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# Translational velocity consensus using distance-only measurements

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*Abstract*— **This paper proposes a strategy to achieve translational velocity consensus in a multi-agent formation using distance-only measurements. Since with agents executing arbitrary motions, distance-only measurement cannot provide enough information for velocity consensus, we postulate that agents engage in a combination of circular motion and linear motion. When energy saving is the first priority, the linear motion component should dominate. On the other hand, when measurement accuracy is the first priority, the circular motion must be more prominent.**

## I. INTRODUCTION

The performance of multi-agent systems in various tasks, e.g. consensus [1], [2], formation shape control [3], [4], cooperative geolocalization [5], etc. has been studied with increasing intensity over recent years. These tasks are usually required to be performed in a decentralised way [6] and using limited information, i.e. each agent should individually identify possible actions, and while such actions are required to help achieve the final goal of the formation, each agent can communicate only with its neighboring agents. Examples of these tasks are retrieving information from an area covered by a sensor network (where the agents are sensors deployed in the area), or moving together in a desired formation shape from one point to another where the agents are ground or aerial vehicles.

In a formation control problem, which is the focus of this paper, each agent tries to contribute to achieve the global goal of the formation using measurements of, typically, relative positions and velocities of its neighbors. Examples of such problems are given in [7]–[10]. These problems become more challenging when the agents cannot instantaneously measure all the information required to apply motion corrections to achieve the final goal of the formation and have to estimate some of this information using their measurements.

An example of such a problem is given in [11], where a formation (shape and translation motion) control method, called stop-and-go, has been devised to control the agents not able to measure the relative positions (both distance and angle) of their neighbors, but only able to measure the distances to their respective neighbors. This measurement restriction makes the control problem significantly harder.

This paper revisits a relevant problem. Agents are required to achieve velocity consensus using distance-only measurements to neighbors. The key is to postulate that the motion of each agent comprises two parts: a translation and a circular motion. The circular motion is around a moving center, and it is the centers of each agent's motion, rather than the agents themselves, which achieve velocity consensus. The purpose of the superimposed circular motion is to allow inter-agent localization and velocity estimation, not using instantaneous measurements, but using distance measurements collected over an interval. We postulate that neighbor agents remain in communication even if they initially have different velocities.

The notion of using deliberate motions of agents to assist in localization was suggested to use in the work [12], in relation to sensor network localization. Their idea is that if each node in a sensor network moves in a small neighbourhood of its original position, it is possible to infer direction information from distance measurements. Our idea is similar; however, the motions in [12] are random while this paper studies the localization problem using distance-only measurements when agents are executing independent circular motions and it further discusses the situation where agents are performing a combination of circular motion and linear motion, with the linear motion components required to achieve consensus. An advantage of having a combination of linear and circular motion over only linear motion as in [11] is that the agents are less likely to travel out of communication range during the localization process. In practical applications, when energy saving is the first priority, the linear motion component can be dominant, provided that communication range constraints remain satisfied. On the other hand, when estimation accuracy is the first priority, the circular motion can be dominant.

The rest of this paper is organised as follows. Section II gives a solution to the localization method using distance-only measurements when two agents are executing independent circular motions. The method is then extended to the case when circular motion and linear motion are combined in Section III. Section IV gives a velocity consensus algorithm for multiple agents and Section V provides some simulation results regarding the strategies developed. Concluding remarks and directions for future research are given in Section VI.

## II. LOCALIZATION WITH ZERO TRANSLATIONAL VELOCITIES

We assume the agents are kinematic points in $\mathbb{R}^2$, and each has its own local coordinate frame. In this section, we restrict the agents to having zero net translational velocity. Consider a pair of agents, 1 and 2. Each agent is moving on a circle with a certain radius, direction and angular velocity. Agent 1 knows the radius and angular velocity of itself and

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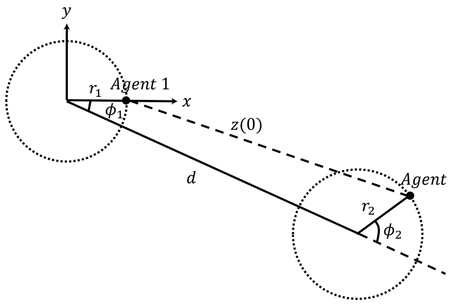


Fig. 1. Set up a coordinate system fixed on agent 1's circle.

can only measure the distance but not bearing of the agent 2. Agent 1 needs to find the circle radius of agent 2, the angular velocity of agent 2 and the position of agent 2's circle centre with respect to agent 1's coordinate frame, as well as agent 2's instantaneous position. Conversely, agent 2 also needs to identify the parameters of agent 1.

Our approach to allowing agent 1 to find the radius of agent 2's motion is as follows. Suppose $r_1$ is the radius of agent 1's motion, $r_2$ is the radius of agent 2's motion, $\omega_1$ is the angular velocity of agent 1, $\omega_2$ is the angular velocity of agent 2, $z(t)$ is the distance at time $t$ between agent 1 and 2 and $d$ is the distance between the two circle centres. At the initial time $t = 0$ as shown in Fig. 1, suppose $\phi_1$ is the angle by which a line joining the circle centres would have to be rotated in a counterclockwise direction to coincide with the line from agent 1's circle centre to agent 1 itself (i.e., $\phi_1$ is the initial phase of agent 1 relative to the line joining the two circle centres), and $\phi_2$ is the initial phase of agent 2. Suppose further that $\omega_1$, $\omega_2$, $r_1$ and $r_2$ are nonzero. Note we set the sign (or equivalently the direction) of $\phi_1$ and $\phi_2$ to be the same as those of $\omega_1$ and $\omega_2$ respectively and $|\phi_1|, |\phi_2| \in [0, 2\pi)$.

Based on Fig. 1, we set up a local coordinate system for agent 1 with origin at the centre of agent 1's circle. Agent 1 is on the $x$ axis when $t = 0$. So the bearing of the circle centre of agent 2's motion (with respect to the direction associated with agent 1's position on its circle) is $-\phi_1$.

For future reference, we note that collision avoidance will be guaranteed if the following condition, not immediately checkable by either agent, is satisfied:

$$d > r_1 + r_2 \tag{1}$$

Assume counterclockwise angular velocities to be positive. Then we can record the kinematic equation of the 2-agent system:

$$\begin{aligned} z^2(t) =& [d + r_2 cos(\omega_2 t + \phi_2) - r_1 cos(\omega_1 t + \phi_1)]^2 \\ &+ [r_2 sin(\omega_2 t + \phi_2) - r_1 sin(\omega_1 t + \phi_1)]^2 \end{aligned} \tag{2}$$

Rewriting (2) using trigonometric function transformations, we have:

$$\begin{aligned} z^2(t) =& (d^2 + r_1^2 + r_2^2) \\ &+ 2dr_2 cos(\omega_2 t + \phi_2) \\ &- 2dr_1 cos(\omega_1 t + \phi_1) \\ &- 2r_1 r_2 cos[(\omega_1 - \omega_2)t + (\phi_1 - \phi_2)] \end{aligned} \tag{3}$$

This equation demonstrates that the time function $z^2$ is a sum of a constant and three sinusoids. By taking the Fourier transform of this function, we obtain a frequency domain function $F(\omega)$. Since $F(\omega)$ is a complex function, we can write

$$F(\omega) = A(\omega) e^{i\Theta(\omega)}$$

$$\begin{aligned} A(\omega) =& (d^2 + r_1^2 + r_2^2)\delta(\omega) \\ &+ dr_1 \delta(\omega - |\omega_1|) \\ &+ dr_2 \delta(\omega - |\omega_2|) \\ &+ r_1 r_2 \delta(\omega - |\omega_1 - \omega_2|) \end{aligned} \tag{4}$$

Note the negative frequency terms in $F(\omega)$ are ignored.

While the Fourier transform above has been calculated, one can also conceive of measuring the time function $z^2$ and numerically obtaining its transform. From the transform, It is then possible to identify the values of $\omega_2$, $r_2$ and $d$ as argued below when the values of $\omega_1$ and $\omega_2$ are generic; in particular, they must be such that no two of the four frequencies 0, $|\omega_1|$, $|\omega_2|$ and $|\omega_1 - \omega_2|$ take the same value. The contrary and special cases will be discussed later in this section.

First, we can use the frequency of $\omega_1$ to identify the term $dr_1\delta(\omega - |\omega_1|)$. From the magnitude of this term and the value of $r_1$, we obtain the value of $d$.

Second, we can identify the zero frequency term $(d^2 + r_1^2 + r_2^2)\delta(\omega)$ and find its magnitude. Because the magnitude of this term equals $d^2 + r_1^2 + r_2^2$ and we already know $r_1$ and $d$, the value of $r_2$ can be found.

After that, we can use the value of $dr_2$ to identify the term $dr_2\delta(\omega - |\omega_2|)$, from which we can obtain $|\omega_2|$. In a 'bad geometry' situation, when $r_1 \approx d$, there is likely to be some difficulty finding $|\omega_2|$ because the magnitudes of terms $dr_2\delta(\omega - |\omega_2|)$ and $r_1 r_2 \delta(\omega - |\omega_1 - \omega_2|)$ would be close or the same. However, this will not happen provided the collision avoidance condition (1) holds.

Once we have obtained the absolute value of $\omega_2$, we need to determine its sign. In (3), we notice that there is a term $-r_1 r_2 cos[(\omega_1 - \omega_2)t + (\phi_1 - \phi_2)]$. As a result, if and only if $\omega_1$ and $\omega_2$ have the same sign, there will be a peak at $|\, |\omega_1| - |\omega_2| \,|$; if and only if $\omega_1$ and $\omega_2$ have different sign, there will be a peak in the transform at $|\, |\omega_1| + |\omega_2| \,|$. Since agent 1 knows the sign of $\omega_1$, it obtains the sign of $\omega_2$.

Next, observe that

$$\phi_1 = \frac{\omega_1}{|\omega_1|}\Theta(|\omega_1|) + \pi \tag{5}$$

$$\phi_2 = \frac{\omega_2}{|\omega_2|}\Theta(|\omega_2|) \tag{6}$$

From (5) and (6) and knowledge of the phase of $F(\omega)$, $\phi_1$ and $\phi_2$ can be found. Note the terms $\frac{\omega_1}{|\omega_1|}$ and $\frac{\omega_2}{|\omega_2|}$ mean that the signs of $\phi_1$ and $\phi_2$ are the same as those of $\omega_1$ and $\omega_2$ respectively.

So the bearing of circle centre of agent 2's motion is given by:

$$-\frac{\omega_1}{|\omega_1|} arg(F(|\omega_1|)) + \pi$$

The solution above is based on a presumption that all four frequency components arising in $F(\omega)$ terms are distinguishable. In special cases, when $\omega_1 = \pm\omega_2$ *or* $\omega_2 = 2\omega_1$, these terms are not distinguishable and it is only possible to obtain the bearing of the circle centre of agent 2's motion. As a result, when controlling a formation using the circular motion strategy for neighbor agent localization, it is very desirable that the angular velocities of agents do not fall into the special cases. For this purpose, we note that if angular velocities can be a priori assigned to agents, we can avoid special cases by just using a limited number of distinct values across all agents. For example, we have:

*Proposition 1:* Suppose $\mathcal{G}$ is a minimally rigid formation [4], [11] in $\mathbb{R}^2$. Then the smallest number of angular velocities needed in order to avoid the special cases described above is four.

*Proof:* In proposition 1 of [11], it is proved that one can colour the vertices of any minimally rigid graph in the plane using four different colours and ensure that there are no two vertices connected by an edge which have the same colour. Now suppose that $\omega_1, \omega_2, \omega_3$ and $\omega_4$ are four angular velocities corresponding to four colours and for any $i \neq j$ there holds $\{\omega_i \neq \pm\omega_j\} \cap \{\omega_j \neq 2\omega_i\}$. Because the colour of each agent is different from the colours of its neighbours, the angular velocities of agents cannot fall into the special cases. ■

## III. COMBINING ROTATION AND TRANSLATION

In formation control problems, velocity consensus is also an important problem. Now that we have discussed the localization problem when the agents have zero translational velocity, we will study a more general case where rotation and translation are combined and velocity consensus is to be achieved. The paper [11] gives a formation control strategy using distance-only measurements while overlooking the velocity consensus problem. It assumes that the leader's velocity is constant and initially unknown to all agents, and the followers take up positions while moving with the same velocity as the leader. In this paper, the new localization method has an advantage over that of [11] that it can be expanded to motion combining rotation and translation. In this case, the velocity consensus of agents' circle centres can be achieved.

### A. Problem statement

Consider two point agents, 1 and 2. Each agent performs a combination of circular and rectilinear motion, so each has a certain radius, direction and angular velocity for the circular motion and velocity for the rectilinear motion. Agent 1 knows its own radius, angular velocity and the translational velocity of its circle centre and can only measure the distance but not bearing of the agent 2. Conversely, agent 2 knows its radius, angular velocity and the velocity of its circle centre and can only measure the distance of the agent 1. The goal is for both agents to localize and sense the velocities of each other for velocity consensus purposes.

As shown in Fig. 2, we set up a global coordinate system with origin still at agent 1's circle centre. Now agent 1 and

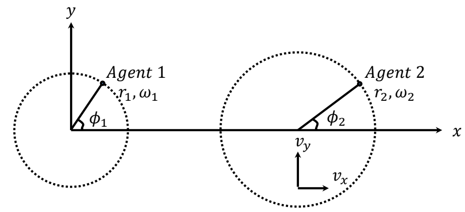


Fig. 2. Set up a coordinate system with respect to agent 1's circle centre

agent 2's circle centres are on the $x$ axis when $t = 0$. The coordinate system is defined by the agent pair, and is used for analysis purposes by us. Its orientation with respect to agent 1's local coordinate basis is not known by agent 1 at this stage though the orientation can be obtained after $\phi_1$ is learnt. The definitions of $r_1$, $r_2$, $\omega_1$, $\omega_2$, $d$, $z$, $\phi_1$ and $\phi_2$ are the same as in Section II. In addition, let $\vec{v}_{c_i}$ be the velocity of agent $i$'s circle centre, $\vec{v}_{ji}$ be the relative velocity of agent $j$'s circle centre with respect to agent $i$'s circle centre, $v_x$ be the $x$ component of the velocity $\vec{v}_{21}$, and $v_y$ be the $y$ component of the velocity $\vec{v}_{21}$. As before, the positive direction of angular velocities is counter-clockwise.

We assume in this paper that $v_x$ and $v_y$ are constant for $kT < t < (k+1)T$, $T > 0$, $k = 0, 1, 2, \cdots$ and may only change at time instants $kT$, perhaps reflecting a discrete-time consensus algorithm. We explain later how to choose $T$.

Now as a replacement for (3), there holds

$$\begin{aligned} z^2(t) =& [d + v_x t + r_2 cos(\omega_2 t + \phi_2) - r_1 cos(\omega_1 t + \phi_1)]^2 \\ &+ [v_y t + r_2 sin(\omega_2 t + \phi_2) - r_1 sin(\omega_1 t + \phi_1)]^2 \end{aligned} \tag{7}$$

Let $d_x = d + v_x t$ and $d_y = v_y t$ and rewrite (7) using easy algebra as:

$$\begin{aligned} z^2(t) =& (d_x^2 + d_y^2 + r_1^2 + r_2^2) \\ &+ 2d_x r_2 cos(\omega_2 t + \phi_2) \\ &+ 2d_y r_2 sin(\omega_2 t + \phi_2) \\ &- 2d_x r_1 cos(\omega_1 t + \phi_1) \\ &- 2d_y r_1 sin(\omega_1 t + \phi_1) \\ &- 2r_1 r_2 cos[(\omega_1 - \omega_2)t + (\phi_1 - \phi_2)] \end{aligned} \tag{8}$$

### B. Finding $d$, $\phi_1$, $v_x$ and $v_y$

In order to identify the value of $d$, $\phi_1$, $v_x$ and $v_y$, we allow agent 1 to measure the distance between the two agents $z(t)$ and analyse the Fourier series of the periodic extension of $z^2(t)$. Lemmas 1 and 2 show the Fourier series of some summands arising in (8) and Lemma 3 will provide the tool to show that these summands are linearly independent and can be identified separately. Theorem 1 gives details of the procedure to identify $d$, $\phi_1$, $v_x$ and $v_y$.

*Lemma 1:* Suppose $d > 0$, $v > 0$, $\omega_1 > 0$ and $T > 0$ are constants with $T = k_1 \frac{2\pi}{\omega_1}$ for some positive integer $k_1$ and $f_1(t) = (d + vt)cos(\omega_1 t)$ $\forall t \in [0, T]$. Note the domain of definition of $f_1(t)$ is bounded. Define $f_1'(t)$ to be the periodic extension of $f_1(t)$. Let $c_n$ be the coefficients of Fourier series

$$f_1'(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi nt/T}$$

Then, if $n = k_1$, there holds

$$c_{k_1} = \frac{1}{2}(d + \frac{1}{2}vT) + \frac{vT}{8\pi k_1}j \tag{9}$$

If $n \neq k_1$ and $n > 0$ there holds

$$c_n = \frac{vTj}{4\pi}(\frac{1}{n-k_1} + \frac{1}{n+k_1}) \tag{10}$$

*Proof:* The lemma above can be proved in a straightforward manner by calculating the value of

$$c_n = \frac{1}{T}\int_0^T (d+vt)cos(\omega_1 t)e^{-jn\frac{2\pi}{T}t}dt \tag{11}$$

■

*Lemma 2:* Suppose $a$, $b$ and $T$ are positive constants. In the following Lemma, $T$ is free but will subsequently be restricted to be consistent with Lemma 1. Define $f_2(t) = at^2 + bt\ \forall t \in [0, T]$. Note the domain of definition of $f_2(t)$ is bounded. Define $f_2'(t)$ to be the periodic extension of $f_2(t)$. Let $c_n$ be the coefficients of the Fourier series

$$f_2'(t) = \sum_{n=-\infty}^{\infty} c_n e^{i2\pi nt/T}$$

Then, for all $n = \pm1, \pm2, \cdots$, there holds

$$c_n = \frac{aT^2}{2\pi^2 n^2} + \frac{aT^2 + Tb}{2\pi n}j$$

($j^2 = -1$)

*Proof:* The lemma can be proved in a straightforward manner by calculating the value of

$$c_n = \frac{1}{T}\int_0^T (at^2 + bt)e^{-jn\frac{2\pi}{T}t}dt \tag{12}$$

■

*Lemma 3:* Suppose $n_1$, $n_2$, $n_3$, $n_4$, $k_1$ and $k_2$ are six different positive integers. Then the matrix

$$\begin{bmatrix} \frac{1}{n_1^2} & \frac{1}{n_1} & \frac{1}{n_1-k_1} + \frac{1}{n_1+k_1} & \frac{1}{n_1-k_2} + \frac{1}{n_1+k_2} \\ \frac{1}{n_2^2} & \frac{1}{n_2} & \frac{1}{n_2-k_1} + \frac{1}{n_2+k_1} & \frac{1}{n_2-k_2} + \frac{1}{n_2+k_2} \\ \frac{1}{n_3^2} & \frac{1}{n_2} & \frac{1}{n_3-k_1} + \frac{1}{n_3+k_1} & \frac{1}{n_3-k_2} + \frac{1}{n_3+k_2} \\ \frac{1}{n_4^2} & \frac{1}{n_2} & \frac{1}{n_4-k_1} + \frac{1}{n_4+k_1} & \frac{1}{n_4-k_2} + \frac{1}{n_4+k_2} \end{bmatrix} \tag{13}$$

is full rank.

*Proof:* The lemma can be proved in a straightforward manner by calculating the value of the matrix determinant.

■

In the following theorem, we show that each agent can estimate the position and translational velocity of the other agent using distance-only measurements over an interval of time $T$. For now we assume that the angular velocities of agents 1 and 2 are commensurate. We later explain what happens if $\omega_1$ and $\omega_2$ are incommensurate.

*Theorem 1:* For a pair of point agents in $\mathbb{R}^2$, if each agent is executing a combination of circular motion and linear motion and the associated angular frequencies are commensurate, each agent can find the position and translational velocity of the other agent by distance-only measurements over an interval.

*Proof:* The definitions of $r_1$, $r_2$, $\omega_1$, $\omega_2$, $d$, $z$, $v_x$, $v_y$, $\phi_1$ and $\phi_2$ are the same as in Section III-A. We choose $T$ so that there exist integers $k_1, k_2$ defining the multiple which $T$ represents of the periods associated with the two angular velocities, i.e. $k_1 = \frac{\omega_1 T}{2\pi}$ and $k_2 = \frac{\omega_2 T}{2\pi}$.

Suppose one continuously measures $z$ for a time period $T$ and finds the Fourier series of the periodic extension of $z^2$. Consider (8) and suppose $c_n$ are the coefficients of Fourier series of the periodic extension of $z^2$, $s_n$ are the coefficients of Fourier series of the periodic extension of $(d_x^2 + d_y^2 + r_1^2 + r_2^2)$, $u_n$ are the coefficients of Fourier series of the periodic extension of $-2d_x r_1 cos(\omega_1 t + \phi_1) - 2d_y r_1 sin(\omega_1 t + \phi_1)$ and $w_n$ are the coefficients of Fourier series of the periodic extension of $2d_x r_2 cos(\omega_2 t + \phi_2) + 2d_y r_2 sin(\omega_2 t + \phi_2)$.

From (8) we know that for any $n > 0 \cap n \neq |k_1 - k_2|$ there holds

$$c_n = s_n + u_n + w_n \tag{14}$$

Note the coefficients of Fourier series of the term $-2r_1 r_2 cos[(\omega_1 - \omega_2)t + (\phi_1 - \phi_2)]$ in (8) are always zero except for the index $n = |k_1 - k_2|$.

Define constants:

$$U = r_1(j\frac{v_x T}{4\pi} + \frac{v_y T}{4\pi})e^{j(\phi_1 + \pi)} \tag{15}$$

and

$$W = r_2(j\frac{v_x T}{4\pi} + \frac{v_y T}{4\pi})e^{j(\phi_2)} \tag{16}$$

Suppose further that $R = \frac{(v_x^2+v_y^2)T^2}{2\pi^2}$ and $I = \frac{(v_x^2+v_y^2)T^2+2v_x dT}{2\pi}$. From Lemma 1, Lemma 2 and (8) we know for any $n > 0 \cap n \neq k_1, k_2$ *or* $|k_1 - k_2|$ there holds

$$\begin{aligned} c_n = &\frac{1}{n^2}R + \frac{1}{n}Ij \\ &+ (\frac{1}{n-k_1} + \frac{1}{n+k_1}) \cdot 2U + (\frac{1}{n-k_2} + \frac{1}{n+k_2}) \cdot 2W \end{aligned} \tag{17}$$

From (17) and Lemma 3 we know that if we have four values of $c_n, n > 0 \cap n \neq k_1, k_2$ *or* $|k_1 - k_2|$, we are able to find the unique solutions of $R$, $I$, $U$ and $W$. Because $d$, $v_x$ and $v_y$ are all real numbers, ideally $R$ and $I$ should also be real numbers. However, sometimes due to noise or error, the $R$ and $I$ obtained from matrix operations may be complex numbers. This will not affect the process below because the values of $R$ and $I$ will not be used in the calculation below.

Now we have the value of $U$ and $W$ and can obtain $u_{k_1}$. Furthermore, from Lemma 1 and (15) we know that

$$u_{k_1} - \frac{U}{2k_1} = \frac{r_1}{2}(d + \frac{1}{2}v_x T - \frac{1}{2}v_y Tj)e^{j(\phi_1+\pi)} \tag{18}$$

$$U = r_1(j\frac{v_x T}{4\pi} + \frac{v_y T}{4\pi})e^{j(\phi_1+\pi)} \tag{19}$$

and $d$, $v_x$, $v_y$ and $\phi_1$ can be found from these equations.

The solutions for $d$ and $\phi_1$ are given by

$$d = \frac{2}{r_1}|u_{k_1} - \frac{U}{2k_1} + \pi jU| \tag{20}$$

$$\phi_1 = arg(u_{k_1} - \frac{U}{2k_1} + \pi jU) + \pi \tag{21}$$

and the solutions for $v_x$ and $x_y$ are given by

$$v_x = \mathrm{Im}(\frac{4\pi U}{Tr_1 e^{j(\phi_1+\pi)}}) \tag{22}$$

$$v_y = \mathrm{Re}(\frac{4\pi U}{Tr_1 e^{j(\phi_1+\pi)}}) \tag{23}$$

■

*Remark 1:* In this method, it does not matter how large $|\vec{v_{12}}|$ is. But in practice, when noise is significant, it is desirable to have relatively small $|\vec{v_{12}}|$. This is because of the fact that if $|\vec{v_{12}}|$ is too large in comparison with $|\omega_1 r_1|$ and $|\omega_2 r_2|$, the circular motions may be too insignificant to be adequately detected and the method might fail. But note that although the relative translational velocity should be small, the absolute translational velocities can still be large to save fuel.

*Remark 2:* In the special situation where there are no rotations and both agents are executing linear motion, the absolute value of the relative velocity between these agents can be obtained from the Fourier series of the term $(d_x^2 + d_y^2 + r_1^2 + r_2^2)$ in (8) but the direction cannot be found. This result is the same as the situation described in Section 5.2.1 of the previous paper [11].

*Remark 3:* When $\omega_1$ and $\omega_2$ are incommensurate, then $z^2(t)$ in (8) is an almost periodic function [13] and one cannot have $T = k_1\frac{2\pi}{\omega_1} = k_2\frac{2\pi}{\omega_2}$ with $k_1/k_2$ a rational number. Thus at least one or maybe both of $k_1$ and $k_2$ are not integers. Now $T$ should be chosen (and, as guaranteed by the theory of almost periodic functions, it can be so chosen by taking it sufficiently large) to ensure that both $k_1, k_2$ are close to integers (and indeed one may be an integer). Then the Fourier coefficients in Lemma 1 and Lemma 2 are different; their expressions have extra additive terms which are small if the deviation of $k_1$ and $k_2$ from integer numbers are small. Thus in Theorem 1 we can still find $d$, $v_x$, $v_y$ and $\phi_1$ with some error which is also small if the deviations of $k_1$ and $k_2$ from integer numbers are small. The longer $T$ is, the more accurate the results are.

## IV. Velocity consensus stability for multiple agents

In this section, we use the estimation algorithm presented in the previous section to solve a multi-agent control problem. Assume there are $N$ agents in $\mathbb{R}^2$ having single-integrator dynamics. Each agent is executing a combination of circular motion and linear motion. The control goal is to achieve velocity consensus of the circle centre of each agent. So in the discussion below, *when we talk about velocity, it is assumed that we are referring to the translational velocity of circle centres.* We assume each agent collects the distance information of its neighbors from time $kT$ until $(k+1)T$ for $k = 0, 1, 2, \cdots$ and updates its estimate of its neighbors' translational velocities at time instants $(k+1)T$. We further assume the translational velocity consensus is performed in a discrete-time manner, i.e. the translational velocities of all agents are constant when $kT < t < (k+1)T \ \forall k = 0, 1, 2, \cdots$ and just changes at time instants $kT$. Let $a_{ij} = 1$ if and only if agents $i, j$ are neighbours of the associated consensus graph and $a_{ij} = 0$ otherwise.

Then according to Theorem 2 of [2], velocity consensus is asymptotically achieved using the consensus algorithm

$$\vec{v}_{c_i}\big((k+1)T\big) = \vec{v}_{c_i}(kT) + \varepsilon \sum_{j=1}^{n} a_{ij}\Big(\vec{v}_{c_j}(kT) - \vec{v}_{c_i}(kT)\Big) \tag{24}$$

for all initial states if $0 < \varepsilon < 1/\Delta$ where $\Delta = \max_i(\sum_{i \neq j} a_{ij})$ is the maximum degree of the network.

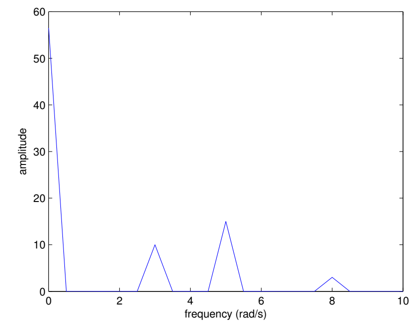


Fig. 3. The fast Fourier transform of $z^2$

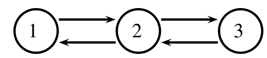


Fig. 4. A example of multi-agent system

## V. Simulation

In this section, we provide simulation results to demonstrate the effectiveness of the localization method in Section II, the velocity identification method in Section III and the translational velocity consensus algorithm in Section IV.

### A. Localization problem

Consider a localization problem in Section II, when $\omega_1 = 5$, $\omega_2 = -3$, $d = 5$, $r_1 = 1$ and $r_2 = 1.5$, the plot of $A(\omega)$ is as Fig.3.

Since Matlab [14] uses a fast discrete Fourier transform to generate a Fourier transform, the frequency resolution $\delta\omega$ no longer approaches zero.

In the simulation result, there are four peaks and the areas under the peaks are equal to $d^2 + r_1^2 + r_2^2$, $dr_1$, $dr_2$ and $r_1 r_2$ respectively.

### B. Velocity consensus problem

Consider a multi-agent system shown in Fig.4, suppose $\omega_i$ is the angular velocity of agent $i$, $T_i$ is the sampling time interval of agent $i$, $(v_{xi}, v_{yi})$ is the translational velocity of agent $i$ and $(p_{xi}, p_{yi})$ is the position of circle centre of agent $i$. In the simulation, we set $\omega_1 = \omega_3 = 5$, $\omega_2 = -3$, $T_1 = T_2 = T_3 = 2\pi$. When $t = 0$, $(v_{x1}, v_{y1}) = (-4, 2)$,

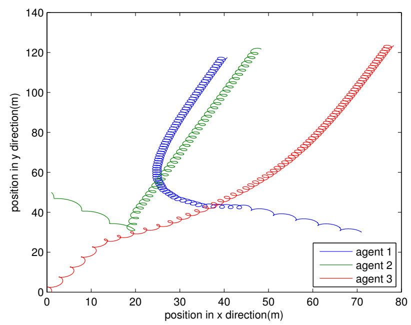


Fig. 5. Results of Theorem 1: The trajectories of agents

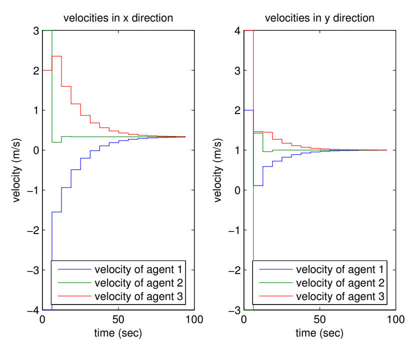


Fig. 6. Results of Theorem 1: The translational velocities of agents

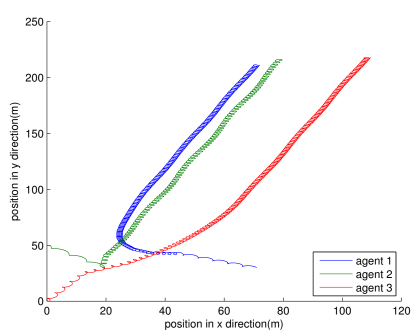


Fig. 7. Results of Remark 3: The trajectories of agents

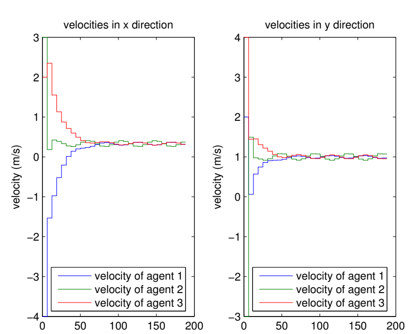


Fig. 8. Results of Remark 3: The translational velocities of agents

$(v_{x2}, v_{y2}) = (3, -2)$, $(v_{x3}, v_{y3}) = (2, 4)$, $(p_{x1}, p_{y1}) = (70, 30)$, $(p_{x2}, p_{y2}) = (0, 50)$ ,$(p_{x3}, p_{y3}) = (0, 0)$ and $\varepsilon = 0.35$. All the parameters are in SI units. The trajectories of the agents are shown in Fig.5 and the velocities of agents are shown in Fig.6.

To illustrate the statement in Remark 3, we change the angular velocity of agent 2 from $-3$ to $-\pi$ rad/s. Then the angular velocities of the agents are not commensurate. The results are shown in Fig.7 and Fig.8.

## VI. Conclusion

In this paper, we proposed a strategy to achieve translational velocity consensus using distance-only measurements for multiple agents. Given the fact that for agents to execute arbitrary motions, distance-only measurements cannot provide enough information for achieving velocity consensus, we studied agents performing a combination of circular motion and linear motion.

In further research, we are looking to achieve formation control and velocity consensus using agents' perturbations, such that agents are not limited to perform a combination of circular motion and linear motion. In addition, it appears very likely that the same strategy as we proposed in this paper can be used in velocity consensus using bearing-only measurements.

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