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Multiagent Self-Localization Using Bearing Only Measurements

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Abstract—**This paper proposes a two stage approach to solving a simple network localization problem arising in the control of multi-vehicle formation shapes using bearing-only measurements. While it is impossible for one agent to localize, in its own coordinate basis, a second agent undergoing arbitrary plane motion using bearing-only measurements, this paper shows how to use a combination of a Fourier Transform and an overdetermined linear system of equations to allow two agents undergoing plane circular motion to localize each other. It is postulated that each agent only knows the parameters fully describing its own motion and must determine enough parameters of the other agent to localize it. A Fourier Transform of the measured bearing is used by each agent to obtain an approximate magnitude of the other agent's angular velocity and a two-dimensional search grid is used in an overdetermined linear equation system to solve the localization problem. The paper investigates the effect of noise in bearing measurements on the accuracy of the proposed method, offering some potential methods of decreasing the effect of noise.**

I. INTRODUCTION

The advantages of autonomous vehicles are being realized with growing speed and many are already employed in various applications, ranging from defence, surveillance, environmental monitoring, industrial condition monitoring to search and rescue. In some applications, use of multiple autonomous agents in a formation can offer many advantages over a single autonomous agent. Formations of agents will be able to cover a search area faster, provide wider coverage and in formation can achieve tasks such as geolocation which may be impossible for a single agent. In addition, they can be of simplified construction with fewer sensors or high-end hardware which results in lower cost and higher reliability. Particularly in geolocation, formation shape preservation can be an important task.

At least two main approaches to formation control exist [1]. Centralized control utilizes a single command agent to manage a formation. The disadvantages associated with centralized control include cost and complexity of the command agent and inability to scale to larger formations. Distributed control, where each agent partakes in formation control, is the more common approach [2]. Most established formation control algorithms aimed at achieving, or stabilizing, a given shape require knowledge by each agent of the bearing and distance of neighbouring agents and it is commonly assumed that both can be measured directly. This is so even when distances alone are the only entities being controlled [4]. Current distributed control laws may also make use of information flow between agents, which is often in the form of signal transmission [5]. In practical situations, there is however often a payload limitation, especially for smaller vehicles. For some military operations it is undesirable to transmit radio signals. These reasons, among others, provide motivation to consider eliminating direct sensing of either the bearing or distance variable and somehow inferring the value of the missing variable. Previous work has proved that it is possible for agents to localize each other using range-only measurements at discrete time intervals and stabilize a formation [6]. This suggests the same possibility might hold with bearing-only measurements. It is known also that localization of agents in a formation is possible using instantaneous bearing only information, together with one distance; however, the algorithms appear to require inter-agent communication [7].

The key contribution of this paper is to show that when bearing data alone is collected over a period of time, when the motion of a pair of neighbour agents has certain characteristics and when each agent has an inertial navigation capability in its own coordinate basis, then the inter-agent distance can also be determined without information exchange. By providing the necessary location information from practical measurements, this method has significant potential for formation shape control. As is reasonably standard in formation control work, each agent is modelled as a point-agent.

The paper is structured as follows. The section immediately following the introduction, section II, builds a framework, provides a geometric representation and establishes the problem definition. Section III details the steps used in obtaining the localization solution at Agent 1. In this section, we assume the Fourier Transform yields a precise value of the magnitude of Agent 2's angular velocity, and the direction is a given piece of data. The modifications needed when at best an approximate value of Agent 2's angular velocity is known and/or the direction is unknown are detailed in Section IV. The section also gives insight to factors affecting solution accuracy and special cases. The inherent problem of noise in practical applications is analysed in Section V to predict robustness along with suggested improvements. Section VI contains concluding remarks.

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II. Framework

As indicated in the introduction, the proposed approach to formation shape control rests on understanding how two agents with bearing measurements between them can localize each other in their respective coordinate bases. This section sets out the framework for considering that task in detail. In an ambient two-dimensional space, each agent of a pair executes simple circular orbits about their respective centres. Their angular velocities and orbit radii are unequal and the centres of the orbits of the two agents are separated by a fixed distance such that the orbits do not cross. Each agent knows only its own angular velocity, initial phase and radius of orbit relative to its own local coordinate system. Agent 1 continuously collects measurements of the bearing between some axis in its local coordinate system and Agent 2. Using this information, Agent 1 must localize Agent 2 in Agent 1's coordinate basis and determine Agent 2's angular velocity, radius of orbit, initial phase and centre of orbit relative to itself (and vice versa for Agent 2). If these values are obtained then the inter-agent distance at any instance in time is obtained and can be used in formation control algorithms. The key is to ensure that each agent's motion can be characterized by a finite dimensional vector, and that this vector is observable (in the usual sense of control theory) over a finite interval using bearing only data. It is possible, and will be appropriate in many practical situations, to postulate that there is a superimposed linear motion component, associated with movement of the whole formation. In fact, it is more straightforward to think of a whole formation which is translating, each agent having the same velocity, and then sinusoidal oscillations by each agent are added around the nominal trajectory. For simplicity, we will dispense with the translation feature here. It may also be postulated that agents undergo circular motion for a specific period of time (or number of revolutions) in order to localize each other, then move on to perform formation shape tasks.

A. Geometric Model

The origin of the local coordinate basis for Agent 1 is positioned at the centre of orbit of Agent 1 and counter-clockwise rotation is taken to be positive. For the localization problem as seen by Agent 1, see Fig. 1.

- $z(t)$ is the absolute distance between Agent 1 and Agent 2 and is a function of time.
- $\theta(t)$ is the bearing angle measured by Agent 1, relative to its *x* axis, and is a function of time.
- d is the fixed linear distance between the centre circles of Agent 1 and Agent 2.
- For $i = 1, 2$:
    - r_i is the orbit radius of Agent i about centre i.
    - ω_i is the angular velocity of Agent i.
    - ϕ_i is the initial phase of Agent i relative to Agent i's *x* axis.
- x_2 is the position of centre 2 along Agent 1's *x* axis.
- y_2 is the position of centre 2 along Agent 1's *y* axis.
- $\sqrt{{x_2}^2 + {y_2}^2} > r_1 + r_2$ must be maintained. (This condition ensures the orbits do not intersect.)

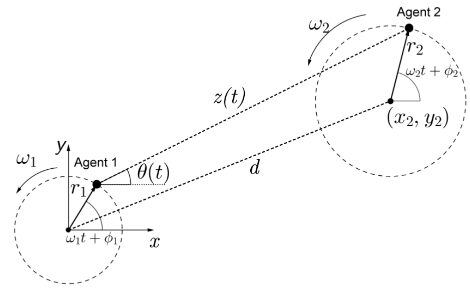

Fig. 1. Geometric Model

B. A Geometric Relation

The following equations can be obtained from the geometric model:

$$x_2 = r_1 \cos(\omega_1 t + \phi_1) + z(t)\cos(\theta(t)) - r_2 \cos(\omega_2 t + \phi_2)$$

$$y_2 = r_1 \sin(\omega_1 t + \phi_1) + z(t)\sin(\theta(t)) - r_2 \sin(\omega_2 t + \phi_2)$$

These can be rearranged to form $\sin(\theta(t))$ and $\cos(\theta(t))$ and $z(t)$ is an unknown, time-varying distance which cannot be measured by Agent 1. Dividing $\sin(\theta(t))$ by $\cos(\theta(t))$ removes $z(t)$ and yields:

$$\tan(\theta(t)) = \frac{y_2 + r_2 \sin(\omega_2 t + \phi_2) - r_1 \sin(\omega_1 t + \phi_1)}{x_2 + r_2 \cos(\omega_2 t + \phi_2) - r_1 \cos(\omega_1 t + \phi_1)} \tag{1}$$

Rearranging for all known values to be on the left hand side:

$$\begin{aligned} r_1[\sin(\omega_1 t + \phi_1) - \tan(\theta(t))\cos(\omega_1 t + \phi_1)] = \\ y_2 - \tan(\theta(t))x_2 + r_2[\sin(\omega_2 t + \phi_2) \\ - \tan(\theta(t))\cos(\omega_2 t + \phi_2)] \end{aligned} \tag{2}$$

This equation will play a key role in the sequel where we describe how to pass from knowledge of $\theta(t)$ over an interval, of ω_1, r_1 and ϕ_1, to knowledge of x_2, y_2, ω_2, r_2 and ϕ_2, and thus the coordinates of Agent 2 in Agent 1's coordinate basis.

C. The Nature Of $\tan(\theta(t))$

When ω_1 and ω_2 are commensurate, i.e. ω_1/ω_2 is rational, then $\tan(\theta(t))$, and thus $\theta(t), \cos(\theta(t))$, etc, is a periodic function.

When ω_1 and ω_2 are incommensurate, the numerator and denominator of $\tan(\theta(t))$ are almost periodic functions [3]. While the term 'almost periodic' is a technical term, it is also descriptive: for any almost periodic function $f(t)$ and any $\epsilon > 0$, there exists a T such that $|f(t) - f(t+T)| < \epsilon$ for all t. This means that, given such a T, use of Fourier techniques, e.g. FFT, on f over an interval of length T will deliver an approximate trigonometric series description of f as a weighted sum of sinusoids of different frequencies and associated phase offsets.

III. Solution Using Fourier Transform And Overdetermined Linear Equation System

The localization problem as seen by Agent 1 is solved in two stages described below in detail. In summary, the initial stage applies the Fast Fourier Transform to $\theta(t)$ measured over a certain time interval in order to determine $|\omega_2|$. At this point, we also assume that the sign of ω_2 is known a priori, so ω_2 is known. (Steps detailing action when the estimate of ω_2 is not exact and/or when the sign of ω_2 is not known, are covered later). Next, the other unknowns y_2, x_2, r_2 and ϕ_2 are found using a search grid of finely spaced values for ϕ_2 and an overdetermined linear system of equations.

A. Fast Fourier Transform

Underpinning the use of a Fast Fourier Transform is the following result.

Theorem 1: The bearing-angle function $\theta(t)$ can be expressed as a linear combination of a constant and an infinite series of sinusoids whose angular frequencies are integer-weighted linear combinations of ω_1 and ω_2.

This result is immediate in the periodic case, but not immediate in the almost periodic case. In outline, it holds because $\theta(t)$ can be obtained as a nonlinear function of $\cos(\omega_i t), \sin(\omega_i t)$ for $i = 1, 2$. It is well known that polynomial expressions of $\cos(\omega_i t), \sin(\omega_i t)$ for $i = 1, 2$ are trigonometric polynomials in which the only frequencies occurring are integer-weighted linear combinations of ω_1 and ω_2. It is nontrivial but not surprising that the result covers more general nonlinear functions. The proof is omitted due to the space limitation.

By taking the Fast Fourier Transform of $\theta(t)$, $|\omega_2|$ can be determined for most situations (some exceptions are discussed later in section IV-B). This is because $|\omega_2|$ is one of two prominent peaks when plotting the FFT of $\theta(t)$, the other being $|\omega_1|$. Depending on the relative values there are likely to be clear peaks also at some integer combinations of ω_1 and ω_2. (Simulations are discussed in the next section.) It is also true that the Fast Fourier Transform of $\tan(\theta(t)), \sin(\theta(t))$ or $\cos(\theta(t)$ contains angular frequencies at linear combinations of ω_1 and ω_2, $n\omega_1 + m\omega_2$, where n and m are integers; such peaks may be more prominent in $\tan(\theta(t)), \sin(\theta(t))$ or $\cos(\theta(t)$ than $\theta(t)$ alone. It then becomes much less obvious which frequency corresponds to ω_2. This will be evident from simulations presented below.

As noted above, at this point we assume that the sign of ω_2 is independently available. Thus the FFT delivers the value of ω_2.

B. Overdetermined System Of Linear Equations

The left hand side of equation (2) is known and if both ω_2 and ϕ_2 are known then the right hand side becomes linear in the unknowns y_2, x_2 and r_2. We are assuming ω_2 is known. Hence the difficulty lies in obtaining ϕ_2. In order to avoid the need to solve non-linear equations involving trigonometric functions we use a one-dimensional grid search based on selecting a finite but closely spaced set of values of ϕ_2. Let:

$$u(t) = r_1[\sin(\omega_1 t + \phi_1) - \tan(\theta(t))\cos(\omega_1 t + \phi_1)]$$
$$v(t) = -\tan(\theta(t))$$
$$w(t) = \sin(\omega_2 t + \phi_2) - \tan(\theta(t))\cos(\omega_2 t + \phi_2)$$

An overdetermined system of n equations can be formed (where $n > 3$ and $n \in \mathbb{Z}$) by using n steps in time, $t_1, t_2, t_3, ..., t_n$ and their respective values of $\tan(\theta(t))$ as follows:

$$\begin{bmatrix} u(t_1) \\ u(t_2) \\ u(t_3) \\ \vdots \\ u(t_n) \end{bmatrix} = \begin{bmatrix} 1 & v(t_1) & w(t_1) \\ 1 & v(t_2) & w(t_2) \\ 1 & v(t_3) & w(t_3) \\ \vdots & \vdots & \vdots \\ 1 & v(t_n) & w(t_n) \end{bmatrix} \times \begin{bmatrix} y_2 \\ x_2 \\ r_2 \end{bmatrix} \quad (3)$$

and expressed as:

$$\boldsymbol{u} = \boldsymbol{A}\boldsymbol{x} \quad (4)$$

An approximate (least squares) solution to the overdetermined system of equations is obtainable. A grid search in the space of $0^\circ \leq \phi_2 \leq 360^\circ$ at a suitable step size (0.5° was found to solve to reasonable accuracy and require little time) is conducted. Further commentary on step size choice is provided in subsection IV-A. Suppose that $\boldsymbol{u} = \boldsymbol{A}\boldsymbol{x}$ cannot be exactly solved by any $\boldsymbol{x}$ (which would require an exact value of ϕ_2). We then look for $\boldsymbol{x}^*$ with the following property:

$$\boldsymbol{x}^* = argmin||\boldsymbol{u} - \boldsymbol{A}\boldsymbol{x}^*|| \quad (5)$$

where:

$$||\boldsymbol{u} - \boldsymbol{A}\boldsymbol{x}^*|| = \sqrt{[\boldsymbol{u} - \boldsymbol{A}\boldsymbol{x}^*]^T[\boldsymbol{u} - \boldsymbol{A}\boldsymbol{x}^*]}$$

As is known,

$$\boldsymbol{x}^* = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{u}$$

At each step in the ϕ_2 search grid, the overdetermined system is solved for $\boldsymbol{x}^*$ and $||\boldsymbol{u} - \boldsymbol{A}\boldsymbol{x}^*||$ is calculated. The unknown parameters of Agent 2 are obtained by finding the $\boldsymbol{x}^*$ and corresponding ϕ_2 which has the minimum $||\boldsymbol{u} - \boldsymbol{A}\boldsymbol{x}^*||$ value with r_2 being positive.

C. Simulation Under Ideal Conditions

All simulations, unless specifically indicated otherwise, used the following values for consistency:

- $y_2 = 1000$ and $x_2 = 2500$
- $r_1 = 200$ and $r_2 = 500$
- $\omega_1 = 10$ rad/s and $\omega_2 = -8$ rad/s
- $\phi_1 = 0$ and $\phi_2 = \frac{\pi}{6}$ rad

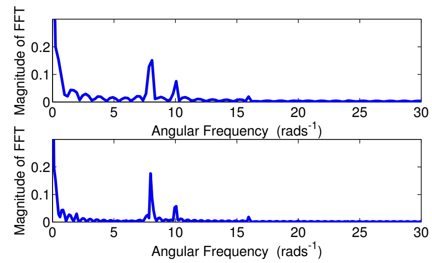

Fig. 2. FFT of $\theta(t)$ sampled over 20 seconds (top) and 40 seconds (bottom)

Figure 2 depicts the Fourier Transform of $\theta(t)$ and the two clearly prominent peaks at $|\omega_1|$ and $|\omega_2|$. As noted, we assume for the moment that the ambiguity in the sign of ω_2 can be resolved, and so we obtain $\omega_2 = -8$. It is true that the peaks in the Fourier Transform have non-trivial width and

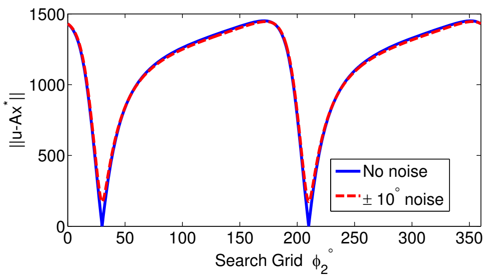

Fig. 3. $||\boldsymbol{u} - \boldsymbol{A}\boldsymbol{x}^{*}||$ over search grid of ϕ_2 with $n = 20$

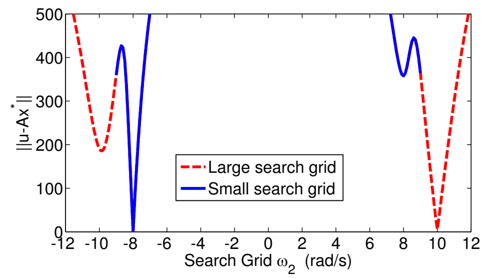

Fig. 4. $||\boldsymbol{u} - \boldsymbol{A}\boldsymbol{x}^{*}||$ over search grid of ω_2

due to sign ambiguity of ω_2 a precise value of ω_2, including direction, cannot be obtained from the plots. This issue is addressed in the following section.

The solid blue line in Fig. 3 illustrates the two values of ϕ_2 which minimise $||\boldsymbol{u} - \boldsymbol{A}\boldsymbol{x}^{*}||$. As consistently observed in all simulations, there are two minima resulting from the grid search, only one of which has $r_2 > 0$. The plot yields $\phi_2 = 30°$, consistent with what was simulated. Corresponding to this value of ϕ_2, $\boldsymbol{x}^{*}$ can be determined and from it, x_2, y_2 and r_2. The computation leads to values which are indistinguishable from the values assumed for the simulation.

IV. Solution When Exact ω_2 Is Unknown

The ideal method used to obtain ω_2 is detailed in III-A. There are two important issues with this. The first is that the ω_2 obtained via FFT is an absolute value and no special feature of the FFT is known which would indicate the rotational direction of Agent 2. Furthermore, to obtain an FFT plot with an accurate value of ω_2, $\theta(t)$ must be measured for a suitably extended period of time. This may not be practical in applications where Agent 1 may need to respond quickly to operationally-driven changes in the motion of Agent 2. When the observation time is limited, the width of the angular frequency bins in the FFT plot of $\theta(t)$ increases and less precise values of $|\omega_2|$ are available. To deal with both issues, a simple modification of the previous solution method can be used, as we now explain. The grid search is extended to a two-dimensional space of values spanning $0° \leq \phi_2 \leq 360°$ and $\left[(-|\omega_2| - \Delta\omega) \leq \pm|\omega_2| \leq (|\omega_2| + \Delta\omega)\right]$ where $\Delta\omega$ is some suitable value which creates a search interval that encompasses the possible value of ω_2, given the approximate magnitude and the peak width is known from the FFT plot. The solid blue line in Fig. 4 illustrates the search grid of ω_2 when $|\omega_2| = 8$ rad/s and $\Delta\omega = 1$ rad/s. The selection of grid spacing as well as $\Delta\omega$ is detailed in IV-A. For each step in ω_2 space, there is a solution $\boldsymbol{x}^{*}$ found associated with the optimum over ϕ_2 space, with some associated value for $||\boldsymbol{u} - \boldsymbol{A}\boldsymbol{x}^{*}||$. The localization problem is solved by finding the $\boldsymbol{x}^{*}$ and the corresponding ω_2 which minimises $||\boldsymbol{u} - \boldsymbol{A}\boldsymbol{x}^{*}||$. The fact that a fairly precise magnitude and direction of ω_2 can be identified is shown by the solid blue line in Fig. 4.

A. Factors Influencing Accuracy of Solution

There are several factors which affect both the accuracy of the unique solution and the computational time required. Of particular note is n, the number of equations in the overdetermined system of equations. The previous section did not define the requirements on n apart from $n > 3$ and $n \in \mathbb{Z}$. In fact, for an appropriately small step size in ϕ_2 and ω_2, and in the absence of noise, using a value of n above 20 has no noticeable impact on solution accuracy. However, computational time increases as n becomes large (by a significant amount if searching over both ω_2 and ϕ_2). A value obeying $20 \leq n \leq 40$ is ideal in most noiseless situations. The step sizes in the search grid for ϕ_2 and ω_2 are strongly linked to computational time but not so much to accuracy. Decreasing the step size when searching over a two-dimensional grid space dramatically increases computational time. Below a certain spacing, further decrease in step size has little effect on solution accuracy. If greater accuracy should be required, a better strategy would be to use a larger initial step size and once the approximate localization solution is established, search in a refined two-dimensional grid for increased accuracy.

The choice for $\Delta\omega$ is made based on the FFT plot. If the measuring time is very low, the FFT will not even display peaks and ω_2 cannot be resolved. Simulations show that several periods of θ must be measured for distinct peaks to be visible. As $\theta(t)$ is measured for longer, the peak of ω_2, and hence the value of $|\omega_2|$, becomes more distinct and $\Delta\omega$ decreases. The effect of increasing the measuring time is shown in Fig. 2. Increasing the size of $\Delta\omega$ increases the solution time significantly. It is therefore best to keep $\Delta\omega$ below 10% of ω_2. Due to the nature of a two-dimensional grid search, it is in fact better to allow $\theta(t)$ to be measured for longer and have a smaller value of $\Delta\omega$ than to measure for a short time and search over a large range of ω_2.

B. Special Cases

The non-generic problem where $\omega_1 = \omega_2$, $r_1 = r_2$, $\phi_1 = \phi_2$ is not solvable by any means as $\theta(t)$ will be a constant. This situation obviously must be avoided. The case where $\omega_1 = \omega_2$ by itself is enough to defeat the algorithm of this paper, and it is in fact unknown if this case is solvable by any other method or by some modification of the method provided. A solution does exist when $\omega_1 = -\omega_2$ (provided the direction of ω_2 is known beforehand) and also when $\omega_1 = n\omega_2$ where $n \in \mathbb{Z}$. In practice, it is desirable that $|\omega_1|, |\omega_2|$ are significantly different. The peaks at ω_1 and ω_2 are the two most prominent peaks for all practical cases. A second different, potentially difficult situation arises if $r_2 \ll r_1$ since ω_2 may not be a prominent peak. This is unlikely to happen in practice but should be avoided. This observation is based on numerous simulations over a range of values.

When searching across the grid space of ω_2, $||\boldsymbol{u} - \boldsymbol{A}\boldsymbol{x}^*||$ is also minimised when $\omega_{2,guess} = \omega_1$. This can present a problem when $|\omega_2| \simeq |\omega_1|$ and two solutions are possible. The dashed line in Fig. 4 shows the scenario when $\Delta\omega$ is too large, the two minima located are at ω_2 and ω_1. Note that if $\Delta\omega$ were suitably smaller, as will occur with a longer measurement interval, the problem would not arise.

C. Reduction To ω_2 Grid Only

It is also possible, when the exact value of ω_2 is unknown, to reduce the search component of the algorithm to a grid of ω_2 only. If we set new constants $a = \cos(\phi_2)$ and $b = \sin(\phi_2)$ then (2) can be rearranged to form:

$$\begin{aligned} r_1\big[\sin(\omega_1 t + \phi_1) - \tan(\theta(t))\cos(\omega_1 t + \phi_1)\big] = \\ y_2 - \tan(\theta(t))x_2 + q\big[\sin(\omega_2 t) - \tan(\theta(t))\cos(\omega_2 t)\big] \\ + p\big[\cos(\omega_2 t) + \tan(\theta(t))\sin(\omega_2 t)\big] \end{aligned} \tag{6}$$

where $q = r_2 a$ and $p = r_2 b$. The new equation is linear in the *four* constants y_2, x_2, q and p. Using a least squares approach as previously demonstrated across a search grid of ω_2 will yield the four constants with r_2 and ϕ_2 following trivially.

A range of techniques, such as the Newton-Raphson method, can be used to reduce the time taken to locate the minima. While these can be applied at the discretion of the user, the basic solution concept remains the same.

V. Situations Involving Noise

In practical applications of multi-agent formation control, a bearing measurement cannot be completely error free. It is not unreasonable to assume that there will be some form of noise or resolution tolerance. This can have a noticeable effect on the accuracy of the solution method detailed in this paper, particularly when $\theta \simeq 90°$ or $\theta \simeq 270°$. This section gives some details on the problems caused by noise including the level of noise which is acceptable. One of the major issues surrounding noise is due to the use in some equations of $\tan(\theta(t))$. The noise is introduced when an agent measures $\theta(t)$ yet the solution method using an overdetermined system employs $\tan(\theta(t))$, and when $|y_2| \gg |x_2|$, $\theta(t)$ will be close to $90°$ or $270°$. Applying tan to such numbers will cause $\tan(\theta(t)) \to \pm\infty$. The result of noise in $\theta(t)$ being magnified in $\tan(\theta(t))$ can be seen in Fig. 5, where $y_2 = 2500$ and $x_2 = 1000$.

A. Simulations With Noise

Noise of varying levels was introduced into $\theta(t)$, following a circular (von Mises) Gaussian distribution [8], which was deemed most appropriate. With a standard deviation of $\sigma = \frac{k}{3}$, the noise is with probability 0.99 less than $k°$. In Fig. 6, this is recorded as the noise limit $\pm1°, \pm2°$, etc. Practical applications involving angular measurements will often have wide bandwidth noise. This was replicated in all simulations

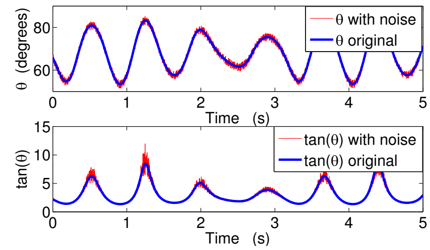

Fig. 5. Effect of noise on $\theta(t)$ and $\tan(\theta(t))$

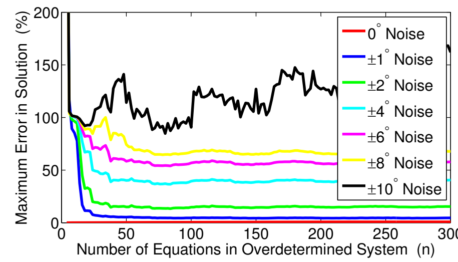

Fig. 6. Error in solution with varying noise levels

using noise with a bandwidth well above the bandwidth of $\theta(t)$. The maximum error in the localization solution was defined as $max\big[e_i = |1 - \frac{i_{Solution}}{i_{Actual}}| \times 100\%\big]$ where $i = r_2, y_2, x_2, \omega_2, \phi_2$. Figure 6 compares the effect of noise to increasing the number of overdetermined equations and clearly there is significant effect on the solution when noise has, say, $\sigma = \frac{4}{3}°$. At $\pm4°$ noise, the error is an unacceptable 25% when using 50 or more equations. In general, noise below $\pm2°$ is acceptable but the number of equations must be increased to $n \geq 50$. The effect of noise can be seen from the dashed line in Fig. 3. It is observed that $||\boldsymbol{u} - \boldsymbol{A}\boldsymbol{x}^*||$ does not approach zero and the final solution is inaccurate. In private communication with the Defence Science and Technology Organisation in Australia it was advised that typical angle error accuracy is between 1 and 10 milliradians (0.06 to 0.6°). In this paper, the angle error used in simulations is higher, corresponding to a lower quality/cheaper sensor. With high quality sensors, it appears that this method is viable.

Agent 1 itself will not know the correct values of ω_2, r_2, x_2, y_2 and ϕ_2, and vice versa for Agent 2. Hence a separate issue is whether one agent can determine the quality of its estimate of the other agent's position. An indication of the accuracy of the localization solution within the perspective of each agent can be obtained by the value, at the estimated values of ω_2, ϕ_2, of $||\boldsymbol{u} - \boldsymbol{A}\boldsymbol{x}^*||$ as compared with $||\boldsymbol{u}||$. The value of $\frac{||\boldsymbol{u}-\boldsymbol{A}\boldsymbol{x}^*||}{||u||}$ follows a similar relationship to n as $max\ e_i$, see the comparison between Fig. 6 and Fig. 7. To a rough approximation, $\frac{||\boldsymbol{u}-\boldsymbol{A}\boldsymbol{x}^*||}{||u||}$ could potentially be used by the agent to estimate the error in solution.

B. Partial Mitigation of Numerical Problems

The problem caused by the function $\tan(\theta(t))$ may be reduced by adjusting the local coordinate system such that θ has a mean of 0 or $180°$. The geometric definition of

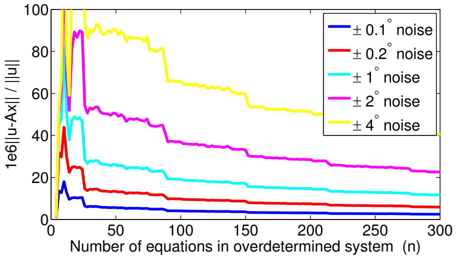

Fig. 7. $\frac{10^6 \times ||\boldsymbol{u} - \boldsymbol{A}\boldsymbol{x}^*||}{||\boldsymbol{u}||}$ due to varying noise

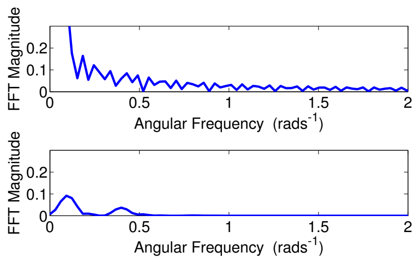

Fig. 8. Original Coordinate Axes (Top) and Rotated Coordinate Axes with Hanning Window (Bottom) measured over 120 seconds

the problem, II-A, involves Agent 1 having some arbitrary, locally defined coordinate system. As Agent 1 is merely trying to localize Agent 2, the local coordinate system can be changed without affecting the final solution. In this case, some algorithm could be implemented to have Agent 1 rotate its local coordinate system such that the centre of Agent 2's orbit is in the $\pm x$ direction. The rotation required can be found by calculating $\bar{\theta}$, the mean value of the measured bearing angle. The angle between the x-axis and the line between the two circle centres can be approximated as $\bar{\theta}$ provided the bearing is measured for an appropriate length of time.

When considering typical orbit radius and speed of real agents (such as UAVs), $|\omega| < 1$ would be realistic. The peaks of ω_1 and ω_2 can therefore be hidden in the leakage around DC in the FFT plot. Both this problem and the exaggeration of noise due to $\tan(\theta)$ can be reduced by application of a coordinate adjustment and window, as demonstrated in Fig. 8. For Fig. 8, the values for $r_1, r_2, x_2, y_2, \phi_1$ and ϕ_2 are unchanged. The angular frequencies are now $\omega_1 = 0.4$ rad/s and $\omega_2 = 0.1$ rad/s with $\pm 2°$ noise. A longer sampling time is required as the period of $\theta(t)$ has increased. Almost certainly, filters can be applied to reduce the level of noise. The time delay involved in obtaining accurate estimates would need to be weighted against the need to ensure stability in formation control.

VI. Conclusion

The solution method presented consists of two main steps. The first is to use a Fourier Transform to obtain an approximate value of $|\omega_2|$ in order to decrease the search grid of ω_2. The final step uses a least squares approach to solving an overdetermined system of equations, combined with gridding so that in fact a set of such systems is solved. This is an entirely numerical approach with the expected balance between accuracy in solution and time to solve. As multi-agent formation control is dynamic, it is desirable to reduce the solution time, including the time for data collection. Greater use of analytical ideas may increase the speed in obtaining the solution. Initial work was focused on the expression for $\tan(\theta(t))$ which proved to be difficult to analyse or express in a convenient form, nor is there a suitable expression for $\theta(t)$. This leads to the belief that a purely analytical solution is unlikely.

The ability to clearly determine peaks of the Fourier Transform depends on the length of the measurement interval. In a dynamically changing system, each agent may only be able to measure $\theta(t)$ for short periods of time and in some instances, ω_2 may not appear as a distinctive peak.

In addition, it was observed that the proposed solution method was affected by noise. Above $\pm 1°$ of circular Gaussian noise the solution obtained was found to have large errors. Potential ways to limit this include some form of filtering at the θ level. Dynamically changing the local coordinates of Agent 1 such that its x-axis is always in line with Agent 2's centre may also reduce the effects of the $\tan$ function nonlinearity at angles close to $90°$ and $270°$. This may not be needed when quality sensors are involved due to the low level of error, though it could still be useful to rotate the axis so that there is no D.C. value in the FFT. The non-generic case of $\omega_1 = \omega_2$, $r_1 = r_2$ $\phi_1 = \phi_2$ must be avoided at all cost. The unique case of $\omega_1 = \omega_2$ needs to be studied in further depth. In the case where there is no solution possible using a modification to this method, it may be prudent to ensure $|\omega_1| \neq |\omega_2|$, especially if the direction of ω_2 is not known a priori.

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