-\sum_{k=1}^{k_m} \int_0^D \eta_k(x)g(D-x)dx\right) k_m g(D-y).\end{aligned}$$

The Lagrangian of our optimization problem becomes

$$\mathcal{L}(\eta(x), \lambda) = \mathcal{M}(D) + \lambda (P_t - \phi_u(D)). \tag{17}$$

The K.K.T. conditions are: $\lambda \geq 0$,

$$\begin{aligned}&\frac{\partial \mathcal{L}(\zeta(x), \lambda)}{\partial \eta(x)} = k_m - \lambda(1 - g(D))\\ &\times \exp\left(-\sum_{k=1}^{k_m} \int_0^D \eta_k(x)g(D-x)dx\right) k_m g(D-y) = 0, \qquad (18)\end{aligned}$$

$$1 - (1 - g(D)) \exp\left(-\sum_{k=1}^{k_m} \int_0^D \eta_k(x)g(D-x)dx\right) \geq P_t, \tag{19}$$

$$\lambda\left(P_t - 1 + (1 - g(D)) \exp\left(-\sum_{k=1}^{k_m} \int_0^D \eta_k(x)g(D-x)dx\right)\right) = 0. \tag{20}$$

According to the condition in Eq. 18, there is $\lambda \neq 0$. Therefore, the constraints $\phi(D) = P_t$ and $\phi(D) \geq P_t$ give the same optimization result. Then from Eq. 20, there is

$$1 - (1 - g(D)) \exp\left(-\sum_{k=1}^{k_m} \int_0^D \eta_k(x)g(D-x)dx\right) = P_t. \tag{21}$$

Substitute Eq. 21 back to Eq. 18. There is

$$k_m - \lambda(1 - P_t)k_m g(D-y) = 0. \tag{22}$$

This condition is feasible for $\lambda > 0$ because $(1 - P_t)k_m g(D - y) > 0$. Finally, from Eq. 21, the objective function is minimised when

$$\sum_{k=1}^{k_m} \int_0^D \eta_k(x)g(D-x)dx = \ln\left(\frac{(1 - g(D))}{1 - P_t}\right). \tag{23}$$

Note that Eq. 23 only gives a requirement on $\eta_k(x)$. To fulfil Eq. 23 while minimising $\mathcal{M}(D) = \sum_{k=1}^{k_m} \int_0^D \eta_k(x)dx$, the function $\eta_k(x)$ needs to have a large value when x is close to D, because $g(z)$ is a non-increasing function of z. Moreover, recall that $\eta_k(x)$ is the k-fold convolution of $\zeta(z) = \rho g(z)p(z)$. Consequently the function $\zeta(z)$ needs a large mean value. This conclusion justifies the previous simulation studies showing that the number of forwardings can be reduced by assigning a larger forwarding probability to a node farther apart from the transmitter [5], [9], [11].

Remark 1. In summary, to achieve a pre-designated probability of successful delivery P_t using the minimum number of forwardings, one should first calculate the requirement on the size of the area under the curve of $\eta_k(x)$ using Eq. 23. Then based on the wireless connection function $g(z)$, design the forwarding probability $p(z)$ in the way that maximises the mean value of $\zeta(z)$. Further discussions on the optimal design of $p(z)$ are provided in the next section.

V. Simulation

This section reports on simulations to validate the analytical results. The Monte Carlo simulations are conducted in a VANET simulator written in C++. Nodes are deployed on a 1D axis following a homogeneous Poisson process with intensity ρ varying from 0.005 to 0.02 veh/m. The UDM and the LSM are used as two typical wireless connection models, where the transmission range under UDM is $r_0 = 250 \sim 500m$ [7] and the parameters of LSM are $\eta = 2$ and $\sigma = 4$ [13].

We first use a typical forwarding probability $p(z) = \min\{p_c \frac{z}{r_0}, 1\}$, which is an increasing function of z, where $p_c \in (0, 1)$ is a tuning parameter. The upper bound on the probability of successful delivery is shown in Fig. 3, plotted together with simulation results for comparison. It can be seen that the gap between simulation and analytical result is smaller under LSM compared with that under UDM; also the gap is smaller when p_c is smaller. This is because randomness reduces the spatial correlation problem introduced in the proof of Theorem 2. Overall, the analytical result is able to capture the impacts of network parameters (i.e. ρ, $g(z)$ and $p(z)$) on the information dissemination process.

Further, it can be seen in Fig. 3 that using the same setting, the network under LSM has a higher probability of successful delivery than that under UDM. This coincides with previous observations [14] that shadowing effects are beneficial to the connectivity of the networks adopting distributed information forwarding schemes.

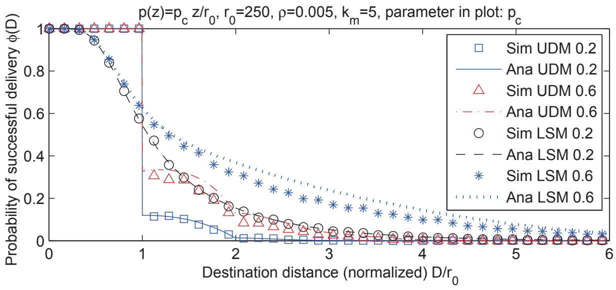

Fig. 3. Simulation (Sim) results and analytical (Ana) upper bound on the probability of successful delivery.

Fig. 4 compares the probability of successful delivery under UDM using four different forwarding probability functions as depicted in Fig. 5. Note that these four functions have the same size of the area under the curve, which means that their convolution results (i.e. $p(z) * p(z)$) also have the same area size under the curve. Therefore, these four functions results in the same expected number of forwarding in each generation. It is interesting to note that Fig. 4 confirms our conjecture given in Remark 1 that the optimal design of $p(z)$ (introduced in Section IV-B) should have the largest mean value, which is a rectangular function (see case 4 in Fig. 5) rather than a linear function or an exponential function of z (considered in related studies (e.g. [5], [9], [11])).

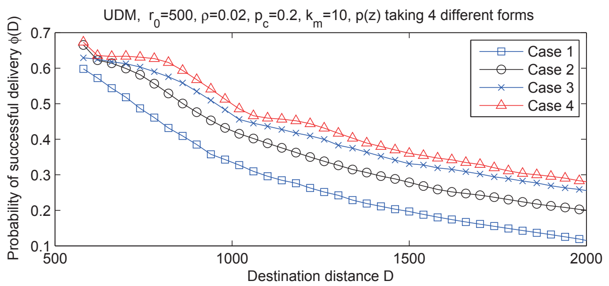

Fig. 4. Simulation results of the probability of successful delivery using 4 different forwarding probability functions $p(z)$ as depicted in Fig. 5.

VI. Conclusion and future work

This paper investigated cooperative information forwarding schemes for VANETs. Theoretical models were established for the information dissemination process under a generic wireless connection model and a generic forwarding probability function. Analytical results on the probability of successful delivery and the total number of forwardings are derived. Further, a conjecture on the optimal information forwarding scheme is provided.

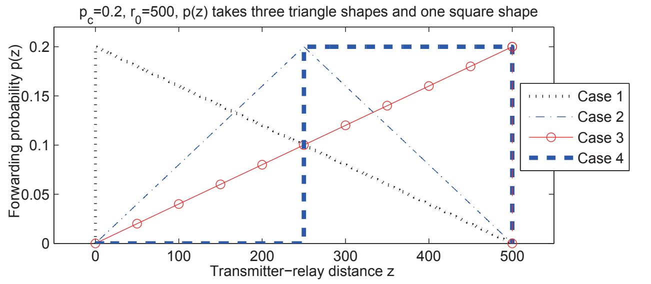

Fig. 5. Four different forwarding probability function for $z \in [0, r_0]$: Case 1 is $p(z) = p_c(1 - \frac{z}{r_0})$; case 2 is $p(z) = p_c z/r_0$ when $0 \leq z \leq r_0/2$ and $p(z) = p_c - 2p_c(z - r_0/2)/r_0$ when $r_0/2 \leq z \leq r_0$; case 3 is $p(z) = p_c \frac{z}{r_0}$ and case 4 is $p(z) = p_c$ when $r_0/2 \leq z \leq r_0$, otherwise $p(z) = 0$. Note that under UDM where $g(z)$ is given by Eq. 1 with $r_0 = 500$, there holds $g(z)p(z) = p(z)$ for either one of these functions.

A complete proof to the conjecture on the optimal forwarding scheme needs to be provided in a future work, which requires a non-trivial analysis on the impact of the spatial correlation problem on the information dissemination process. Further, network coding techniques can be employed to reduce the redundancy in packet forwardings, where each relay combines its own packet and its received packet into a coded packet and forwards to other vehicles.

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