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Bias Reduction Based on Maximum Likelihood Estimates with Application in Scan-based Localization

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Abstract: In this paper, a novel bias reduction method is proposed to analytically express and reduce the bias arising in localization problems, thereby improving the localization accuracy. The proposed bias reduction method mixes Taylor series and a maximum likelihood estimate, and leads to an easily calculated analytical bias expression in terms of a known maximum likelihood cost function. In the simulations we apply the proposed method to the scan-based localization problem. Monte Carlo simulation results demonstrate the performance of the proposed method in this context.

Key Words: Bias; Maximum likelihood; Scan-based localization; Geolocation; Passive Localization

1 Introduction

The localization problem has been widely investigated in various areas for a long time and many localization algorithms have been proposed [1–3]. It is obvious that one of the most important conditions in the localization problem is the accuracy of estimated target position.

However in practical situations, errors in measurements are inevitable. A localization process normally includes an application of a nonlinear transformation of noisy measurements to produce a desired estimated target position. Due to the interaction between the noisy measurements (the measurement noise is usually, including here, assumed to be zero-mean) and the nonlinear transformation, a systematic error, or bias, arises in the estimated target position[1]. Therefore many techniques have been sought and on occasion been presented to reduce bias, enhancing the localization estimation accuracy.

Gavish et al. [6] present approximate analytical expressions for the covariance matrix of the estimation error and the bias for two well-known bearing-only localization algorithms, viz. the maximum likelihood and the Stansfield estimators. While the analytical expression for the bias is independent of the type of localization problem, other than it must be one the solution of which is expressible as the extremizing value of a maximum likelihood function, the use of three different truncated Taylor series expansions in determining the bias expression may lead to quite imprecise results. Further the process to obtain the bias expression is not direct and obvious. There is not any systematic simulation analysis in the paper. In [7], the bias in non-linear systems with noisy observables is treated as an illustrative example for applying tensor algebra. The method expands the non-linear localization function, which maps measurements to target positions, to second order in the noise using a Taylor series. The expected value of the second order term is considered as the analytic expression of bias. However the main focus of this paper is how to use tensor algebra, rather than bias analysis.

In our previous work [8], we present a bias reduction approach, which mixes Taylor series and Jacobian matrices, to formulate the bias analytically in an easy way. The proposed method, termed here the Taylor-Jacobian bias reduction method, is general in the sense that it can apply to many localization problems, being largely independent of the type of measurements and the functional relationship between target position and measurements. However when the number of usable measurements is much greater than the number of dimensions of the ambient space, the process of introducing extra variables will become more complex or even computationally unaffordable.

In this paper we propose a novel bias reduction method associated with maximum likelihood estimates. First of all we expand the localization mapping $\boldsymbol{g}$ (which maps the vector of measurements to a position vector estimate) by a Taylor series truncated at the second-order term in the measurement noise. Assuming the errors in the measurements are all zero mean and independent to each other, the expected value of the second-order term can be considered as the bias which is in terms of derivatives of $\boldsymbol{g}$. However, in a localization problem it may be very hard or even impossible to analytically express the localization mapping $\boldsymbol{g}$ and its derivatives. In our previous work [8], to overcome this problem, we adopted one property of the Jacobian matrix to formulate the bias analytically by using the inverse mapping of $\boldsymbol{g}$ (call it $\boldsymbol{f}$) which maps the (estimated) target position to the (noisy) measurements. In contrast to what is often the case with $\boldsymbol{g}$, the function $\boldsymbol{f}$ typically is available analytically, and this observation lies behind the calculations. However, when the number of obtained independent usable measurements is greater than the number of dimensions of the ambient space we need to introduce extra variables to solve the resulting overdetermined problem. This process becomes more complex or even possibly impractical when the number of usable measurements is much greater than the number of dimensions, for example when in two dimensional space with ten measurements. In the new method we compute the derivatives of $\boldsymbol{g}$ by using the maximum likelihood cost function C (which is related to $\boldsymbol{f}$) and its derivatives avoiding there-

[1]There are actually two kinds of bias in localization problem. The first one is measurement bias which means that bias exists in the measurement set. This is caused by the environmental conditions. We do not focus on this type of bias in our research work. More details can be obtained in [4, 5]. Without any further notation, we assume there is no bias in the measurements. The other type of bias arises in the estimation process, with bias free but noisy measurements being processed nonlinearly, which is the subject of this paper.

by introduction of any extra variables. To demonstrate the performance of the proposed method, we apply it to a scan-based localization algorithm. The Monte Carlo simulation results illustrate the performance of the proposed method.

The rest of the paper is organized as follows. In Section II some background aspects relevant to our work are presented. In Section III we propose the new bias reduction method which is associated with maximum likelihood estimates and briefly review the scan-based localization problem. The results of Monte Carlo simulation are provided in Section IV. Section V summarizes the paper and comments on future work.

2 Background

In this paper the Einstein summation convention is used: A repeated subscript and superscript index is an implied summation, e.g. $u_i v^i = \sum_i u_i v^i = u^T v$. Without any further notice throughout this paper vectors and matrices are denoted by lower case bold symbols and upper case bold symbols respectively. A noisy parameter will have a tilde on top. D denotes the number of dimensions of the ambient space and N denotes the number of usable measurements.

In this section, we will first explain the reasons which cause the estimation bias in localization. Consider an experiment in which a target's location $\boldsymbol{x}$ ($\boldsymbol{x} \in R^n$ denotes the location vector of a target and x^α denotes the component of the location vector ($\alpha = 1, 2, ..., D$)) is estimated from some measurements $\boldsymbol{\theta}$ ($\boldsymbol{\theta} \in R^N$ denotes the noiseless measurement vector and θ^i denotes the entry of the $\boldsymbol{\theta}$ ($i = 1, 2, ..., N$)). For each set of noisy measurements $\tilde{\boldsymbol{\theta}}$ (the measurement noise is assumed to be zero mean) the target's location is estimated from the formula $\tilde{\boldsymbol{x}} = \boldsymbol{x} + \delta\boldsymbol{x} = \boldsymbol{g}(\tilde{\boldsymbol{\theta}})$ (here $\boldsymbol{g} : R^N \rightarrow R^D$ is the localization mapping from the (noisy) measurements to the target position (estimates)). If this experiment is repeated many times one might expect that the average value of $\tilde{\boldsymbol{x}}$ is the true value $\boldsymbol{x}$. However when the mapping from target measurements to the target's location is non-linear the expected value target's location is not the true target's location, i.e. a bias occurs:

$$E[\tilde{\boldsymbol{x}}] = E[\boldsymbol{g}(\tilde{\boldsymbol{\theta}})] \neq \boldsymbol{g}(E[\tilde{\boldsymbol{\theta}}]) = \boldsymbol{g}(\boldsymbol{\theta}) = \boldsymbol{x} \tag{1}$$

This result can be summarized as follows: 'The average of the mappings of each set of noisy measurements (the measurement noise is assumed to be zero mean) is not the map of the average of all sets of noisy measurements'. We define the bias as the difference between the expected value of $\boldsymbol{x}$ based on multiple mappings of measurements and the true target location:

$$E[\delta\boldsymbol{x}] = E[\boldsymbol{g}(\tilde{\boldsymbol{\theta}})] - \boldsymbol{x} \tag{2}$$

Next we present the formulation of bias analytically. In a practical situation, the noise will always exist in the measurements and thus the estimate will:

$$\boldsymbol{x} + \delta\boldsymbol{x} = \boldsymbol{g}(\boldsymbol{\theta} + \delta\boldsymbol{\theta}) \tag{3}$$

where $\delta\boldsymbol{x}$ denotes the error in estimated target position.

To determine the bias here we consider $x^\alpha = g^\alpha(\theta^i + \delta\theta^i)$. Assume the localization mapping $\boldsymbol{g}$ is well-defined for each point and there are derivatives of any order of g^α. Now we can expand the g^α by a Taylor series and truncating it at the second order term, we have:

$$\begin{aligned} x^\alpha + \delta x^\alpha &= g^\alpha(\theta^i + \delta\theta^i) \\ &\approx g^\alpha(\theta^i) + g^\alpha_{,i}\delta\theta^i + \tfrac{1}{2} g^\alpha_{,ij}\delta\theta^i\delta\theta^j \end{aligned}$$

where $g^\alpha_{,i} = \frac{\partial g^\alpha}{\partial \theta^i}$ and $g^\alpha_{,ij} = \frac{\partial^2 g^\alpha}{\partial\theta^i\partial\theta^j}$. Hence the difference between the true position and the estimate is approximately:

$$\delta x^\alpha = g^\alpha_{,i}\delta\theta^i + \frac{1}{2} g^\alpha_{,ij}\delta\theta^i\delta\theta^j \tag{4}$$

Because the errors in measurements are independent of each other and have zero mean with covariance matrix $\boldsymbol{\Sigma}$, taking the expectation results in:

$$E(\delta x^\alpha) = \frac{1}{2} g^\alpha_{,ij}\Sigma^{ij}_\theta \tag{5}$$

where Σ^{ij}_θ denotes the ij entry of the measurement errors covariance matrix. Here the expected value of δx^α which is $E(\delta x^\alpha)$ can be considered as the bias in localization.

According to equation (5), it seems we have already obtained the analytical expression for the bias. However there is actually a difficulty buried inside: that is the analytic determination of g^α, which is apparently required if its Hessian matrix is to be computed. Though for range-measurement localization in 2-dimensional space, it is not very difficult to formulate g^α analytically, when considering, e.g. a scenario in R^3 involving a mixture of range and bearing measurements, to obtain the analytical expression of g^α becomes very challenging. Therefore, it is essential for us to present an alternative method to analytically express the derivatives of g^α, to allow computation of the bias and its consequent reduction.

3 Bias Associated with Maximum Likelihood Estimates

3.1 Bias Associated with Maximum Likelihood Estimates

From the last section, we know that the problem we are facing is how to analytically express the localization mapping $\boldsymbol{g}$ and its derivatives. In order to solve this problem, we first briefly review the localization problem. In the noiseless case, one can obtain the position of a target by solving the following formulation:

$$\boldsymbol{\theta} = \boldsymbol{f}(\boldsymbol{x}) \tag{6}$$

where the function $\boldsymbol{f} : R^D \rightarrow R^N$ is the mapping from the target position (estimates) to the (noisy) measurements and can be obtained analytically according to the geometry of the targets and anchors[2].

In practical situations, errors in measurements are inevitable. Though when $N = n$, one still can obtain a target position estimate in effect by solving $\tilde{\boldsymbol{\theta}} = \boldsymbol{f}(\tilde{\boldsymbol{x}})$. However, generally when $N \geq n + 1$, this equation will have no solution in the noisy case. In order to obtain an approximate position estimate, various methods have been proposed such as

[2]Though 'anchor' is often used as a term in wireless sensor networks denoting a sensor whose position is already known via GPS, in this paper without further notice the term 'anchor' is applied to denote a device/object whose location is already known.

maximum likelihood, least squares, etc [9], [10]. The main idea of these approaches is similar: convert the localization problem to an optimization problem as follows:

$$\tilde{\boldsymbol{x}} = \arg\min_{\boldsymbol{x}} C(\boldsymbol{x}, \tilde{\boldsymbol{d}}) \quad (7)$$

where the cost function C is related to $\boldsymbol{f}$ and can be possibly formulated as follows:

$$C = \sum_{i=1}^{N} [f_i^2 - \tilde{d}_i^2]^2 \quad (8)$$

By solving the minimization problem, we can obtain estimated positions of the unlocalized sensors. Note that algorithms for solving the minimization problem are not necessarily straightforward or of low complexity [9, 10]; this however is not the concern of this paper.

From the above analysis, we can conclude that the mapping $\boldsymbol{f}$ can be analytically written down, thus the cost function C which is related to $\boldsymbol{f}$ also can be analytically expressed. Therefore a question arises naturally: can we express the second derivative of g^α evaluated at some $\boldsymbol{\theta}$ in some other simple way avoiding introducing extra variables? The answer is yes. Next we will present how to formulate $g^\alpha_{,ij}$ in terms of derivatives of the known cost function C, evaluated at $(\boldsymbol{x}, \boldsymbol{\theta})$, where $\boldsymbol{x} = \boldsymbol{g}(\boldsymbol{\theta})$.

Theorem 1: Let $C : R^D \times R^N \to R^D$ be a known smooth cost function for which a unique solution of the optimization problem (7) is available for all $\boldsymbol{\theta} \in \Theta \subset R^N$. Let $\boldsymbol{g}(\boldsymbol{\theta})$ denote the minimizing value of $\boldsymbol{x}$. Let $\boldsymbol{w}(\boldsymbol{x}, \boldsymbol{\theta}) = \frac{\partial C}{\partial \boldsymbol{x}^T}$. The following equations hold at every point $(\boldsymbol{x}, \boldsymbol{\theta}) = (\boldsymbol{g}(\boldsymbol{\theta}), \boldsymbol{\theta})$ for $\boldsymbol{\theta} \in \Theta$ and for all $a = 1, 2, ..., D$, $\alpha = 1, 2, ..., D$, $\beta = 1, 2, ..., D$, $i = 1, 2, ..., N$ and $j = 1, 2, ..., N$:

$$w^a_{,\alpha} g^\alpha_{,i} + w^a_{,i} = 0 \quad (9)$$

and by further differentiating equation (10) in respect to θ^j and can obtain:

$$\begin{aligned} w^a_{,\alpha} g^\alpha_{,ij} &= -[w^a_{,\alpha\beta} g^\alpha_i g^\beta_j + w^a_{,\alpha j} g^\alpha_{,i} + w^a_{,\beta i} g^\beta_{,j} + w^a_{,ij}] \\ &= -[g^\alpha_{,i} \;\; 1] \begin{bmatrix} w^a_{,\alpha\beta} & w^a_{,\alpha j} \\ w^a_{,\beta i} & w^a_{,ij} \end{bmatrix} \begin{bmatrix} g^\beta_{,j} \\ 1 \end{bmatrix} \end{aligned} \quad (10)$$

Assuming invertibility for all $\boldsymbol{\theta} \in \Theta$ of the Jacobian matrix with (a, α) entry $w^a_{,\alpha}$, first derivatives of g^α can be expressed in terms of first derivatives of w^a by (9), and second derivatives of g^α can be expressed in terms of first and second derivatives of w^a by (10).

Proof: Because $\boldsymbol{g}(\boldsymbol{\theta})$ is a minimizer with respect to $\boldsymbol{x}$ of $C(\boldsymbol{x}, \boldsymbol{\theta})$, it is a zero of $\frac{\partial C}{\partial \boldsymbol{x}} = \boldsymbol{w}(\boldsymbol{x}, \boldsymbol{\theta})$, i.e.

$$\boldsymbol{w}(\boldsymbol{g}(\boldsymbol{\theta}), \boldsymbol{\theta}) = \boldsymbol{0} \; \forall \theta^i \quad (11)$$

Differentiating with respect to θ^i yields (9). Equation (10) results from differentiation of (9) with respect to θ^j. **END**

The condition of the theorem statement that the Jacobian matrix be nonsingular is not unreasonable; it is a sufficient, though admittedly not necessary, condition for the existence of a unique function $\boldsymbol{g}$.

From equation (10), we obtain the analytical expressions for the second derivatives of g^α. Substituting the formulas into equation (5) we can finally obtain the easily-calculated expressions for the bias.

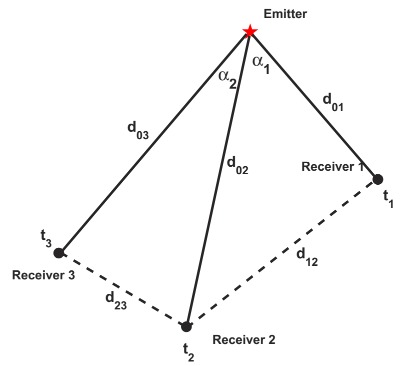

Fig. 1: An emitter scanning across three receivers; The red star indicates the radar location whereas the filled circles indicate the locations of the receivers.

In practical situations, we can obtain the inaccurate estimated position of the target by using existing localization algorithms. Then we can input the inaccurate target location into the obtained analytical expression for the bias. Finally we can improve the accuracy of the localization by subtracting the obtained bias, viz. $\tilde{\boldsymbol{x}} - bias_{\tilde{\boldsymbol{x}}}$. If necessary, one can input this target location into the expression for the bias as the argument at which second derivatives are evaluated, to improve the accuracy of the computed bias.

3.2 Brief Review of Scan-Based Localization

In order to demonstrate the performance of the proposed bias reduction method, we will apply the method to scan-based localization problem. Therefore in this subsection we will present a brief description of scan-based localization. The target in scan-based localization is normally assumed to be an emitter such as a radar, and a multi-agent system will be applied to cooperatively localize the emitter. In such a situation the emitter is always assumed to have a (generally mechanically) rotating radar antenna with a narrow beam; the scan direction and scan rate are assumed constant and and indeed known to each agent (as a receiver) in the multi-agent system, which records the time instants at which the rotating beam passes the agent [10].

For ease of exposition, here we consider a situation with one emitter and three receiving agents, and assume that all lie in a plane. Fig. 1 shows an emitter (whose location is $x = (x_1, x_2)^T$) scanning across three receivers at time t_1, t_2 and t_3. The emitter is scanning clockwise. For scan-based localization, the separate time values are not important, but rather their differences, $t_{12} = t_2 - t_1$ and $t_{23} = t_3 - t_2$. In fact, we treat t_{12}, t_{23} as quasi-measurements. Each quasi-measurement (together with knowledge of the scan rate and direction) in the noiseless case defines a circle of computable center and radius, and indeed an arc of such a circle on which the emitter must lie. The pair of sensors determining the time difference lies at the end points of the arc. The intersection of two such circular arcs defines the emitter position; thus there is a vector function $\boldsymbol{g} = (g_1, g_2)^T$, the localization mapping, of the two variables t_{12}, t_{23}, with $\boldsymbol{g}$ embodying a formula for the intersection of two circular arcs. Again, the localization mapping $\boldsymbol{g}$ in scan-based localization problems is hard or even impossible to be analytically written down directly. Therefore finding an analytic expression for the derivatives of mapping is a significant challenge.

According to the proposed bias reduction method in the last subsection, we know that we can apply the cost function C to denote the derivatives of $\boldsymbol{g}$ thus resulting in an analytical expression of bias. Since C is simply related to $\boldsymbol{f}$ and obtained noisy measurements (according to equation (8)), now our aim is to obtain an analytic form for the mapping $\boldsymbol{f}$. Again take the situation shown in Fig. 1 for example. For ease of exposition here we present the scalar forms which is much more clearly and easily understood. Let α_1, α_2 denote the angles subtended at the emitter by the lines joining it to the two pairs of physical sensors, see Fig. 1 Since the scan rate ω is a known constant, we can obtain the following equations:

$$\alpha_i = \omega t_{i,i+1} \qquad i = 1, 2 \tag{12}$$

Given the three receivers at known locations $p_1 = (x_1, y_1)$, $p_2 = (x_2, y_2)$ and $p_3 = (x_3, y_3)$, it is straightforward to see that

$$\alpha_i = \arccos \frac{d_{0,i}^2 + d_{0,i+1}^2 - d_{i,i+1}^2}{2 d_{0,i} d_{0,i+1}} \qquad i = 1, 2 \tag{13}$$

where

$$d_{0,i} = \sqrt{(x - x_i)^2 + (y - y_i)^2}, \quad i = 1, 2, 3$$
$$d_{i,i+1} = \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2}, \quad i = 1, 2$$

Substituting equations (13) into (12), we can obtain the following formulas:

$$t_{i,i+1} = f_i(x, y) = \arccos \frac{d_{0,i}^2 + d_{0,i+1}^2 - d_{i,i+1}^2}{2 d_{0,i} d_{0,i+1} \omega} \qquad i = 1, 2 \tag{14}$$

Equation (14) provides an analytic formula for $\boldsymbol{f}$.

Further we should note that in scan-based localization (in the noisy situation), the physical measurements are quasi-measurements $\tilde{t}_{12} = \tilde{t}_2 - \tilde{t}_1$ and $\tilde{t}_{23} = \tilde{t}_3 - \tilde{t}_2$. This leads to

$$\tilde{t}_{12} = \tilde{t}_2 - \tilde{t}_1 = t_{12} + \delta t_{12}$$
$$\tilde{t}_{23} = \tilde{t}_3 - \tilde{t}_2 = t_{23} + \delta t_{23}$$

Though the noises in $\tilde{t}_1$, $\tilde{t}_2$ and $\tilde{t}_3$ are all assumed to be i.i.d Gaussian noise with zero mean and known variance σ^2, δt_{12} and δt_{23} are no longer independent. The covariance matrix Σ is given by

$$\Sigma = 2\sigma^2 \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix} \tag{15}$$

where $2\sigma^2$ is the variance of an individual time-difference measurement. Note that the means of δt_{12} and δt_{23} remain zero.

Now we have obtained the analytical expression of $\boldsymbol{f}$ and the covariance matrix. The following process to compute the estimation bias is the same as the one described in last subsection. We do not show the details here. In next section we will apply Monte Carlo simulation to demonstrate the performance of the proposed bias reduction method in scan-based localization.

4 Simulation Results

In this section, the simulation results will be shown to demonstrate the performance of the proposed bias reduction method in scan-based localization problems. All the simulated data are provided by the Defence Science Technology Organization (DSTO) Australia.

The simulations were done using DSTO's synthetic integration lab (SIL). This accurately simulates existing radar and receiver systems, it includes accurate, senor, emitter, terrain and propagation models. The fidelity of the SIL means that there is no significant difference between its simulation results and those generated by field tests.

Simulation results are provided in 2-dimensional space with two scenarios: (1) three receivers and one emitter (2) four receivers and one emitter. Different emitter positions are considered.

The simulation set-up is as follows:

- The measurement error for each usable measurement is produced by a Gaussian noise with zero mean and known variance σ^2. The level of noise (the standard deviation σ) is adjusted in the simulation set as 0.05, 0.1 or 0.15 seconds for time measurements t_i.
- All the simulation results are obtained from 1000 Monte Carlo experiments.
- In the simulations the bias is considered as the average absolute distance (average of 1000 experimental results) between the true emitter position and the estimated emitter position. In the simulation figures it is termed the 'average absolute distance error'.[3]
- The scan rate is $\frac{4\pi}{5}$ radians per second.
- The distance unit used in the simulations is kilometers (km).
- The time unit and the angle unit used in the simulations are seconds and radians.
- 'Analytical bias' denotes the bias value computed by using the analytical expression derived from the proposed method
- 'Without bias reduction method' denotes the bias arising in localization without using any bias reduction.
- 'After bias reduction' denotes the bias value which is equal to the bias without using any bias reduction method minus the analytical bias computed by the proposed bias reduction method[4]

1) Three Receivers Scenario

[3] In practice, the bias is a vector whose entries can be negative or positive. Here we only focus on how large the bias is. Therefore the absolute distance between the estimated target position and the true position is used to evaluate the bias.

[4] The values of the bias without using any bias reduction method and analytical bias are Euclidean norms of two 2-vectors (in two-dimensional ambient space). The 'after bias reduction' value is the Euclidean norm of the difference of these two 2-vectors.

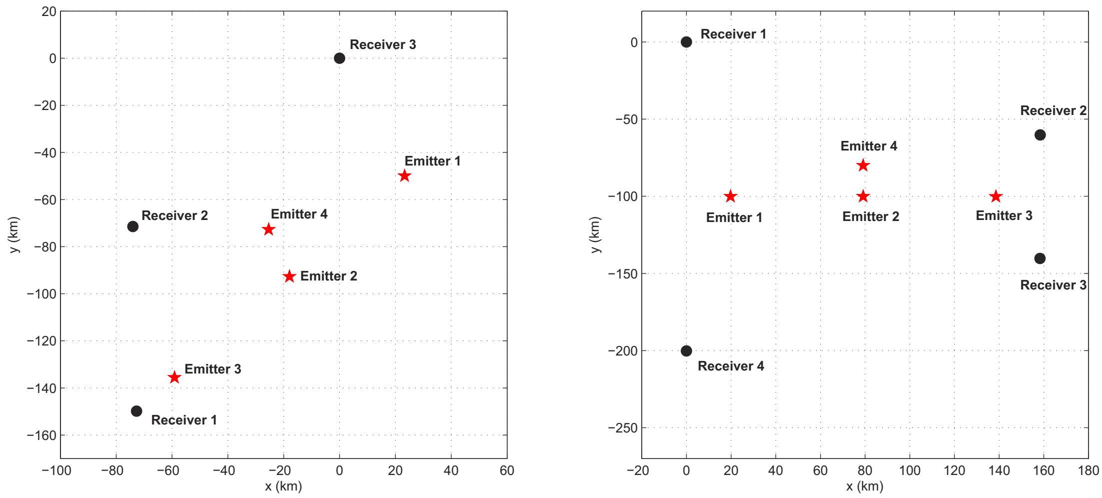

Fig. 2: Positions of emitters and receivers (a) Three receivers situation (b) Four receivers situation

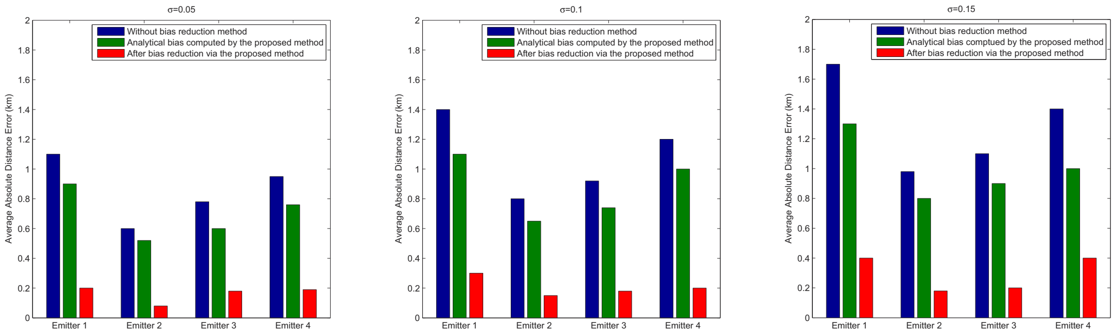

Fig. 3: Three receivers scenario with different noise level (a) σ=0.05 (b) σ=0.10 (c) σ=0.15

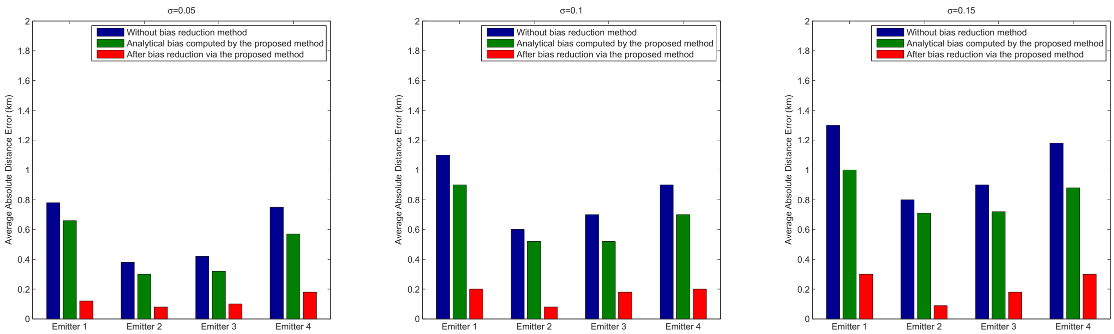

Fig. 4: Four receivers scenario with different noise level (a) σ=0.05 (b) σ=0.10 (c) σ=0.15

In this situation, three receivers give rise to two quasi-measurements (t_{12} and t_{23}). Therefore the ambient space dimension n is equal to the number of measurements N.

Fig. 2(a) depicts the three receivers and the three different emitter positions. The three receivers are located at (-72.74, -149.86), (-74.05, -71.46) and (0, 0). The four different emitter positions are (23.32,-49.98), (-17.937,-92.719), (-59.096,-135.554) and (-25.364, -72.736).

Fig. 3(a) illustrates the comparison of the bias in three situations: without any bias reduction method (blue bar), analytical bias computed by the proposed method (green bar), and after bias reduction via the proposed method (red bar). The standard deviation of the measurement error is 0.05 seconds ($\sigma = 0.05$). Evidently, the proposed bias reduction methods can reduce the bias (the red bar is much lower than the blue bar). Further from the figure we can see that the height of the green bar is almost always very close to the blue bar, which means that the analytical bias computed via the proposed method is almost equal to the original bias before bias reduction. This result, from another point of view, verifies the performance of the proposed bias reduction method. Moreover from the figure we can see the proposed method can reduce the localization bias by typically between 70% and 80% for any of the four emitter positions.

The effect of differently adjusting the level of noise to 0.1 seconds ($\sigma = 0.1$) and 0.15 seconds ($\sigma = 0.15$) is depicted in Fig. 3(b) and Fig. 3(c) respectively, from which we can conclude that though the bias is enlarged when the level of noise increases the proposed bias reduction method still performs very well (the analytical bias is still close to the original bias and after reduction the bias is no more than 0.4 km). The simulation results demonstrate that the proposed bias reduction method is robust to the noise level.

2) Four Receivers Scenario

In this situation, there are four receivers, and we obtain three independent quasi-measurements. We denote these quasi-measurements as t_{12}, t_{23} and t_{34}. The ambient space dimension $n = 2$ is less than the number of quasi-measurements, which corresponds to the $N > n$ situation.

Fig. 2(b) shows the location of the four receivers and the four emitters positions. The four receivers are positioned at (0, 0), (158.41,-60.21), (158.29,-140.26) and (0,-200.24). There are four different emitter positions: (19.77,-100.12), (79.17,-100.15), (138.58,-100.21) and (79.19,-80.08).

Fig. 4(a) shows the simulation results in 2-dimensional space with three quasi-measurements. Again, from the figure we can see that the proposed method reduces the bias very effectively (reducing it by up to 75%). Further the analytical bias computed by the proposed bias reduction method (the green bar) is always close to the original bias (the blue bar). Moreover, by comparing to the simulation results for the three receivers scenario (Fig. 3(a)) we can see that introducing an extra usable measurement, unsurprisingly, improves the accuracy of the localization.

Fig. 4(b) and Fig. 4(c) illustrate the performance of the proposed bias reduction method with different levels of noise ($\sigma = 0.1$ and $\sigma = 0.15$). Similar to the three receivers scenario, the proposed method is effective in reducing bias even with a high level of noise. Again, comparing to the three receiver scenario (Fig. 3(b) and Fig. 3(c)) the accuracy of the localization is enhanced by introducing one more usable measurement.

5 Conclusion

In this paper, a novel bias reduction method based on a maximum likelihood estimate is proposed. We first expand the localization mapping $\boldsymbol{g}$ by a Taylor series truncated at the second order term. The expected value of the second order term is considered as the bias. However to analytically express $\boldsymbol{g}$ and its derivatives is very hard or even impossible, and so we need to develop a method to overcome this problem to evaluate the bias. In our previous work [8], we adopt one property of Jacobian matrix to express the bias analytically by using the inverse mapping of the localization mapping. However, when the number of measurements is much greater than the number of dimensions of the ambient space, extra variables are required which increases the computational burden. In the proposed new method this is circumvented by working with derivatives of the maximum likelihood cost function (which must be evaluated at the maximum likelihood estimate). In order to demonstrate the performance of the proposed bias reduction method we apply it to the scan-based localization problem. Monte Carlo simulation results demonstrate the performance of the proposed method. Our future work is to seek to improve the performance of the current bias reduction method through truncating the Taylor series at higher-order terms.

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