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CONDITIONS FOR LOCAL STABILITY AND ROBUSTNESS OF ADAPTIVE CONTROL SYSTEMS

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Abstract

This paper reports some preliminary results concerning robustness properties of adaptive control systems to unmodeled dynamics and bounded disturbances. The analysis is conducted from the viewpoint of input/output stability theory. Generic representations are proposed for both continuous-time and discrete-time adaptive systems and conditions for stability and robustness are developed for each case. These conditions require varying degrees of a priori knowledge about the plant, e.g., global conditions involving minimal knowledge and local conditions involving more restrictive assumptions.

I. Continuous-Time Case

A. Global Analysis

A large class of continuous-time adaptive systems can be represented by the nonlinear system (Fig. 1):

$$e_t = e_t^* - H(p)v_t, \quad v_t = \xi_t' \pi_t$$

$$\xi_t = \xi_t^* - G(p)v_t \qquad (S_c)$$

$$\dot{\hat{\pi}}_t = A(\xi_t, \omega_t), \quad \omega_t = \xi_t e_t$$

where e_t, e_t^*, $v_t \in R$, and π_t, ξ_t, $\xi_t^* \in R^n$. The operators $H(p)$ and $G(p)$ are proper rational functions with real coefficients in the differential operator p, i.e., $(px)_t := \dot{x}_t$. We will refer to e_t as the output error, π_t as the parameter error, v_t as the control error, ξ_t as the regressor and $A(.,.)$ as the adaptation gain. In general, only e_t and ξ_t are available as measurements. The parameter error $\pi_t := \hat{\pi}_t - \pi^*$, where $\hat{\pi}_t$ is the adaptive estimate of the true, but unknown, parameter π^*. The signals e_t^* and ξ_t^* are referred to as the tuned output error and regressor, respectively, meaning that these signals are generated from an 'ideal' system with the desired parameters π^*, i.e., $\hat{\pi}_t = \pi^*$. Details on the relation between (S_c) and the actual system (unknown plant + adaptive controller) can be found elsewhere, e.g., [1]-[3]. In general, the unknown plant is imbedded in $G(p)$ and $H(p)$, which, incidentally, are also functions of the true parameter π^*.

Since π^* as well as the plant are unknown, it follows that $H(p)$ and $G(p)$ are unknown. However, in order to establish conditions for stability of (S_c), it is necessary to know something about $H(p)$ and $G(p)$. The same remark holds for knowledge about the tuned signals e_t^* and ξ_t^*. The following theorem gives conditions for global stability of (S_c). The term 'global' refers to the intention of requiring minimal, but reasonable, restrictions on $H(p)$, $G(p)$, e_t^* and ξ_t^*. Proof of Theorem 1 is given in [2].

Theorem 1: Global Stability. For the system (S_c) assume that:

(A1) The elements of $G(p)$ are strictly proper and exponentially stable (all poles strictly inside the left half plane)

(A2) $H(p)$ is strictly positive real (SPR), i.e., the elements of $H(p)$ are strictly proper, exponentially stable, and Re $[H(j\omega)]$ is positive for all $\omega \in [0,\infty)$.

(i) Suppose that the adaptation gain is constant, i.e.,

$$A(\xi_t, \omega_t) = A_o \omega_t, \ A_o = A_o' > 0 \qquad (1)$$

Under these conditions, if

$$e^*, \dot{e}^* \in L_2 \cap L_\infty \ (\Rightarrow e_t^* \to 0), \ \text{and} \ \xi^*, \dot{\xi}^* \in L_\infty^n$$

then:

(i-a) $\pi \in L_\infty^n$, $\dot{\pi}$, $\ddot{\pi} \in L_2^n \cap L_\infty^n$, and $\dot{\pi}_t \to 0$ at the rate $e_t^* \to 0$.

(i-b) e, $\dot{e} \in L_2 \cap L_\infty$, and $e_t - e_t^* \to 0$ exp.

(i-c) $v \in L_2 \cap L_\infty$, $\dot{v} \in L_\infty$, and $v_t \to 0$.

(i-d) ξ, $\dot{\xi} \in L_\infty^n$; $\xi - \xi^*$, $\dot{\xi} - \dot{\xi}^* \in L_2^n \cap L_\infty^n$, and $\xi_t - \xi_t^* \to 0$ exp.

(ii) Suppose that ξ_t is persistently exciting [4], i.e., $\exists$ constants α_1, α_2, $\alpha_3 > 0$ such that

$$\alpha_1 I_n \le \int_s^{s+\alpha_3} \xi_t \xi_t' \, dt \le \alpha_2 I_n, \ \forall s \in R^+ \qquad (2)$$

(ii-a) Result (i) holds, and in addition, π_t, $v_t \to 0$ exp.

(ii-b) If the elements of e^*, $\dot{e}^*$, ξ^*, and $\dot{\xi}^*$ are all in L_∞, then the elements of π, $\hat{\pi}$, $\ddot{\pi}$, e, $\dot{e}$, v, $\dot{v}$, ξ, and $\dot{\xi}$ are all in L_∞.

(iii) Suppose that the adaptation gain is <u>retarded</u>, [5], i.e.,

$$A(\xi_t, \omega_t) = \begin{cases} A_o\omega_t \,, & |\hat{\pi}_t| < c, \quad c \geq \max |\pi^*| \\ A_o(\omega_t - (1 - |\hat{\pi}_t|/c)^2\hat{\pi}_t), & |\pi_t| \geq c \end{cases} \qquad (3)$$

(iii-a) Result (i) holds
(iii-b) Result (ii-b) holds.

<u>Remarks.</u> The major difficulties in applying Theorem 1 are that, in the first place, $H(p) \in SPR$ (condition (A2)) is an unlikely event in actual systems, due to the effect of unmodeled dynamics [6,7]. Secondly, the conditions on e^* as given in (i) are also unlikely, namely $e^*_t \to 0$. This condition rules out the presence of unmeasurable bounded disturbances. Thus, the conditions on e^*_t in (ii-b) remain the only realistic case insofar as the tuned signals are concerned. But, this raises another problem: ensuring that either ξ_t is persistently exciting (2) or that the adaptation gain is retarded (3). Notwithstanding the difficulty with the SPR condition on $H(p)$, there are specific problems related to (2) and (3). For example:

<u>Persistent Excitation (PE)</u>: With bounded disturbances (conditions (ii-b)) it is not known how to guarantee that $\xi_t \in PE$. Recall that ξ_t is generated <u>inside</u> the adaptive loop, and thus, can only be controlled from the input, i.e., from either e^*_t or, more likely, from ξ^*_t. Since we do not know $G(p)$ and $H(p)$ it is not possible to conclude beforehand if $\xi_t \in PE$ even if $\xi^*_t \in PE$. In the special (unrealistic) case of no unmodeled dynamics and no bounded disturbances, $e^*_t = 0$, (ii-a) holds and $\xi^*_t \in PE \Rightarrow \xi_t \in PE$. Even though this latter situation is easily ruled out, it certainly makes sense that $\xi^*_t \in PE$ implies a 'local' result. That is, with certain suitable restrictions on signal size and so forth, the system is robust. These arguments were formalized to some extent in [3] and will be slightly extended here.

<u>Retarded Update</u>: In [5] it is suggested that the update algorithm be retarded as given by (3). Likewise, in [8], a slow or 'leaky' integrator is added. Although both these schemes (as well as similar ones) do give B.I.B.O. results, they both require additional information abut the plant, e.g., as in (3) an upper bound on $|\pi^*|$. These results can also be considered as 'local' results.

<u>Slow Variations</u>: Together with a retarded update, another mechanism for ensuring B.I.B.O. stability is to slow the variations in ξ_t (see [3]). The idea follows by examining the simple constant gain retarded algorithm,

$$\begin{aligned} \dot{\pi}_t &= A_o\omega_t - \alpha\,\pi_t \qquad (\alpha > 0) \\ &= -A_o\xi_t\, H(p)\xi_t'\pi_t + A_o\xi_t e_t^* - \alpha\,\pi_t \end{aligned}$$

If ξ_t is constant then exponential stability can be assured by direct LTI techniques. Thus, if ξ_t varies slowly enough with respect to the dynamics of $H(p)$ it is reasonable to expect a similar result. We will examine this more closely. However, control of ξ_t introduces the same difficulties as in requiring $\xi_t \in PE$, i.e., only 'local' results can be obtained.

B. Local Analysis.

The system (S_c) can be transformed to a more useful form for local analysis:

$$\tilde{x} = \tilde{x}_L - \tilde{x}_{NL} \qquad (\tilde{S}_c)$$

$$\tilde{x}_{NL} = F\, f(\tilde{x})$$

where:

$$\tilde{x} := (\pi, \tilde{e}, \tilde{\xi}) := (\pi-\pi^*, e-e^*, \xi-\xi^*)$$

$$\tilde{x}_L := (\pi_L, \tilde{e}_L, \tilde{\xi}_L) \;; \quad f(\tilde{x}) := (\tilde{\xi}'\pi, \tilde{\xi}\tilde{e})$$

The system $(\tilde{S}_c)$ is obtained from (S_c) by linearization of (S_c) about e^*_t, ξ^*_t, and π^*, resulting in the linearized perturbation response $\tilde{x}_L$. The remaining nonlinear terms $\tilde{x}_{NL}$ are contained in $f(\tilde{x})$ where F is a time-varying linear operator. The characteristics of F, as well as those of $\tilde{x}_L$, depend on the adaptation gain and the behavior of the tuned signals, e^*_t and ξ^*_t (see [3]).

Consider the constant gain algorithm (1) with a retarded update. Fig. 2 depicts the resulting system $(\tilde{S}_c)$ where:

$$L := \frac{1}{p+\alpha}A_o, \quad \alpha = \begin{cases} >0, & \text{with (3)} \\ 0, & \text{otherwise} \end{cases}$$

$$M := \xi^*\, H(p)\, \xi^{*\prime} + e^* G(p)\xi^*$$
(4)

$$N := \xi^*\, H(p) + e^* G(p)$$

Thus, Fig. 2 reveals that $\tilde{x}_L$ is the response to $\mu_L := (\xi^* e^*, 0, 0)$ with $\pi_o \neq 0$, whereas $\tilde{x}_{NL}$ is the response to $\mu_{NL} := (0, \tilde{\xi}\tilde{e}, \tilde{\xi}'\pi)$ with $\pi_o = 0$. Clearly, boundedness of the linearized response $\tilde{x}_L$ and stability of the operator F require stability of the map η, π_o into π, indicated in Fig. 2 by $K(\pi_o)$. It is shown in [3] that stability of $K(\pi_o): \eta \mapsto \pi$ ensures the existence of conditions for local stability of the adaptive system (S_c) or $(\bar{S}_c)$.

Of particular interest is the degree to which it is possible to maintain stability despite arbitrary dynamics $H(p)$ and $G(p)$, i.e., robustness to model error. Primary consideration is given to unmodeled dynamics in $H(p)$. Let,

$$H(p) := \bar{H}(p) + \Delta_H(p) \tag{5}$$

where $\bar{H}$ denote the nominal dynamics obtained under ideal conditions, $\hat{\pi}_t = \pi^*$; consequently, we may consider $\bar{H}$ to be a fixed transfer function which is independent of π^*. All errors will be lumped into Δ_H. The desired result of the local analysis is to obtain a quantitative bound on the worst case model error for which stability of $(\bar{S}_c)$ is guaranteed. We will do this by analyzing the stability robustness properties of the map $K(\pi_o)$, thus, the results obtained will only verify that local conditions exist.

B.1 Local Stability by Persistent Excitation. Assume that

$$\begin{aligned} &\xi_t^* \in PE \\ &\bar{H}(p) \in SPR \end{aligned} \tag{6}$$

Under these conditions, it follows from [4] that the system

$$\dot{x}_t = -\xi_t^* \, \bar{H}(p) \, \xi_t^{*\prime} x_t \tag{7}$$

is exponentially stable, i.e., there exists constants $m, \lambda > 0$ such that

$$|x_t| \le m\, e^{-\lambda t} |x_o| \tag{8}$$

The following result gives a coarse bound on the model error Δ_H.

Theorem 2. The system $K(\pi_o)$ in Fig. 2 is L_∞-stable if:

$$\lambda/m > \sigma := \|e^*\|_\infty \|\xi^*\|_\infty \, \gamma_\infty(G) \tag{9}$$

and

$$\gamma_\infty(\Delta_H) < (\lambda/m - \sigma)/\|\xi^*\|_\infty^2 \tag{10}$$

Proof: Follows directly from small gain theory (see e.g. [9]); details are in [3].

Remarks: Although sharper bounds can be obtained [10], the significance of Theorem 2 is that $H(p)$ need not be SPR if $\xi_t^* \in PE$. The conditions of Theorem 2 can be determined experimentally by simulating $K(\pi_o)$ for a variety of $\pi_o \in R^n$, $\mu \in L_\infty^n$, and $\xi_t^* \in PE$. This procedure can only yield an estimate.

B.2 Local Stability by Slow Variations with Retarded Update. In this case we will assume that ξ_t^* is not PE, but varies slowly, in a defined way, in relation to the known dynamics of $\bar{H}(p)$. Let ξ_τ^* denote ξ_t^* frozen at time $t = \tau$. Let $\bar{K}_\tau(p)$ denote the linear time-invariant operator given by,

$$\bar{K}_\tau(p) := [I_n + L(p)\bar{N}_\tau(p)]^{-1} L(p) \tag{11}$$

where

$$\bar{N}_\tau(p) := \xi_\tau^* \, \bar{H}(p) \, \xi_\tau^{*\prime} \tag{12}$$

Let R_τ denote the linear time-varying operator

$$R_\tau := \xi_t^* H(p) \xi_t^{*\prime} + e_t^* G(p) \xi_t^{*\prime} - \bar{N}_\tau(p) \tag{13}$$

The operator $\bar{K}_\tau(p)$ is simply $K(\pi_o)$ with $N = \bar{N}_\tau(p)$, i.e., ξ_t^* fixed at ξ_τ^*. Thus, R_τ represents the effect of how far ξ_t^* is from other values ξ_τ^* under the dynamics of $H(p)$ and $G(p)$.

Suppose that $\bar{K}_\tau(p)$ is exponentially stable, i.e., there exists constants $m, \lambda > 0$ such that,

$$|(\bar{K}_\tau(p)u)_t| \le m \int_0^t e^{-\lambda(t-s)} |u_s| ds, \quad u \in L_\infty[0,t], \ \tau \in R^+ \tag{14}$$

The following result is analogous to Theorem 2.

Theorem 3: The system $K(\pi_o)$ is L_∞-stable if:

$$\lambda/m > \sigma := \|e^*\|_\infty \|\xi^*\|_\infty \, \gamma_\infty(G) + \sup_{\tau \ge 0} \gamma_\infty(\xi^* \bar{H} \xi^{*\prime} - \xi_\tau^* \bar{H} \xi_\tau^{*\prime}) \tag{15}$$

and

$$\gamma_\infty(\Delta_H) < (\lambda/m - \sigma)/\|\xi^*\|_\infty^2 \tag{16}$$

Proof: Follows directly from small gain theory; details in [3],[10].

Remarks: *As in Theorem 2, the conditions here* for local stability do not depend on $H \in SPR$, and in this case do not depend on $\xi_t^* \in PE$. Thus, Theorem 3 is weaker than Theorem 2. The key is to establish (14), i.e., exponential stability of $\bar{K}_\tau(p)$. Note that with $\bar{H} \in SPR$, and $L(p)$ given by (4) with $\alpha > 0$, (14) is established by passivity arguments. Tighter *bounds on the norm operations can be obtained* [10]. Also, the norms themselves can be estimated by simulating candidate actual systems (Fig. 2).

II. Discrete-Time Case

A. Global Analysis

The discrete-time version of S_c is somewhat *different*, due to the inherent system delay $k \geq 1$. The following discrete-time nonlinear system is representative of most discrete-time adaptive systems:

$$e_t = e_t^* - H_1(q^{-1})v_{1,t} - H_2(q^{-1})v_{2,t}$$

$$\xi_t = \xi_t^* - G_1(q^{-1})v_{1,t} - G_2(q^{-1})v_{2,t}$$

$$\pi_t = \pi_{t-1} + A(\xi_t, \omega_t), \quad \omega_t := \xi_t e_t \qquad (S_{d,k})$$

with

$$v_{1,t} := \xi_t' \pi_{t-1}, \quad v_{2,t} := \xi_t' \pi_{t-k}$$

The signal and operator dimensions are the same as those defined in (S_c), with q^{-1} the backward shift operator $(q^{-1}x)_t = x_{t-1}$. In general, with a unit delay $(k = 1)$, (S_d) collapses to the form of (S_c). Specifically, the unit delay adaptive system is:

$$e_t = e_t^* - H(q^{-1})v_t, \quad v_t := \xi_t' \pi_{t-1}$$

$$\xi_t = \xi_t^* - G(q^{-1})v_t \qquad (S_{d,1})$$

$$\pi_t = \pi_{t-1} + A(\xi_t, \omega_t), \quad \omega_t := \xi_t e_t$$

In this paper we will only examine $S_{d,1}$. Details on $S_{d,k}$ can be found in [10].

A.1. Adaptation Algorithms. It is an understatement to say that the choice of discrete-time algorithms is overwhelming. However, following [11],[12] they more or less belong to the following almost generic types:

Projection

$$A(\xi_t, \omega_t) = (1 + |\xi_t|^2)^{-1} \omega_t \qquad (P)$$

Recursive Least Squares

$$A(\xi_t, \omega_t) = S_t \omega_t$$

$$S_t^{-1} = S_{t-1}^{-1} + \xi_t \xi_t', \quad S_o = S_o' > 0 \qquad (RLS)$$

Stochastic Approximation

$$A(\xi_t, \omega_t) = s_t \omega_t$$

$$s_t^{-1} = s_{t-1}^{-1} + |\xi_t|^2, \quad s_o > 0 \qquad (SA)$$

Available stability results have dealt almost exclusively with $(S_{d,k})$ where, in the deterministic case, $e_t^* = 0$, with either $H_1(q^{-1}) = 1$ (or positive constant) and $H_2(q^{-1}) = 0$, or vice versa, e.g., [11]. (The stochastic version assumes e_t^* has zero mean with bounded variance, e.g., [12].) The following theorem extends the deterministic results to the case where $e^* \in \ell_2$. Thus, e_t^* approaches zero asymptotically, but is not identical to zero. Proof of theorem 2 is in [10].

Theorem 4: Global Stability. For the system $(S_{d,1})$ assume that:

(A1) The elements of $G(q^{-1})$ are proper and exponentially stable (all poles strictly inside the unit disc)

(A2) $H(q^{-1})$ is proper, exponentially stable, and for some constant $\delta > 0$,

$$| H(q^{-1}) - 1 | \leq \delta, \quad \forall |q| = 1 \qquad (17)$$

Under these conditions, if $e^* \in \ell_2$ ($\Rightarrow e_t \to 0$) and $\xi^* \in \ell_\infty^n$ then using adaptation algorithm (P), (RLS), or (SA) results in $e, v \in \ell_2$ and $\xi, \pi \in \ell_\infty^n$ provided that

$$\delta < 1 \qquad (18)$$

Remarks.

(1) Theorem 4 offers no more than part (i) of Theorem 1 for continuous-time systems, in that it is not possible to insure an arbitrarily large model error. The bound (18) of $\delta < 1$ is as unrealistic as the requirement that $H(p) \in$ SPR in Theorem 1 (in fact, $H(p) \in$ SPR implies that $\delta < 1$; see [2]). Similar restrictive results for discrete-time adaptive systems have been reported in [13] and [14].

(2) It can be shown [10] that Theorem 2 is valid if, in (A2), H is either an LTI operator in the sector:

$$| H(q^{-1}) - \bar{H}(q^{-1}) | \leq \delta, \quad \forall |q| = 1$$

where $\qquad (19)$

$$\bar{H}(q^{-1}) \in \text{SPR}$$

or if $H - \bar{H}(q^{-1})$ is a slope-restricted memoryless nonlinearity, i.e.,

$$| (Hv)_t - \bar{H}(q^{-1})v_t | \leq \delta \, | v_t | \qquad (20)$$

(3) With arbitrary sector conditions on H, Theorem 2 holds for $\delta < 1$ if the adaptation gain is modified, e.g.,

$$A(\xi_t, \omega_t) = m_t \omega_t, \quad m_t = (1 + \|\xi\|_{t\infty}^2)^{-1/2} \qquad (21)$$

Other modifications like this can be constructed, provided m_t satisfies certain conditions, e.g., if m_t is a positive nonincreasing function, then sector properties on H apply to the operator $m_t H m_t^{-1}$. The required properties of m_t relate to the noncausal multiplier theory described in [9]. Picking the right multiplier – which is only needed in the proof of stability – is an artform akin to selecting a suitable Lyapunov function for a nonlinear system. The multiplier requirements do, however, motivate a myriad of modifications to adaptation gains (as proposed in (21)), for which multiplier selection is more easily facilitated, see e.g. [14]. It is unclear at this time whether these modifications can acheive practical sector conditions on H for global stability, i.e., where $\delta >> 1$.

B. Local Analysis

Stability results dependent on persistent excitation or retarded update have a more 'local' character than their discontinuous-time counterparts, and thus, have been left out of the global analysis. As remarked before after Theorem 1, these are the known means to insure ℓ_∞-stability, which we have argued is the case most related to the actual system environment.

The local stability analysis for continuous-time systems can be developed analogously for the discrete-time case, with only minor modifications. Thus, Theorems 2-3 have their discrete-time counterparts. One major difference, however, is that the nonlinear term in $(\tilde{S}_c)$ is more complicated due to the complexity of the adaptation gain algorithms, e.g., (P) or (RLS). Other than that, similar results follow for the discrete-time case [10].

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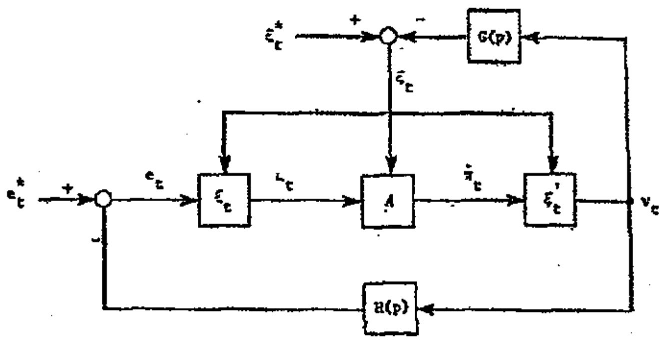

Fig. 1. Adaptive System *A* is the parameter update algorithm.

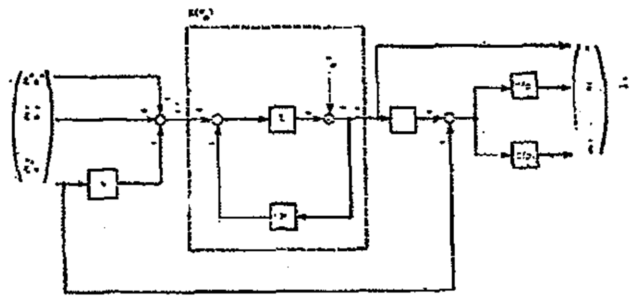

Fig. 2. Local System.