Localization and Circumnavigation of a Group of Targets by a Single Agent Using Bearing Measurements

Lu Xia, Mohammad Deghat, Brian D. O. Anderson and Yiguang Hong

***Abstract*— This paper considers the problem of localization and circumnavigation by a single agent of a group of targets which are either stationary or moving slowly with unknown speed. The system inputs include the agent's position and the bearing angles of the targets. There are two modes of signal reception for the agent: continuous and segmented. With continuous signal reception, the agent is able to receive bearing information from all targets at the same time. With segmented reception, bearing information is only received from one target at a time. The concept of a virtual center or target for the agent to circle around is introduced as well as a method of calculating it. An estimator is proposed, initially for the continuous reception case with stationary targets, to localize the virtual center as well as a control law that forces the agent to move on a circular trajectory around the virtual center such that both the estimator and the control system convergence to the correct trajectory are exponentially stable. Then the case where signal is received by the agent in a segmented manner is studied in the same way. The algorithms can be used when the targets are not stationary, and simulations show that the algorithms remain useful when the targets do not move very fast.**

I. INTRODUCTION

A common surveillance problem is to arrange for one or possibly several agents to navigate around a target in a circular trajectory of prescribed radius. Prior literature dealing with such problems includes but is not limited to [1]–[5]. One of the simplest scenarios is that there is one stationary target to be monitored, there is a single agent, and the agent knows the initial position of the target. The task is to find a control law that causes the agent to move to and then around a circle in an agreed sense (i.e. clockwise or counterclockwise) with prescribed radius centered on the target.

There are various ways in which the problem can be made more complex, and in some of these more complex problems ideas of adaptive control or dual control [6] can appear. Simultaneous control and estimation are required, and the quality of estimation can heavily depend on the control used. A typical example is that the target has an unknown initial position. In this case, an estimator of the target position as well as a control algorithm which forces the agent to move to and then on the required trajectory are needed for the surveillance task. Problems like this with a single agent localization algorithm or a multi-agent collaborative localization algorithm have been studied in e.g. [7]–[10]. In terms of the measured data available to allow estimation and control, distance measurements between the agent and target are commonly used. For example, [11] and [12] have studied the problem where an agent continuously measures the distance between itself and the target at some unknown position and tries to circumnavigate it. The situation where bearing measurements rather than distance measurements are used is also studied in [13]. One reason for using bearing measurements is that they usually can be achieved with a passive technique, i.e. they do not require the agents to transmit signals [13].

In this paper, we investigate the case where there is a single agent but multiple targets which can be either stationary or moving. We propose estimation and control algorithms using bearing measurements for determining the estimated positions of the targets and making the agent circle around them. The algorithms are inspired by the algorithms in [13] which are for the single target case. However, the algorithms proposed here are not trivial extensions of the single target case where the target itself acts as the center of the circle around which the agent moves. In the multiple targets case, the center of the circle is not naturally or automatically defined; therefore we need to add a center estimation algorithm which changes the structure of the control algorithm and adds another layer of complexity to the problem. The algorithms we propose in this paper use bearing but not range measurements and, as is usual, avoid using derivatives of measurements, i.e. do not measure angular velocity, to avoid high-frequency noise effect. In the stationary target case, they exhibit exponentially fast convergence. In terms of robustness against noise and system uncertainties, although filtering algorithms like EKF can be applied when the measurements are noisy or when the target is moving slowly, the proposed method, without using any filtering algorithm, can tolerate measurement noise and slow movement of the target. This results from the fact that exponentially stable systems are robust against system uncertainties.

The rest of this paper is structured as follows. In section II, the problem is formally defined and the proposed solution is provided in the immediately following section. Section IV contains results from Matlab simulations demonstrating the feasibility of our algorithm. The last section gives conclusions and proposals for future works. Full proofs are not provided for the results in this paper; it can be expected that generalization of the approach in [13] will be the key to

Lu Xia is with the Australian National University, Canberra ACT 0200 (e-mail: u4488603@anu.edu.au).

Mohammad Deghat and Brian D. O. Anderson are with Research School of Information Sciences and Engineering, The Australian National University, Canberra ACT 0200 and National ICT Australia (NICTA). (e-mail: {Mohammad.Deghat, Brian.Anderson}@anu.edu.au).

Yiguang Hong is with the Key Laboratory of Systems and Control, Institute of Systems Science, Chinese Academy of Sciences, Beijing 100190, China (e-mail: yghong@iss.ac.cn).

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establishing formal proofs.

II. Problem Statement

Suppose there are N targets with unknown positions $x_i(t) \in \mathbb{R}^2$, $i \in 1, 2, 3, ..., N$ at time t and there is also an agent moving on a known trajectory $y(s) \in \mathbb{R}^2$ for $s \le t$. **Until further notice**, we shall assume all $x_i(t)$ are constant. Both the targets and the agent are assumed to be modellable as points. Let d be the desired radius of the circle enclosing all targets and on which the agent should move, let $\varphi_i(t)$, $i \in 1, 2, 3, ..., N$ be a unit vector in the direction of the line going from $y_i(t)$ to x_i, that is

$$\varphi_i(t) = \frac{x_i(t) - y(t)}{\|x_i(t) - y(t)\|} = \frac{x_i(t) - y(t)}{D_i(t)} \tag{1}$$

and let $\bar{\varphi}_i(t)$ be the unit vector obtained by $\pi/2$ clockwise rotation of $\varphi_i(t)$. The center of the circle is also relevant, and discussion of where this should lie appears below. Figure 1 shows the situation when $N = 3$ and $s = t$.

Suppose the agent can measure the bearing angles to the targets. In terms of bearing measurements, we consider two scenarios which might be applicable under different situations:

1) *Continuous measurements*: The agent measures all bearing angles $\theta_i(t), i \in 1, 2, 3, ..., N$ to the targets at the same time and for all t.
2) *Segmented measurements*: The agent measures one bearing angle at a time with a fixed interval length being used for each bearing. Let τ and $T = N\tau$ be the time taken to measure the bearing angle to one target and to all targets, respectively. The value of τ will be discussed in section III-B. The agent measures $\theta_1(t)$ for a period of τ and then successively measures $\theta_2(t)$, $\theta_3(t)$, ... each for the same period of time. When the agent has measured the bearing angle to all targets, the agent then starts a new cycle and measures $\theta_1(t)$ again.

There are two tasks. The first is to find an estimator that estimates the position of a virtual center $x(t)$ where $x(t)$ is the center of the circle that the agent is going to move around. The estimate of $x(t)$ is denoted by $\hat{x}(t)$. The choice of $x(t)$ is described below. Note that when the separate target positions $x_i(t)$ are constant, $x(t)$ will also be taken as constant. The second task is to find a control algorithm that controls the agent to move toward and then on the desired circle. Furthermore, there are two convergence criteria. First, the estimation error as given in (2) should go to zero exponentially fast when the target is stationary:

$$\tilde{x}(t) = \hat{x}(t) - x(t) \tag{2}$$

Second, it is required that $D(t) = ||y(t) - x(t)||$, where $||.||$ denotes the Euclidean norm, goes to d (which must be sufficiently large that all targets can fit in a circle of diameter $2d$) exponentially fast in the stationary target case. We term this exponential stability of the controller. Note that with exponential stability of both estimator and controller in the stationary case, one can expect robustness in the presence of target motion, i.e. if the target does not move too fast, then one would anticipate that the agent will approximately circumnavigate a moving virtual center.

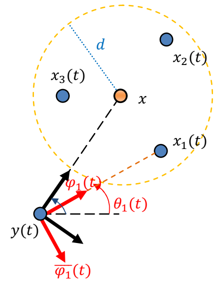

Fig. 1. An illustration of the problem and the relationships between variables.

III. Proposed Solution

We previously studied the case where there is only one target and one agent (see [13]). It is shown in [13] that both estimator and controller are exponentially stable. Before considering the case where there is more than one target, we first recall the estimator and controller equations for the case where there is only one target located at $x(t)$.

The estimator and the controller can be defined as

$$\dot{\hat{x}}(t) = k\big(I - \varphi(t)\varphi^T(t)\big)\big(y(t) - \hat{x}(t)\big) \tag{3}$$

and

$$\dot{y}(t) = \big(\hat{D}(t) - d\big)\varphi(t) + \alpha\bar{\varphi}(t) \tag{4}$$

where I is a 2×2 identity matrix, $\hat{D}(t) = ||y(t) - \hat{x}(t)||$, and k and α are positive constants. When the estimator converges, $\hat{D}(t) \to D(t)$ and according to (4), the agent moves toward the desired circle if $\hat{D}(t) \ne d$. Once it reaches the circle, it moves with the speed of α on the circle. Note that these equations give exponential convergence when $x(t)$ is constant, and robust behavior for slow motion of the target.

When there is more than one target, the first step is to find $x(t)$ which is the point (virtual center) around which the agent is going to move. The point $x(t)$ should be inside the convex hull defined by the targets. The radius of the desired circle d should also be larger than the maximum distance between $x(t)$ and the targets. We consider two different possibilities for $x(t)$ and first propose our solution for the case where continuous measurements are taken and then extend the result to the case where segmented measurements are used.

A. Continuous measurements

We continue to assume the targets are all stationary. We will later consider the case where the targets move slowly in Section IV. Our first approach is to estimate all target positions at the same time and then use the individual estimates $\hat{x}_i(t)$ to calculate $\hat{x}(t)$ as

$$\hat{x}(t) = \frac{1}{N}\sum_i \hat{x}_i(t). \tag{5}$$

When all the estimators converge, $\hat{x}(t)$ also converges to $x = \frac{1}{N}\sum_i x_i$. Similarly to (3), the estimator for target i can be written as

$$\dot{\hat{x}}_i(t) = k\big(I - \varphi_i(t)\varphi_i^T(t)\big)(y(t) - \hat{x}_i(t)) \quad (6)$$

Let $\varphi(t) = \frac{\hat{x}(t)-y(t)}{\|\hat{x}(t)-y(t)\|}$. Then we can apply the same controller as (4) in which $\hat{D}(t) = ||y(t) - \hat{x}(t)||$ and $\bar{\varphi}(t)$ is the unit vector obtained by $\pi/2$ clockwise rotation of $\varphi(t)$. Note that the unit vector $\varphi(t)$ is not a measurement of the bearing angle of an actual target anymore, but represents the bearing angle of $\hat{x}(t)$ which is the estimated position of the virtual target x, the center of mass of the individual targets. There is an important distinction now which should be made. For the single target case, the unit vector $\varphi(t)$ in (4) always points toward the target but in the multi-target case explained above, it points to $\hat{x}(t)$ rather than $x(t)$. Hence in the multi-target case with the additional level of complexity, it is harder to estimate the circle center. Thus the motion of the target circle maybe persists for longer, or is a bigger motion (i.e. the center of the target circle wanders more before convergence). In Figure 5, an example of agent movement is shown where the circling trajectory (blue line) shows the movement of the agent and the irregular trajectory (green line) shows the movement of the estimated center ($\hat{x}(t)$). The estimated center for the single target case also moves, but because the agent measures the bearing to the center, in general the motion is less irregular and more quickly convergent.

Theorem 1: Adopt the notation above and assume that all targets are stationary. Let $x = \frac{1}{N}\sum_i x_i$. Then by using the estimator (6) *and the controller* (4) *and considering* (5)*, the estimation error converges to zero exponentially fast and $\|y(t) - x\|$ goes to d exponentially fast as $t \to \infty$.*

***Proof (outline only)*:** If the estimator is stable, then the problem is similar to the single target case where we have a stationary target and we want to circumnavigate it. Then the controller is similar to the one in [13] which is stable. So we only need to show that the estimator is stable. A sufficient condition that makes the estimation error converge to zero exponentially fast is that the agent moves such that for some $T > 0$, it has a positive average speed over T along the unit vectors $\bar{\varphi}_i(t)$ (see the proof of Lemma 3 in [13] for the details). This is obvious since, similarly to (4), there is always a term along $\bar{\varphi}_i(t)$ that drags the agent in the direction perpendicular to $\varphi_i(t)$. ■

For the purposes of implementing the control law, we need the direction of x, or an estimate of this, so that $\varphi(t)$ can be constructed. There is a second possible way to do this, viz forming a unit vector which is the average of all unit vectors from the agent to the individual targets (or their position estimates). Thus one might reasonably choose:

$$\varphi(t) = \frac{\sum_{i=1}^{N}\varphi_i}{||\sum_{i=1}^{N}\varphi_i||} \quad (7)$$

The problem with the above choice of $\varphi(t)$ in (7) is that it does not always point to a fixed virtual target x. Although all targets are stationary, the unit vector $\varphi(t)$ almost always points to a different x as the agent moves. As shown in Figure 2, the estimated position of the target does not converge to a fixed point and the trajectory of the robot looks more like an ellipse than a circle. When $N = 2$, the choice of $\varphi(t)$ in (7) is the angular bisector of $\varphi_1(t)$ and $\varphi_2(t)$.

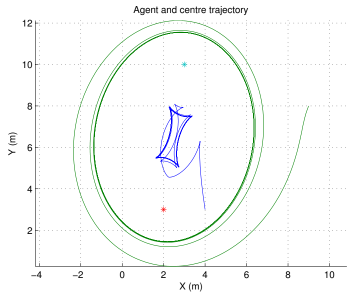

Fig. 2. The ellipses are the agent trajectory and the other trace shows the movement of center for circumnaviagation.

A similar method of calculating $\varphi(t)$ as (7) has also been considered which only uses the mid-point of the maximum and minimum bearing angles at time t, say $\varphi_{max}(t)$ and $\varphi_{min}(t)$. The problem is, apart from the similar issue with (7), $\varphi_{max}(t) - \varphi_{min}(t)$ might not be differentiable, although it is continuous, which may result in difficulties in the stability proof of the estimator and controller.

After simulation of both methods, we found that even though (7) gives a more straightforward *instantaneous* computation of a $\varphi(t)$, the estimated position of the virtual target as shown in Figure 2 does not converge to a fixed point. Thus the desired trajectory around the virtual target might not be a circle (it might be an ellipse as in Figure 2). For the above reason, we have chosen the center of gravity as our virtual center. The detailed simulations are presented in Section IV. Therefore, the estimator for each target remains the same as in (6) but the control law now becomes:

$$\dot{y}(t) = \Big(||y(t) - \frac{\sum_{i=1}^{N}\hat{x}_i(t)}{N}|| - d\Big)\varphi(t) + \alpha\bar{\varphi}(t) \quad (8)$$

where

$$\varphi(t) = \frac{\sum_{i=1}^{N}\hat{x}_i(t) - Ny(t)}{||\sum_{i=1}^{N}\hat{x}_i(t) - Ny(t)||} \quad (9)$$

and $\bar{\varphi}(t)$ is the unit vector obtained by $\pi/2$ clockwise rotation of $\varphi(t)$.

B. Segmented measurements

In this case, the agent can only measure one bearing angle at a time. It will switch between the targets at intervals of τ. If the period τ is too large, then the agent might estimate one target position unnecessarily accurately while it remains totally unaware of other targets. If τ is too small, then the agent might not have enough data to estimate the position of the target accurately and therefore give rise to errors which in a worst case may result in a collision. So there will be some kind of range of acceptable τ, in part influenced by noise, and if targets are not stationary, by the speed of their motion. Let $n = 0, 1, 2, ...$ index the number of the complete switching cycles and suppose that from time $nN\tau + i\tau - \tau$ to $nN\tau + i\tau$ the agent is measuring the bearing angle to target i and the estimated position of this target, $\hat{x}_i(t)$, will be updated. Any of the estimated position values remains the same as the value at the end of the last interval on which it was estimated while the value of that estimated position is not updated.

During the switching cycle n, the estimator of each target is given by

$$\begin{cases} \dot{\hat{x}}_i(t) = k\big(I - \varphi_i(t)\varphi_i^T(t)\big)(y(t) - \hat{x}_i(t)\}); \quad for \\ (nN + i - 1)\tau \leq t \leq (nN + i)\tau \\ \dot{\hat{x}}_i(t) = 0; \quad for \\ t < (nN + i - 1)\tau; t > (nN + i)\tau \end{cases} \tag{10}$$

After a certain number of switchings, the estimated position of the virtual target in (5) converges to x and the agent becomes stable on the prescribed circular trajectory.

Note that among $\hat{x}_1, ..., \hat{x}_N$, only one of the estimated positions is changing at any time. The center of gravity of all agents gets updated as long as the position of any target gets updated and the control algorithm is the same as (8).

C. Both measurement methods for targets with slow movement

We have discussed above how our algorithms work for the case of continuous measurements and segmented measurements. Note that $x_i(t)$ was assumed constant and now we would like to show that when the targets are moving slowly, the estimation error $\tilde{x}(t)$ converges to a neighborhood of zero. Note that there should be an upper bound for the distance between any two targets for the reason of keeping all targets within prescribed circumnavigation radius.

Theorem 2: Adopt the notation above and assume the targets are moving on differentiable trajectories and there exists a sufficiently small ε such that $\|\dot{x}_i(t)\| < \varepsilon$ and assume $\alpha - \varepsilon > 0$. Also assume that for all $t > 0$ the relative distance between agents are such that all agents lie inside a circle with radius d. Then by using estimator (6) *and controller* (8) *for the case where the measurements are taken continuously, or by using estimator* (10) *and controller* (8) *for the case where the measurements are taken in segments, the estimation error $\tilde{x}(t)$ converges to a neighborhood of zero exponentially fast and the circling radius $D(t)$ converges to a neighborhood of d exponentially fast.*

***Proof (outline only)*:** It is clear that the speed of the agent should be sufficiently larger than the speed of the target. Also, the speed of the agent along the unit vector $\bar{\varphi}(t)$ should satisfy $\alpha - \varepsilon > 0$. This guarantees that the estimator works properly. The constraint that all agents lie inside a circle with radius d ensures that regardless of the movement of the targets, there is always a circle with radius d that could include all targets. Since the estimator and the controller are exponentially stable for the stationary targets case, then having targets moving is like having the estimation and control error dynamics perturbed by a nonvanishing but bounded perturbation. So, similarly to the proof in [13], we can conclude the above theorem.■

IV. Simulations

In this section, we present the simulation results for both continuous measurements and segmented measurements. We also add slow movement to the targets and show the result in simulations. In Figure 3 and 4, we assumed there are 3 stationary targets and set the circling radius as 3. It can be seen that the estimation error exponentially goes to zero and the circling radius exponentially goes to d. In the case where measurements are taken in segments (Figure 5), we simulated 2 targets with a signal switching period of 1 second and the prescribed radius 2. As shown in Figure 6, there is some transient error in the first 20 seconds but it is decreasing and finally, the estimation error exponentially goes to zero and the circling radius exponentially goes to d which is showing that the control algorithm is working well.

Figure 7 and Figure 11 show the trajectories of the agent when the targets are moving slowly. The speeds of the targets in Figure 11 are larger than in Figure 7. It can be seen that the errors of the estimator and the controller for both cases do not converge to zero, but converge to neighborhoods of zero. The size of these neighborhoods depends on the speeds of the targets. For faster targets, the neighborhood is larger and therefore, the errors are larger (see Figure 8 and 12). Figure 9 and 10 show the case where the targets are moving slowly and the measurements are segmented. It can be seen again that the errors of the estimator and controller go to neighborhoods of zero as time increases.

V. Conclusion and Future Works

In this paper, we have extended the work in [13] to the case where there are multiple targets with either continuous bearing measurements or segmented bearing measurements. We have analyzed different ways of calculating a virtual center for circumnavigation and selected the center of gravity for our algorithm. Then we proposed estimator and control algorithms for both cases and showed that with stationary targets, the estimation errors go exponentially fast to zero and the circling radius goes exponentially fast to the prescribed distance. It is shown in simulations that for the moving target case, the estimator and controller can tolerate slow motion of targets with only modest affects on accuracy. It also appears that the larger the speed of the targets, the larger the estimation error.

Future directions of research include general collision avoidance; having three or more agents forming a polygon formation circling a target/multiple targets; and also estimating the speed of targets if they are moving at an unknown constant speed; use of method like EKF or IPDA-FR [14] to further reduce the effect of noise and to increase accuracy.

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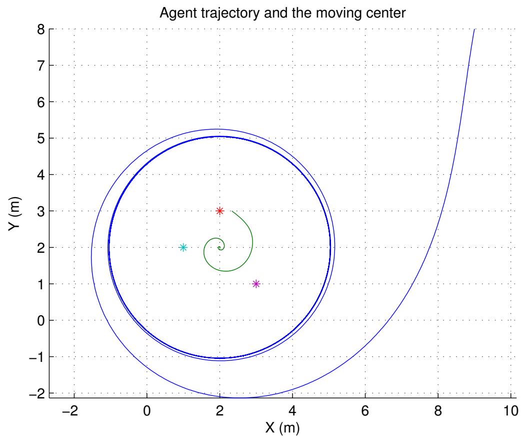

Fig. 3. Agent trajectory and estimated center trajectory in X-Y plane.

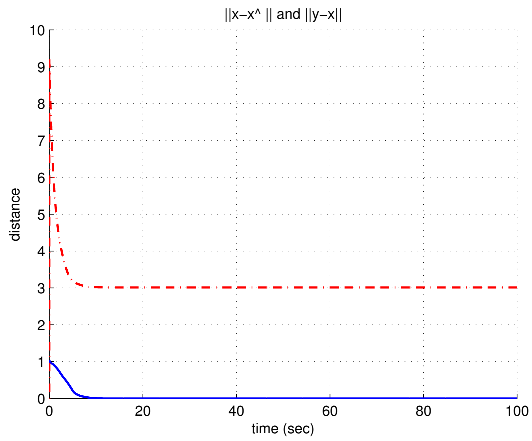

Fig. 4. $\|y(t)-x(t)\|$ (upper red line) and $\|x(t)-\hat{x}(t)\|$ (lower blue line) for the case where the measurements are taken continuously.

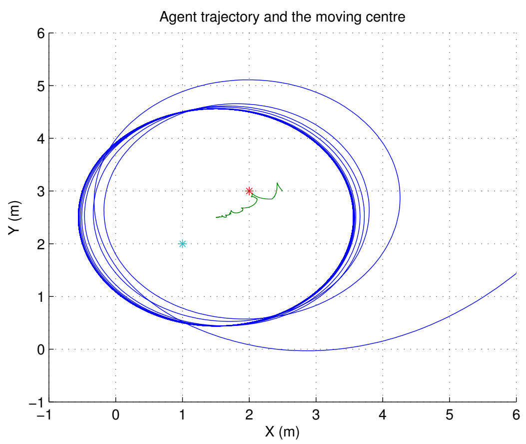

Fig. 5. Agent trajectory and estimated center trajectory in X-Y plane for the case where measurements are taken in segments.

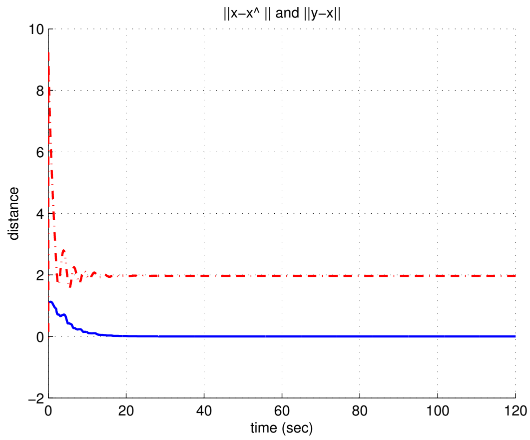

Fig. 6. $\|y(t)-x(t)\|$ (upper red line) and $\|x(t)-\hat{x}(t)\|$ (lower blue line) for the case where measurements are taken in segments.

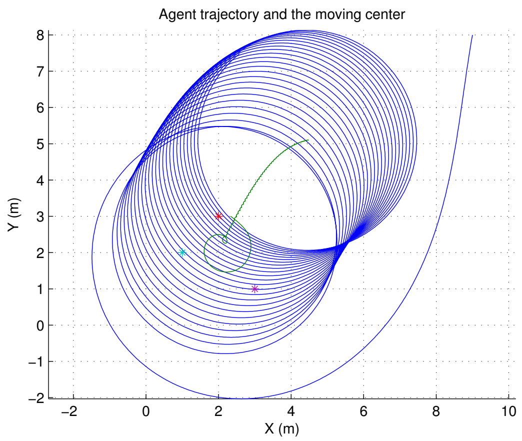

Fig. 7. Agent trajectory and estimated center trajectory in X-Y plane when targets are moving and measurements are taken continuously.

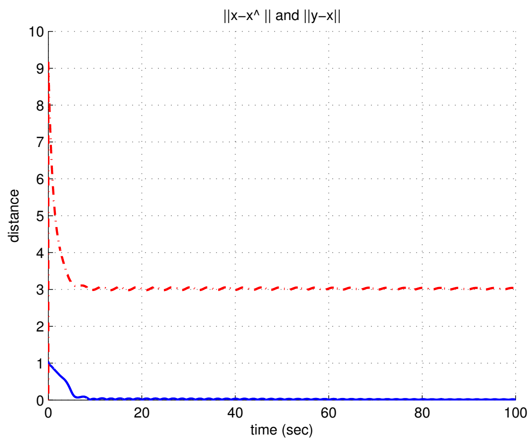

Fig. 8. $\|y(t)-x(t)\|$ (upper red line) and $\|x(t)-\hat{x}(t)\|$ (lower blue line) for the case where measurements are taken continuously and targets are moving.

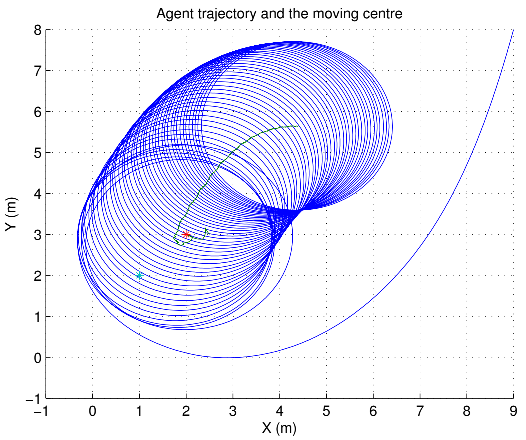

Fig. 9. Agent trajectory and estimated center trajectory in X-Y plane for the case where measurements are taken in segments and targets are moving.

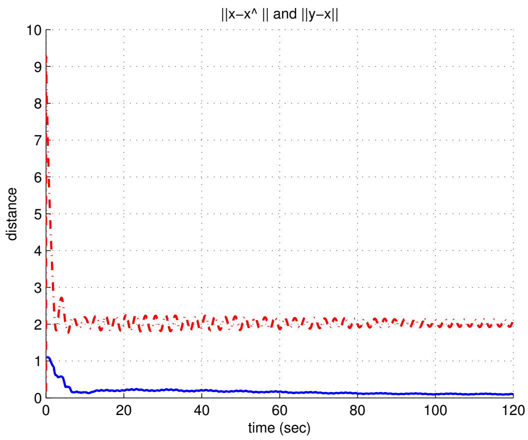

Fig. 10. $\|y(t) - x(t)\|$ (upper red line) and $\|x(t) - \hat{x}(t)\|$ (lower blue line) for the case where measurements are taken in segments and targets are moving.

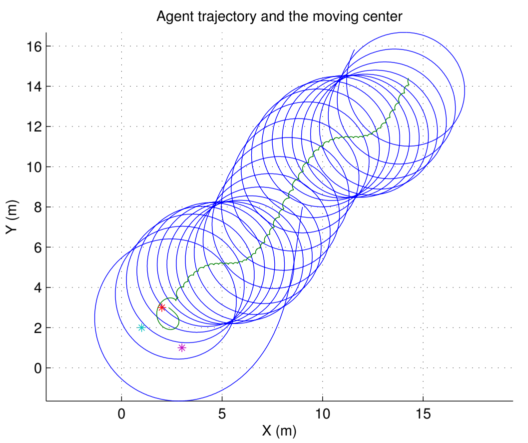

Fig. 11. Agent trajectory and estimated center trajectory in X-Y plane when targets are moving with greater speed and measurements are taken continuously.

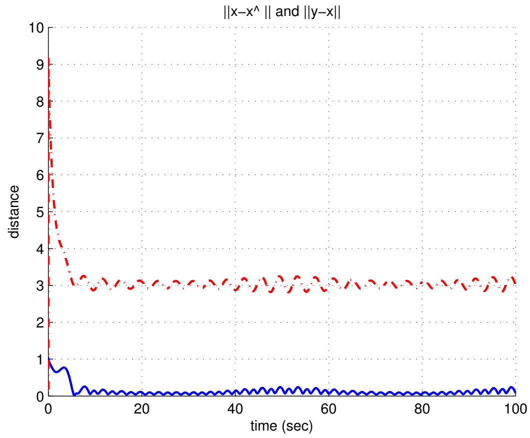

Fig. 12. $\|y(t) - x(t)\|$ (upper red line) and $\|x(t) - \hat{x}(t)\|$ (lower blue line) for the case where measurements are taken continuously and targets are moving with greater speed.

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