LOW SENSITIVITY ACTIVE FILTERS: FUNDAMENTAL REQUIREMENTS AND SYNTHESIS PROCEDURES

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Abstract

A new theory and synthesis procedure are advanced for low sensitivity active filters, based on structural passivity of building blocks. Key to the theory and synthesis methods are the concepts of bounded reality and lossless bounded real property. Wave-active filters which are well known for low passband sensitivity turn out to be special cases falling out of the new synthesis procedure. The synthesis procedure is direct, and does not start from a lumped LC prototype network.

Introduction

There has been considerable interest in the development of active RC filter structures with low sensitivity to parameter variations, during the last two decades [1]-[2]. The most significant contribution in this direction was the observation by Orchard [3] that certain suitably double terminated lossless ladder two-ports have inherently low passband sensitivity with respect to the element values. This led to numerous approaches, such as the inductance simulation approaches [4]-[6], FDNR based schemes [7] and the flowgraph based approaches [8]-[9], for developing active RC filter structures that simulate in some sense the topological properties of the double terminated lossless two-ports. Even though all of these methods do lead to very low sensitivity active RC filter structures, they are basically limited to implementing transfer functions that are realizable as doubly-terminated lossless ladders. However, there are cases where the transfer function of interest may not be implementable in such form, and hence, the question is whether low sensitivity realizations are possible for such transfer functions. If so, the next problem is how to realize such structures.

Fundamental Requirements for Low Sensitivity

Let the analog transfer function H(s) to be realized be a real rational function with the following properties:

(i) H(s) is stable, and

(ii) $|H(j\omega)| \leq 1 \quad \forall\ \omega$ (1)

Such a function will be called a bounded-real (br) function. Assume that $|H(j\omega)| = 1$ at a frequency $\omega = \omega_i$ in the passband. Figure 1 shows a typical magnitude response of such a transfer function. Note that any stable transfer function can be made into a br function simply by scaling. Assume also that the structure implementing H(s) is such that $|H(j\omega)|$ does not exceed unity for any ω regardless of the actual values of the circuit parameters α_r as long as they are in the immediate neighborhood of their nominal values α_{0r} and as long as the structure remains stable. Now, let a parameter α_r be perturbed incrementally from its nominal value:

$$\alpha_r \to \alpha_{0r} + \Delta\alpha_r \tag{2}$$

Then, $|H(j\omega_i)|$ cannot increase as a result of this perturbation, and therefore, the plot of $|H(j\omega_i)|$ as a function of α_r takes the form shown in Fig. 2 in the vicinity of α_{0r}. Thus we have zero first-order sensitivity, i.e.,

$$\frac{\alpha_r}{|H(j\omega)|}\ \frac{\partial|H(j\omega)|}{\partial\alpha_r} = 0 \tag{3}$$

for $\omega = \omega_i$ and $\alpha = \alpha_{0r}$.

If there are a number of closely spaced maxima of $|H(j\omega)|$ of unity value in the passband, we can expect very low sensitivity properties in this band. Summarizing, if the following properties are satisfied by the structure and its associated transfer function, then the structure is guaranteed to exhibit low passband sensitivity:

P(1): $|H(j\omega)| \leq 1 \quad \forall\ \omega$

P(2): $|H(j\omega_i)| = 1 \quad i = 1, 2, \ldots, N$

P(3): Property P(1) holds regardless of the exact values of the circuit parameters as long as they remain in the immediate neighborhood of their nominal values.

Property P(3) implies that the boundedness property P(1) is "structure-induced." Now, P(1) is equivalent to the condition:

$$|Y(j\omega)| \leq |X(j\omega)| \quad \text{for all input } x(t) \tag{4}$$

where $Y(j\omega)$ and $X(j\omega)$ are, respectively, the Fourier transforms of the response y(t) and the excitation x(t). Applying Parseval's relation, we thus obtain from Eqn. (4):

$$\int_{-\infty}^{\infty} y^2(t)dt \leq \int_{-\infty}^{\infty} x^2(t)dt \tag{5}$$

or in other words, the output energy delivered by the network is at most equal to the input energy delivered to the network by the source. Moreover, if Eqns. (4) and (5) are satisfied with equality sign for all inputs, then the structure is said to be lossless. As a result, a stable real rational transfer function H(s) for which $|H(j\omega)| = 1$ for all values of ω, will be called a lossless bounded-real (lbr) function.

A useful extension of the above concept is a lossless bounded-real 2 x 2 stable transfer matrix $\mathcal{T}(s) = [T_{ij}(s)]$ defined by the condition:

$$\tilde{\mathcal{T}}(s)\mathcal{T}(s) = I \tag{6}$$

where $\tilde{\mathcal{T}}(s) = \mathcal{T}^t(-s)$ and I is the 2 x 2 identity matrix. Condition (6) implies that the output energy is equal to the input energy, i.e.,

$$\int_{-\infty}^{\infty} y_1^2(t)dt + \int_{-\infty}^{\infty} y_2^2(t)dt = \int_{-\infty}^{\infty} x_1^2(t)dt + \int_{-\infty}^{\infty} x_2^2(t)dt \tag{7}$$

or equivalently to the condition

$$|T_{1j}(j\omega)|^2 + |T_{2j}(j\omega)|^2 = 1 \quad , \quad j = 1, 2 \tag{8}$$

indicating that the elements $T_{ij}(s)$ of the transfer matrix $\mathcal{T}(s)$ are scalar br functions but not necessarily lbr. The two-input, two-output network (two-pair) realizing an lbr transfer matrix will be called an lbr two-pair if it ensures the lbr property of the transfer matrix independent of the actual values of the circuit parameters. Before proceeding further, we state a few properties of br functions which are useful in the development of a synthesis procedure.

Property I. For a br function $G(s)$ with $G(0) = \pm 1$, the quantity $G(s) \frac{dG(s)}{ds}$ evaluated at $s = 0$ is negative.

Property II. For a br function $G(s)$ with $G(\infty) = \pm 1$, the quantity $G(s) \frac{dG(s)}{ds^{-1}}$ evaluated at $s = \infty$ is negative.

Property III. For a br function $G(s)$ with $G(j\omega_0) = \pm 1$, the quantity $G(s) \frac{dG(s)}{ds}$ evaluated at $s = j\omega_0$ is real and negative.

Property IV. If $G(s)$ is br then so is $G(F(s))$ where $F(s)$ is positive real. In particular, if $G(s)$ is lbr and $F(s)$ is lpr, then $G(F(s))$ is lbr. More generally, if $\mathcal{T}(s)$ is an lbr two-pair then $\mathcal{T}(F(s))$ is an lbr two pair whenever $F(s)$ is a reactance function. Proofs of these properties follow along similar lines as in the case of digital br functions [10], and are therefore omitted here.

The Synthesis Approach

The basic requirements for low sensitivity analog networks is quite similar to that recently developed for digital filters [10]-[12] where the boundedness conditions of the transfer function $G(z)$ was imposed on its magnitude on the unit circle $z = e^{j\omega}$. Following a similar approach as outlined in these works, we develop here a synthesis procedure by which an m-th order br transfer function $G_m(s)$ is realized by "extracting" an lbr two-pair such that the constraining transfer function $G_{m-1}(s)$ (Fig. 3) is br and is of lower order. This process is continued, and after successive extractions, we are then left with a cascade of lbr two-pairs constrained by a constant transfer function of magnitude less than one. The overall cascade is of the form shown in Fig. 6 and we call it the Π-cascade. It can be shown that the Π-cascade of two lbr two-pairs is an lbr two-pair and therefore, the entire structure is an lbr two-pair terminated in a br constant.

LBR Two-Pair Structures

A two-pair (Fig. 4) can be described by a chain matrix, defined as

$$\begin{bmatrix} X_1(s) \\ Y_1(s) \end{bmatrix} = \begin{bmatrix} A(s) & B(s) \\ C(s) & D(s) \end{bmatrix} \begin{bmatrix} Y_2(s) \\ X_2(s) \end{bmatrix} = \Pi(s) \begin{bmatrix} Y_2(s) \\ X_2(s) \end{bmatrix} \quad (9)$$

where the chain parameters $A(s)$, $B(s)$, $C(s)$ and $D(s)$ are related to the transfer-matrix parameters by

$$A = \frac{1}{T_{21}},\ B = -\frac{T_{22}}{T_{21}},\ C = \frac{T_{11}}{T_{21}},\ D = \frac{T_{12}T_{21} - T_{11}T_{22}}{T_{21}} \quad (10a)$$

$$T_{11} = \frac{C}{A},\ T_{12} = \frac{AD - BC}{A},\ T_{21} = \frac{1}{A},\ T_{22} = -\frac{B}{A} \quad (10b)$$

In this paper we deal exclusively with reciprocal two-pairs, characterized by $T_{12}(s) = T_{21}(s)$ or equivalently $A(s)D(s) - B(s)C(s) = 1$ for all s. The 'extraction' of a two-pair from a function $G_m(s)$ (Fig. 3) leaves behind a remainder $G_{m-1}(s)$ where G_m and G_{m-1} are related as:

$$G_{m-1} = \frac{C - AG_m}{BG_m - D},\ G_m = \frac{C + DG_{m-1}}{A + BG_{m-1}} \quad (11)$$

It can be shown that a reciprocal two-pair is lbr if and only if

$$A(s) = D(-s) = \tilde{D}(s) \quad (12a)$$
$$B(s) = C(-s) = \tilde{C}(s) \quad \text{for all } s \quad (12b)$$
$$A(s)D(s) - B(s)C(s) = 1 \quad (12c)$$

with all zeroes of $A(s)$ strictly in the left half s-plane. With the help of Eqn. (12) it can be shown that a first order reciprocal lbr two-pair must take one of the following two forms:

Type 1

Chain Parameters:

$$A = \frac{s + a}{x_1},\ B = \frac{cs + b}{x_1},\ C = \tilde{B},\ D = \tilde{A} \quad (13a)$$

Transfer Parameters:

$$T_{11} = \frac{-cs + b}{s + a},\ T_{12} = T_{21} = \frac{x_1}{s + a},\ T_{22} = -\frac{cs + b}{s + a} \quad (13b)$$

where

$$c = \pm 1 \text{ and } x_1 = \pm\sqrt{a^2 - b^2} \text{ with } \frac{b^2}{a^2} < 1.$$

Type 2

Chain Parameters:

$$A = \frac{s + a}{x_0 s},\ B = \frac{cs + b}{x_0 s},\ C = \tilde{B},\ D = \tilde{A} \quad (14a)$$

Transfer Parameters:

$$T_{11} = \frac{cs - b}{s + a},\ T_{12} = T_{21} = \frac{x_0 s}{s + a},\ T_{22} = -\frac{cs + b}{s + a} \quad (14b)$$

with $x_0 = \pm\sqrt{1 - c^2}$, $b = \pm a$ and $c^2 < 1$.

Note that Type 1 two-pair has a transmission zero (zeroes of T_{12} and T_{21}) at $s = \infty$ whereas Type 2 two-pair has a transmission zero at $s = 0$. For the lbr-based synthesis, with suitable choices of b, c and x_1, we arrive at four different forms of first order lbr two-pairs, as tabulated in Table I. Note that Type 1A has $T_{11}(\infty) = -1$ whereas Type 1B has $T_{11}(\infty) = 1$. Similarly Type 2A has $T_{11}(0) = -1$ whereas Type 2B has $T_{11}(0) = +1$. Fig. 5 shows a Type 1A two-pair.

Property IV of lbr functions as given in the previous section can be used to develop second (and higher) order lbr two pairs from the two pairs listed in Table I. Thus the following reactance transformation:

$$s \rightarrow \frac{s^2 + \beta}{s} \quad (15)$$

where $\beta = \omega_0^2$, which maps the point $s = 0$ onto $s = \pm j\omega_0$ can be used to derive the lbr two-pairs shown in Table II. Specifically, Type 3A of Table II is obtained by applying this transformation on Type 2B and Type 3B is obtained by transforming Type 2A. Tables I and II give us a complete set of lbr two-pairs that are needed in the synthesis of a br function.

The Basic Two-Pair Extraction Procedure.

Basic to our synthesis procedure is the extraction of an lbr two-pair from a br function, such that the remainder is a lower order br function. Given a br function $G_m(s)$ let us assume that it is scaled such that $|G_m| = 1$ for some s on the imaginary axis. Then there are four possible cases:

Case I: $G_m(0) = \pm 1$

Case II: $G_m(\infty) = \pm 1$

Case III: $G_m(\pm j\omega_0) = \pm 1$ for $0 < \omega_0 < \infty$

Case IV: $|G_m(\pm j\omega_0)| = 1$ but $G_m(\pm j\omega_0)$ is complex

For each of these cases, we outline "lbr extraction rules", leading to order-reduction.

Case I. Let $G_m(0) = 1$. The lbr two pair Type 2B of Table I has the property $T_{11}(0) = 1$, and $T_{12}(0) = 0$ and is clearly the most appropriate two-pair to be extracted. However, a suitable value of 'a' should be found that ensures a reduced-order br remainder $G_{m-1}(s)$. From Eqn. (11) we have

$$G_{m-1} = (a - (s + a)G_m)/(- aG_m - (s - a)) \qquad (16)$$

Setting s = 0 we find

$$G_{m-1}(0) = \frac{a - aG_m(0)}{-aG_m(0) + a} = \frac{a - a}{a - a} \qquad (17)$$

which shows that, there is a cancellation of the factor s between the numerator and denominator of Eqn. (16). G_{m-1} therefore has at most an order of m. An order reduction can be obtained by one more cancellation of the factor s. Applying L'Hospital's rule to Eqn. (16) we get

$$\lim_{s \to 0} G_{m-1} = \frac{-G_m - (s + a)G'_m}{-aG'_m - 1} = \frac{-1 - aG'_m(0)}{-1 - aG'_m(0)} \qquad (18)$$

In order to obtain the desired cancellation we have to choose $a = -1/G'_m(0)$. It can be shown that, with this choice of 'a', G_{m-1} continues to be bounded real.

If $G_m(0)$ were equal to -1 rather than 1, we can insert a negative sign ahead and proceed as above. Equivalently we can extract a modified two-pair Type 2A, (Table I) with $a = 1/G'_m(0)$. Case II can be handled in a similar manner, leading to the extraction of Type 1A or Type 1B lbr two-pairs, with values of 'a' as given in Table I.

Case III. Let $G_m(\pm j\omega_0) = 1$, $0 < \omega_0 < \infty$. We now wish to extract a second order two-pair such that the remainder has an order less than that of $G_m(s)$ by 2. The two-pair Type 3A, Table II, has $T_{11}(\pm j\omega_0) = 1$ and is the most appropriate lbr two-pair to be extracted. Once again, it remains to find the right value of 'a'. For this, we note,

$$G_{m-1}(s) = \frac{as - (s^2 + as + \beta)G_m}{-asG_m - (s^2 - as + \beta)} \qquad (19)$$

setting $s = \pm j\omega_0$ and remembering $\beta = \omega_0^2$

$$G_{m-1}(\pm j\omega_0) = \frac{a\omega_0 - a\omega_0}{a\omega_0 - a\omega_0}$$

which shows that there is a cancellation of the factors $(s \pm j\omega_0)$, and this implies that the order of $G_{m-1}(s)$ is at most that of $G_m(s)$. It can be shown that, further order reduction by 2 results by choosing $a = -2/G'_m(s)$ at $s = j\omega_0$. Note that by property III, 'a' is real and positive, regardless of whether $s = j\omega_0$ or $s = -j\omega_0$ is used. The case where $G_m(\pm j\omega_0) = -1$ can be handled by extracting the two-pair Type 3B. Note that in view of properties I, II and III, the quantity 'a' appearing in Tables I and II is always positive. Thus, as long as parameter fluctuations in an implementation do not change the sign of 'a', the lbr property is retained and this is the key for the low sensitivity properties.

Two points are worth noting before proceeding to case IV. First, referring back to Eqn. (11), if G_m has order m, and if A, B, C, D are of first order, then G_{m-1} cannot have an order smaller than m-1. Thus, after a first order lbr two-pair is extracted according to Table I, the order of the remainder is precisely one less than that of G_m. An equivalent statement holds for the second order extraction also. Second, it can be easily verified that if an lbr two-pair of a given type (say Type 1A) is cascaded to another lbr two-pair of the same type, the resulting lbr two-pair is of the same type and therefore the same order. As a consequence, if $G_m(s_0) = +1$ (where $s_0 = 0$ or ∞ or $\pm j\omega_0$) and an appropriate two-pair is extracted, the remainder G_{m-1} of lower order cannot have a value '1' at $s = s_0$. (For if it did, another two-pair of the same type could be extracted to give a remainder G_{m-2} of still lower order, and at the same time the two extracted two-pairs can be combined into one, of the same order!) Thus, the two-pair extraction has completely 'removed' the 'one' from G_m to produce G_{m-1}.

The two-pair extractions of Types 1B, 2B and 3A can therefore be considered as '1' removal operations from $s = \infty$, $s = 0$ and $s = \pm j\omega_0$ whereas those of Type 1A, 2A and 3B can be considered as '-1' removal operations. If the br function $G_m(s)$ is such that $G_m(j\omega)$ is neither '1' nor '-1' for any ω, then there is no more '1' or '-1' to be removed, and this leads to Case IV.

Case IV: If $|G_m(\pm j\omega_0)| = 1$ but $G_m(\pm j\omega_0)$ itself is not real, then we can extract a first order two-pair of the form given in Table I to obtain a remainder G_{m-1} such that $G_{m-1}(\pm j\omega_0)$ is real and $G_{m-1}(\pm j\omega_0) = 1$. For example in order to force $G_{m-1}(\pm j\omega_0) = 1$, Type 1A or Type 2A can be used, whereas to force $G_{m-1}(\pm j\omega_0) = -1$, Type 1B or Type 2B can be extracted. This extraction typically leads to a remainder G_{m-1} whose order is one higher than G_m. To consider a specific case, let us extract Type 1A. Then, in order to force $G_{m-1}(\pm j\omega_0) = 1$, 'a' should be chosen such that

$$[- s - (s + a)G_m(s)]/[sG_m(s) + s - a] = 1 \qquad (20)$$

at $s = \pm j\omega_0$, which gives,

$$a = 2j\omega_0[1 + G_m(j\omega_0)]/[1 - G_m(j\omega_0)] \qquad (21)$$

In Table III we tabulate various cases, and call the corresponding first order two-pairs, Type 4. For a br function $G_m(s)$ with $|G_m(j\omega_0)| = 1$, it can be shown that 'a' appearing in Table III are always real.

Now, we can extract a Type 3A two pair from G_{m-1} in order to get an order reduction by 2. Let the remainder function be denoted G_{m-2}. Then G_{m-2} has order one lower than G_m. Thus, further extraction of a first order two-pair is necessary to obtain a final remainder G_{m-3} of order two less than G_m. In order to see how to accomplish this, note that a Type 4 two-pair gives rise to a transmission zero at $s = 0$ (Type 4B and D) or at $s = \infty$ (Type 4A and C) which tends to force G_m to be equal to ±1 for this value of s. In order to avoid this unintentional behavior, the extraction of the first order two pair from G_{m-2} should be such as to cancel the transmission zero created by the Type 4 two-pair. The following two rules of extraction cover all possible synthesis requirements falling under case IV:

(1) If extraction of Type 4A with $a = a_1$ is followed by that of Type 3A with $a = a_2$, then a Type 4A two pair with 'a' given by $a_3 = -(a_1 + 4a_2)$ should follow. This gives an overall remainder whose order is two lower than that of G_m.

(2) Similarly if extraction of the Type 4C with $a = a_1$ is followed by that of 3B with $a = a_2$ then a two-pair of the Type 4C with 'a' given by $a_3 = -(a_1 + 4a_2)$ should follow.

The proofs are omitted here for brevity. This completes the set of all rules required for br synthesis. A numerical example is next given, to illustrate the synthesis procedure.

An Example

Consider

$$G_4 = (-27s^3 + 11s^2 - 17s + 3)/(27s^3 + 16s^2 + 19s + 6)$$

It can be verified that $G_4(s)$ is br; also $G_4(\infty) = -1$. Extracting the lbr two-pair Type 1A with suitable value of 'a' will therefore result in a second order br remainder. The appropriate value of 'a' is found from Table I to be equal to 1. Extracting

$$A = s + 1, \; B = s, \; C = -s, \; D = -s + 1$$

leads to

$$G_3 = (13s^2 - 8s + 3)/(14s^2 + 10s + 6)$$

Clearly $G_3(0) = 1/2$ and $G_3(\infty) = 13/14$, which means, we cannot extract any more 1st order lbr two-pairs for order reduction. However, $G_3(\pm j1) = \frac{-10 - 8j}{-8 + 10j}$ which means $G_3(\pm j1) \neq \pm 1$ but has magnitude unity. Let us therefore extract a Type 4C two pair with

$$a_3 = 2.j1. \frac{1-G_3(j1)}{1+G_3(j1)} = 2$$

The two-pair to be extracted is

$$A = \frac{s + 2}{2}, \quad B = \frac{-s}{2}, \quad C = \frac{s}{2}, \quad D = \frac{-s + 2}{2}$$

This leads to the remainder function

$$G_2(s) = (s^3 - 8s^2 + 19s - 6)/(s^3 - 10s^2 - 17s - 12)$$

Now $G_2(j1) = -1$ as expected. Extracting therefore a Type 3B two-pair with $\beta = 1$ and $a_2 = 1$ we get the remainder function $G_1(s) = (s - 6)/(s - 12)$. To compensate for the unwanted transmission zero introduced by the Type 4C two-pair, we now extract another Type 4C two-pair with $a_1 = -(a_3 + 4a_2) = -(2 + 4) = -6$. This gives the remainder $G_0 = 1/2$ which is a br constant. The complete realization is shown in Fig. 6.

Conclusive Remarks

It should be noticed that even though the building blocks presented in this paper are derived in an independent manner, some of them resemble the circuits presented in [13]. The main advantage of the new scheme proposed in this paper is that, we do not start from classical circuit elements, but obtain all synthesis starting from some fundamental considerations. The viewpoint advanced here has led to several other interesting results based on passivity, including new and elegant stability-test procedures, to be presented elsewhere.

Table I

First Order Reciprocal LBR Two-Pairs

Type	Parameters	When to Use
1A	$A = \tilde{D} = \frac{s + a}{a}$, $B = \tilde{C} = \frac{s}{a}$; $a = dG_m/ds^{-1}$ at $s = \infty$;	$G_m(\infty) = -1$
1B	$A = \tilde{D} = \frac{s + a}{a}$, $B = \tilde{C} = -\frac{s}{a}$; $a = -dG_m/ds^{-1}$ at $s = \infty$;	$G_m(\infty) = 1$
2A	$A = \tilde{D} = \frac{s + a}{s}$, $B = \tilde{C} = \frac{-s}{a}$; $a = 1/(dG_m/ds)$ at $s = 0$;	$G_m(0) = -1$
2B	$A = \tilde{D} = \frac{s + a}{s}$, $B = \tilde{C} = \frac{-a}{s}$; $a = -1/(dG_m/ds)$ at $s = 0$	$G_m(0) = 1$

Table II

Second Order Reciprocal LBR Two-Pairs

Type	Parameters	When To Use
3A	$A = \frac{s^2 + as + \beta}{s^2 + \beta}$; $B = \frac{-as}{s^2 + \beta}$; $D = \tilde{A}$, $C = \tilde{B}$; $a = -2/(dG_m/ds)$ at $s = j\omega_0$	$G_m(j\omega_0) = 1$
3B	$A = \frac{s^2 + as + \beta}{s^2 + \beta}$; $B = \frac{as}{s^2 + \beta}$; $D = \tilde{A}$, $C = \tilde{B}$; $a = 2/(dG_m/ds)$ at $s = j\omega_0$	$G_m(j\omega_0) = -1$

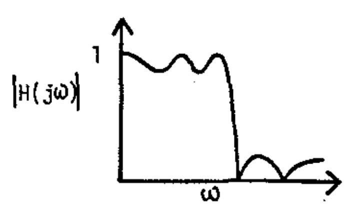

Fig.1. A Typical Response

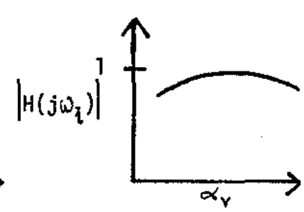

Fig.2. Zero Sensitivity..

Table III

Two-Pairs to be Extracted when $G_m(j\omega_0)$ is Complex

Type	Parameters	Use to Force
4A	$A = \tilde{D} = \frac{s+a}{a}$, $B = \tilde{C} = \frac{s}{a}$; $a = 2s(1 + G_m)/(1 - G_m)$ at $s = j\omega_0$	$G_{m-1}(j\omega_0) = 1$
4B	$A = \tilde{D} = \frac{s+a}{a}$, $B = \tilde{C} = \frac{a}{s}$; $a = \frac{s}{2}(1 - G_m)/(1 + G_m)$ at $s = j\omega_0$	$G_{m-1}(j\omega_0) = 1$
4C	$A = \tilde{D} = \frac{s+a}{a}$, $B = \tilde{C} = \frac{-s}{a}$; $a = 2s(1 - G_m)/(1 + G_m)$ at $s = j\omega_0$	$G_{m-1}(j\omega_0) = -1$
4D	$A = \tilde{D} = \frac{s+a}{a}$, $B = \tilde{C} = \frac{-a}{s}$; $a = \frac{s}{2}(1 + G_m)/(1 - G_m)$ at $s = j\omega_0$	$G_{m-1}(j\omega_0) = -1$

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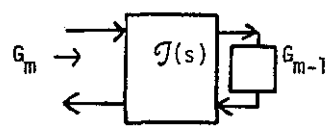

Fig.3 Two-Pair Extraction

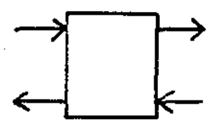

Fig. 4: Two-Pair.

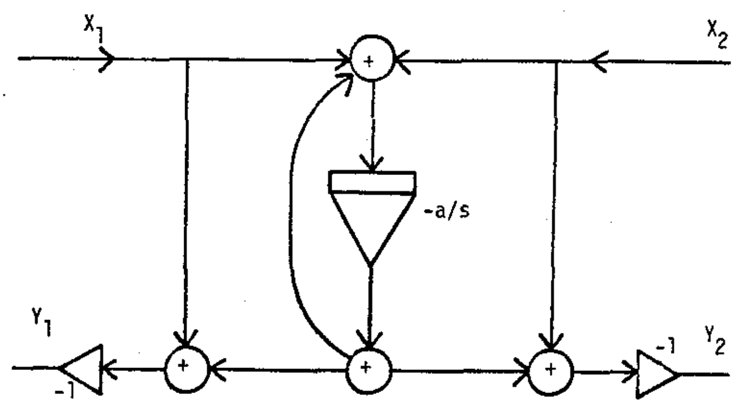

Fig.5 Type 1A LBR Two-Pair Structure.

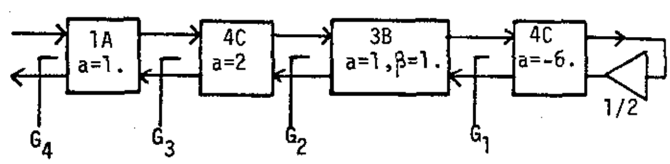

Fig.6. Synthesis Example.