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A Generic Bias-Correction Method with Application to Scan-Based Localization

Yiming Ji, Changbin Yu Brian D.O. Anderson and Samuel P. Drake

Abstract—**In previous work a method was proposed to determine the bias in localization algorithms using range or bearing data. In this paper the method is extended to be more generic; in particular, different types of measurement data are permitted, and there may be more measurements than there are variables to estimate. The method combines the Taylor series and Jacobian matrices to determine the bias, and leads to an easily calculated analytical bias expression, despite the general unavailability of analytic expressions for the solution of most localization problems. The method is used to estimate the bias in scan-based localization. Monte Carlo simulation results verify the performance of the proposed method in this context.**

Keywords: **Bias; Taylor series; Scan-based localization; Geolocation; Passive Localization; Targeting; Tracking**

I. Introduction

Bias is a term in estimation theory and is defined as the difference between the expected value of a parameter estimate and its true value [1]. If the bias of a particular estimation scheme can be estimated then it can be removed. Of course, in a particular single instance of the estimation problem, the correction may worsen the quality of the estimate; but over a number of experiments, it may be expected to improve the quality of the estimate, by lowering the mean square error.

A common class of estimation problems are those localization estimates, i.e. determination of the position of a target, e.g. a sound emitter, or an electromagnetic wave emitter, using some form of measurements, such as range, time difference of arrival, bearing, etc. In almost all practical localization situations, measurement errors are inevitable, and these lead to errors in estimating the true target position. In [2] Picard et al. discussed several models for estimating the bias in the range measurements, and presented a set of iterative algorithms that minimize the bias and provide maximum likelihood position estimates. In [3], Gavish et al. presented analytical expressions of bias which permit performance comparison for two well known bearing-only location techniques, viz. the maximum likelihood and the Stansfield estimators. Further in [4], Drake et al. presented an introduction to tensor algebra with some application examples in estimation theory. One of the tensor algebra applications proposed in the paper treats the bias in estimating a nonlinear function of a variable of which one has an observation contaminated by zero mean additive noise. The method involves expanding the nonlinear function around the noiseless observation value using a Taylor series which is truncated at the second order. The expected value of the first order term is zero and the expected value of the second order term introduces a bias. As an example the localization of a target is considered using noisy range and bearing information from a monostatic radar; with independent and gaussian zero mean noises contaminating each measurement, there is a systematic bias in the target estimate, such that the estimated position on average is closer to the radar than the true target position.

In our previous work [5, 6, 7], we proposed a method to determine and correct the bias in localization algorithms. We hypothesized the existence of a localization mapping $\mathbf{g}$ (which maps the vector of measurements to a position vector estimate). Following the lead of [4], we viewed this mapping using a Taylor series expansion around the nominal noiseless measurements that in principle are associated with the correct target position. The expansion is to second-order in the measurement noise as the first order term has zero mean and the expected values of the second-order term as expressible in terms of the derivatives of $\mathbf{g}$. However, in a localization problem it may actually be very hard to calculate the derivatives of $\mathbf{g}$ analytically. In contrast, the inverse mapping of $\mathbf{g}$ (call it $\mathbf{f}$) which maps the target position to a (noiseless) sensor measurement can often be obtained analytically, together with its derivatives. Therefore, we introduce the Jacobian matrix of $\mathbf{f}$ to compute the derivatives of the localization mapping $\mathbf{g}$ in terms of the derivatives of $\mathbf{f}$, resulting in a simple calculation of bias. In comparison with the approach presented in [3], the Monte Carlo simulation results demonstrate a clearly better performance for our bias correction method [7].

In our previous work however, the analysis of the proposed method was restricted to localization problems using only range or bearing measurements. In this paper we present the bias correction method in a more generic way allowing an arbitrary number of noisy measurements which are not restricted to being either range or bearing. To demonstrate the performance of the proposed method, the generic bias correction method is applied by way of example to scan-based localization algorithms, which have not been analyzed in our previous work. In the process of applying the proposed method to scan-based localization, the original bias correction method needs to be adjusted in a minor way to allow for certain correlations in the measurement errors.

The rest of this paper is organized as follows. In Section II the background and motivation of our work are presented. The generic bias correction method is proposed in Section III. In Section IV, we apply the bias correction method to the scan-based localization problem. Monte Carlo simulation results are also provided in Section IV. Section V summarizes the main results of the paper.

II. Background and Motivation

A. Background

A brief review of estimation bias will be presented in this section.

Let $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T$ and $\mathbf{\Theta} = (\theta_1, \theta_2, \ldots, \theta_N)^T$ denote the target position and noiseless measurement vector respectively. Let $\mathbf{f}\colon \mathbf{x} \to \mathbf{\Theta}$ denote the associated mapping, which is almost always analytically computable. Let $\mathbf{g}\colon \mathbf{\Theta} \to \mathbf{x}$ denote the inverse localization mapping; thus with $\mathbf{\Theta} =$

Y. Ji and B. Anderson are with National ICT Ltd. Australia-NICTA and Research School of Engineering, The Australian National University Canberra ACT 2601, Australia. Email: {yiming.ji, brian.anderson}@anu.edu.au

C. Yu is with Research School of Engineering, The Australian National University Canberra ACT 2601, Australia. Email: brad.yu@anu.edu.au

S. P. Drake is with Electronic Warfare and Radar Division, Defence Science and Technology Organization (DSTO) Edinburgh, SA 5111, Australia. Email: sam.drake@dsto.defence.gov.au

$\mathbf{f}(\mathbf{x})$, there holds $\mathbf{x} = \mathbf{g}(\boldsymbol{\Theta})$. In order that this mapping can be well defined, it is necessary that $N \geq n$. In case $N > n$, usually the set of equations $\mathbf{f}(\mathbf{x}) = \boldsymbol{\Theta}$ yielding $\mathbf{x}$ from $\boldsymbol{\Theta}$ will be overdetermined, while in case $N = n$, there may be two or more solutions (but for generic geometries only a finite number); in this case, the selection of the correct solution requires some additional information regarding the target position.

In practice, noise in the measurements is inevitable. We denote this noise by $\delta\boldsymbol{\Theta} = (\delta\theta_1, \delta\theta_2, ..., \delta\theta_N)^T$. The $\delta\theta_i$ are generally assumed to be independent Gaussian random variables with zero mean and known variances σ_i^2, which may be the same. The error in the target position resulting from an estimation procedure using noisy data is denoted as $\delta\mathbf{x}$. If we define $\tilde{\boldsymbol{\Theta}} = \boldsymbol{\Theta} + \delta\boldsymbol{\Theta}$ and $\tilde{\mathbf{x}} = \mathbf{x} + \delta\mathbf{x}$ the localization amounts to solving $\mathbf{f}(\tilde{\mathbf{x}}) = \tilde{\Theta}$ for $\tilde{\mathbf{x}}$. If the function $\mathbf{g}$ is known, then in effect we are implementing the following equation

$$\tilde{\mathbf{x}} = \mathbf{x} + \delta\mathbf{x} = \mathbf{g}(\Theta + \delta\Theta) = \mathbf{g}(\tilde{\Theta}) \tag{1}$$

If $\mathbf{g}$ is a non-linear function then this process will lead to bias in the target position estimate [8]. Suppose, by way of a thought experiment, that in estimating the value of x the measurement process equation (1) was repeated M times. As $M \to \infty$, the average of the estimates would go to :

$$E[\tilde{x}_i] = E[g_i(\tilde{\Theta})] \tag{2}$$

Now note that if g_i is nonlinear we would have:

$$E[\tilde{x}_i] = E[g_i(\tilde{\Theta})] \neq g_i(E[\tilde{\Theta}]) = g_i(\Theta) = x_i$$

The bias in our estimated of x_i is defined as the difference between the expected value of x_i and the true value of x_i, i.e.

$$Bias_{x_i} = E[\tilde{x}_i] - x_i = E[g(\tilde{x}_i)] - x_i$$

If computable, the bias can be used to systematically correct any single estimate from any single measurement set.

B. Motivation

From the above analysis, we can see that if the estimation mapping $\mathbf{g}$ is nonlinear and the sensor measurements are noisy, bias is present. In practice, these two conditions are presented in most localization scenarios. Since bias is a systematic and possibly computable error, it is desirable to remove it.

In [4], Drake et al. gave a short introduction to tensor algebra and provided a few sample applications. One such application was concerned with bias arising in non-linear estimation problems with noisy measurements. To determine the bias they considered, as we do, $\tilde{x}_i = g_i(\tilde{\theta})$. Assuming the estimator mapping $\mathbf{g}$ is well defined, they expanded the function g_i by a Taylor series and truncated it at second order:

$$\begin{aligned} x_i + \delta x_i &= g_i(\tilde{\theta}_1, \tilde{\theta}_2, ..., \tilde{\theta}_N) \\ &= g_i(\theta_1 + \delta\theta_1, \theta_2 + \delta\theta_2, ..., \theta_N + \delta\theta_N) \\ &\approx g_i(\theta_1, \theta_2, ..., \theta_N) + \sum_{j=1}^{N} \frac{\partial g_i}{\partial \theta_j}\delta\theta_j \\ &\quad + \frac{1}{2!}\sum_{j=1}^{N}\sum_{l=1}^{N} \delta\theta_j\delta\theta_l \frac{\partial^2 g_i}{\partial\theta_j\partial\theta_l} \end{aligned}$$

Assuming as is often the case that the measurement errors are independent Gaussian random variables with zero mean and known variance (the variance of measurement errors would have to be obtained from manufacturer or experimental data), the approximate bias expression is:

$$E(\delta x_i) = \frac{1}{2!}\sum_{j=1}^{N} \sigma_j^2 \frac{\partial^2 g_i}{\partial \theta_j^2} \tag{3}$$

For some estimators, the mapping $\mathbf{g}$ can be obtained analytically. However in some situations including many localization problems, finding the analytic $\mathbf{g}$ analytically is very hard or even impossible. If $\mathbf{g}$ can not be obtained then equation (3) can not be evaluated, and we need a new method to analytically obtain the derivatives. The bias in our estimated of x_i is defined as the difference between the expected value of x_i and the true value of x_i.

The key to do this is to notice that $\mathbf{g}$ is the inverse of the mapping $\mathbf{f}$ for which often an analytic form is known. Below we show how to use the mapping $\mathbf{f}$ and its derivatives to calculate the derivatives of $\mathbf{g}$ using the Jacobian identity, ultimately resulting in an estimate of the bias.

III. Bias Correction Method

To begin, we first assume $N = n$ (the number of sensor measurements N is equal to the dimension of position coordinates of the target n). We assume further that $\mathbf{f}$ is a known analytic function. Because $\mathbf{f}$ and $\mathbf{g}$ are inverse mappings, the Jacobian identity holds:

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ . & \cdots & . \\ . & \cdots & . \\ . & \cdots & . \\ \frac{\partial f_N}{\partial x_1} & \cdots & \frac{\partial f_N}{\partial x_n} \end{bmatrix} \begin{bmatrix} \frac{\partial g_1}{\partial \theta_1} & \cdots & \frac{\partial g_1}{\partial \theta_N} \\ . & \cdots & . \\ . & \cdots & . \\ . & \cdots & . \\ \frac{\partial g_n}{\partial \theta_1} & \cdots & \frac{\partial g_n}{\partial \theta_N} \end{bmatrix} = \mathbf{I}_n \tag{4}$$

By rearranging the equation set (4) we can obtain analytical expressions for the $\frac{\partial g_i}{\partial \theta_j}$ $(i = 1, 2, ..., n; j = 1, 2, ..., N)$ in terms of $\frac{\partial f_i}{\partial x_j}$, and thus as analytic functions of the x_i. For ease of exposition we use g_j^i to denote the expressions of $\frac{\partial g_i}{\partial \theta_j}$ as functions of $x_1, x_2, ..., x_n$. To obtain second derivatives, let us use $\frac{\partial g_1}{\partial \theta_1}$ as an example to illustrate the general approach. Starting with

$$\frac{\partial g_1}{\partial \theta_1} = g_1^1 \tag{5}$$

we differentiate with respect to x_1 first, and thereby obtain $\frac{\partial g_1}{\partial {\theta_1}^2}\frac{\partial f_1}{\partial x_1} + ... + \frac{\partial g_1}{\partial\theta_1\partial\theta_i}\frac{\partial f_i}{\partial x_1} + ... + \frac{\partial g_1}{\partial\theta_1\partial\theta_N}\frac{\partial f_N}{\partial x_1} = \frac{\partial g_1^1}{\partial x_1}$. If we further differentiate the equation (5) with respect to $x_2, ..., x_n$ we can obtain an equation set as follows:

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_N}{\partial x_1} \\ . & \cdots & . \\ . & \cdots & . \\ . & \cdots & . \\ \frac{\partial f_1}{\partial x_n} & \cdots & \frac{\partial f_N}{\partial x_n} \end{bmatrix} \begin{bmatrix} \frac{\partial^2 g_1}{\partial {\theta_1}^2} \\ . \\ . \\ . \\ \frac{\partial^2 g_1}{\partial\theta_1\partial\theta_N} \end{bmatrix} = \begin{bmatrix} \frac{\partial g_1^1}{\partial x_1} \\ . \\ . \\ . \\ \frac{\partial g_1^1}{\partial x_n} \end{bmatrix} \tag{6}$$

Note that the quantities on the right side of this equation are all expressible analytically as functions of $x_1, x_2, ..., x_n$. Likewise the entries $\frac{\partial f_i}{\partial x_j}$ in the matrix on the left are known functions of $x_1, x_2, ...x_n$. Hence by solving the equation set (6), we can obtain a formula for $\frac{\partial^2 g_1}{\partial \theta_1^2}$ as a function of $x_1, x_2, ..., x_n$. The formulas for $\frac{\partial^2 g_i}{\partial \theta_j^2}$ for all i, j can be obtained in the same way. Substituting the formulas

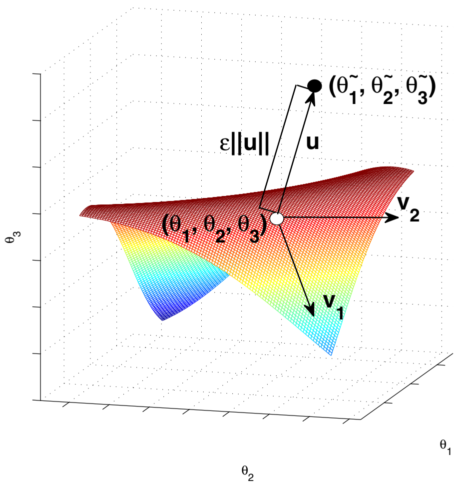

Fig. 1. Introduce one extra variable (Here N=3 and n=2). The surface is the set of points $(\theta_1, \theta_2, \theta_3) = (f_1(x, y), f_2(x, y), f_3(x, y))$ obtained as x, y vary.

into equation (3) we can finally obtain the easily-calculated expressions for the bias in terms of f_i and its derivatives.

The calculation is used as follows. We first obtain the estimated position of the target by using an existing localization algorithm. Then we can input the estimated target location into the obtained analytical expression for the bias. Finally we can improve (on a statistical basis at least) the accuracy of the localization by subtracting the obtained bias, viz. $\tilde{x_i} - bias_{x_i}(i = 1, 2, ..., n)$. (The process could in fact be iterated, but the effect of even one more iteration on the corrected position estimate will almost always be marginal).

If the number of measurements N is greater than the number of scalar position variables n, we cannot obtain the equations (4) and (6), and so we cannot straightforwardly express the bias using the derivatives of $\mathbf{f}$. Indeed, while the noise-free equation $\mathbf{f}(\mathbf{x}) = \Theta$ is overdetermined, the noisy equation $\mathbf{f}(\tilde{\mathbf{x}}) = \tilde{\Theta}$ in general will have no solution. The localization problem is typically solved by something like a least squares approach[1], and for the purposes of bias determination, we build on this approach too, to introduce $N - n$ extra variables into $\mathbf{f}$ thereby making $n = N$.

For ease of exposition, here we take $N = n + 1$, which means just one extra variable needs to be introduced. Consider N-dimensional space, with axes corresponding to the N measurements. Assume an $(N-1)$-dimensional hypersurface (illustrated in Figure 1 for the case $N = 3$) consists of points which correspond to all sets of noiseless measurements $(\theta_1, \theta_2, ..., \theta_N)$, i.e. $\theta_i = f_i(x_1, x_2, ...x_n)$ for $i = 1, 2, ..., N$. According to the least squares method, we can consider choosing $x_1, x_2, ..., x_n$ to minimize the following cost function:

$$F_{\text{cost-function}}(\mathbf{x}, \tilde{\Theta}) = \sum_{i=1}^{N}(f_i - \tilde{\theta}_i)^2 = \sum_{i=1}^{N} \delta\theta_i^2 \quad (7)$$

In fact, the least squares method attempts to find a point $(\theta_1, \theta_2, ..., \theta_N)$ (the white point in Figure 1) on the surface corresponding to an obtained set of noisy measurements $(\tilde{\theta}_1, \tilde{\theta}_2, ..., \tilde{\theta}_N)$ (the black point in Figure 1 which is generically off the surface) to minimize the distance between the two points. Hence the white point must be the orthogonal projection of the black one onto the surface, or the black point must be on the normal vector to a tangent plane of the surface passing through the white one.

The n tangent vectors at any point on the n dimensional surface are given by vectors as follows:

$$\mathbf{v}_i = [\frac{\partial f_1}{\partial x_i}, \frac{\partial f_2}{\partial x_i}, ..., \frac{\partial f_N}{\partial x_i}]^T \quad i = 1, 2, ..., n \quad (8)$$

The normal vector to the surface ($\mathbf{u}$) is formed from the cross product of the tangent vectors ($\mathbf{v}_i$), that is: $\mathbf{u}$ [9] of the surface through the white point:

$$\mathbf{u} = [u_1, u_2, ..., u_N]^T = \mathbf{v}_1 \times \mathbf{v}_2 ... \times \mathbf{v}_n \quad (9)$$

Note that in the noiseless case $\Theta = \mathbf{f}(x_1, x_2, ...x_n)$ where $\mathbf{f}$ can be written down easily according to the geometry of the sensors and target. The black point can be regarded as an image obtained with a new analytical mapping $\mathbf{F} = (F_1, F_2, ..., F_N)^T : R^N \to R^N$ corresponding to moving from $\mathbf{f}$, which is a known function of $x_1, x_2, ...x_n$ along the normal vector for a distance $\varepsilon||\mathbf{u}||$. The new mapping $\mathbf{F}$ is a known analytic function of $x_1, x_2, ...x_n$ and ε:

$$\tilde{\Theta} = \mathbf{F}(\tilde{\mathbf{x}}, \varepsilon) = \mathbf{f}(\tilde{\mathbf{x}}) + \varepsilon\mathbf{u} \quad (10)$$

(Of course, $\mathbf{u}$ is an analytic function of $x_1, x_2, ...x_n$).

Introduction of the extra variable ϵ means that equation (10) is not an overdetermined equation set, and $\mathbf{F}$ is in principle invertible. Therefore we can consider the new localization mapping (call it $\mathbf{G}$) as the inverse mapping of $\mathbf{F}$. We can then proceed along the same lines as previously to determine the bias.

When the number of input measurements N exceeds $n+1$, the situation is similar to the case $N = n + 1$. More details can be obtained in [6, 7].

IV. Application to the Scan-Based Localization Problem

In order to demonstrate the effectiveness of the proposed bias-correction method, we apply the proposed method to the scan-based localization problem, which is studied in [10].

A. *Review of Scan-Based Localization*

The target is assumed to be an emitter with a (generally mechanically) rotating radar antenna with a narrow beam; the scan direction and scan rate are assumed constant and indeed known to each receiver, which records the time instants at which the rotating beam passes the receiver [10]. For ease of exposition, here we consider a situation with one emitter and three receiving sensors, and assume that all lie in a plane. Figure 2 shows an emitter scanning across three receivers (i.e., receiver 1, receiver 2 and receiver 3) at the times t_1, t_2 and t_3. The emitter is scanning clockwise. For localization, the separate time values are not important, but rather their differences, $t_{12} = t_2 - t_1$ and $t_{23} = t_3 - t_2$, are. In fact, we treat t_{12}, t_{23} as quasi-measurements. Each quasi-measurement (together with knowledge of the scan rate and direction) in the noiseless case defines a circle of computable center and radius, and indeed an arc of such a circle on which the emitter must lie. The pair of sensors determining the time difference lie at the end points of the arc. The intersection of two such circular arcs defines the emitter position; thus there is a vector function $\mathbf{g} = (g_1, g_2)^T$, the localization mapping, of the two variables t_{12}, t_{23}, with $\mathbf{g}$ embodying a formula for the intersection of two circular arcs. Finding an analytic expression for the mapping is a significant challenge.

[1] The least squares approach is equivalent to a maximum likelihood approach when all noise variances are the same (and measurement noises are independent zero mean Gaussian random variables). Weighted least squares can capture variations on this.

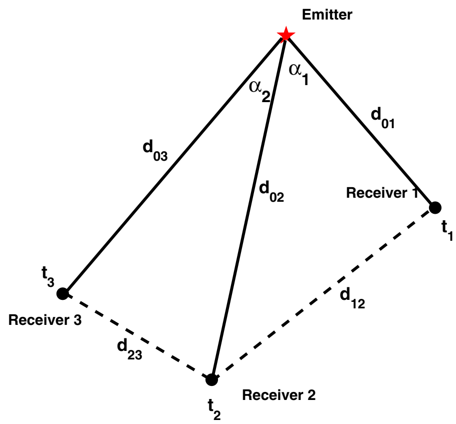

Fig. 2. An emitter scanning across three receivers; The red star indicates the radar's location whereas the filled circles indicates the location of the receivers.

The two factors causing bias to arise in an estimate, viz. nonlinearity of the estimation mapping and noise in the measurements, are present in any practical scan-based localization problem. Accordingly, we seek to apply our method to use the inverse of **g** (viz. the mapping **f** from target position to measurements) to obtain a formula for the bias.

B. Analytic expression for the mapping from emitter position to quasi-measurements

Now we aim to obtain an analytic form for the function **f** which maps the emitter position (x, y) to the quasi-measurements (t_{12}, t_{23}). Let α_1, α_2 denote the angles subtended at the emitter by the lines joining it to the two pairs of physical sensors, see Figure 2. Since the scan rate ω is a known constant, we can obtain the following equations:

$$\alpha_i = \omega t_{i,i+1} \qquad i = 1, 2 \tag{11}$$

Given the three receivers at known locations $p_1 = (x_1, y_1)$, $p_2 = (x_2, y_2)$ and $p_3 = (x_3, y_3)$, it is straightforward to see that

$$\alpha_i = \arccos \frac{d_{0,i}^2 + d_{0,i+1}^2 - d_{i,i+1}^2}{2d_{0,i}d_{0,i+1}} \qquad i = 1, 2 \tag{12}$$

where

$$d_{0,i} = \sqrt{(x - x_i)^2 + (y - y_i)^2}, \quad i = 1, 2, 3$$
$$d_{i,i+1} = \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2}, \quad i = 1, 2$$

Substituting equations (12) into (11), we can obtain the following formulas:

$$t_{i,i+1} = f_i(x, y) = \arccos \frac{d_{0,i}^2 + d_{0,i+1}^2 - d_{i,i+1}^2}{2d_{0,i}d_{0,i+1}\omega} \qquad i = 1, 2 \tag{13}$$

These last equations provide an analytic formula for $\mathbf{f} = (f_1, f_2)$, which is the inverse mapping of localization process $\mathbf{g} = (g_1, g_2)$.

When the number of receivers is larger than 3, we can use the least-squares based method proposed in Section III to introduce extra variables.

C. Bias-Correction in Scan-Based Localization

As the first step in obtaining an expression for the bias, we Taylor expand the localization mappings g_1 and g_2 to the second order terms, as described in Section III. However, in the scan-based localization problem, equation (3) requires adjustment. To see why, note that noise in the time-of-arrival (TOA) measurements can be modelled as follows:

$$\tilde{t}_i = t_i + \delta t_i \quad i = 1, 2, 3 \tag{14}$$

where the δt_i are assumed to be i.i.d Gaussian random variables with zero mean and known variance σ^2.

However in scan-based localization, the physical measurements are replaced by quasi-measurements $\tilde{t}_{12} = \tilde{t}_2 - \tilde{t}_1$ and $\tilde{t}_{23} = \tilde{t}_3 - \tilde{t}_2$. This leads to

$$\tilde{t}_{12} = \tilde{t}_2 - \tilde{t}_1 = t_{12} + \delta t_{12} \tag{15}$$
$$\tilde{t}_{23} = \tilde{t}_3 - \tilde{t}_2 = t_{23} + \delta t_{23} \tag{16}$$

where δt_{12} and δt_{23} are no longer independent and have covariance matrix Σ given by

$$\Sigma = 2\sigma^2 \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix} \tag{17}$$

where $2\sigma^2$ is the variance of an individual time-difference measurement. Note that the means of δt_{12} and δt_{23} remain zero.

Now the approximate bias expression for three receivers is as follows:

$$E(\delta x) \quad = \quad \frac{1}{2!}[2\sigma^2 \frac{\partial^2 g_1}{\partial t_{12}^2} - 2\sigma^2 \frac{\partial^2 g_1}{\partial t_{12} \partial t_{23}} + 2\sigma^2 \frac{\partial^2 g_1}{\partial t_{23}^2}] \tag{18}$$

$E(\delta y)$ can be obtained in the same way. The remaining calculations are the same as those described in Section III.

D. Simulation Results

In this subsection, the simulation results will be shown to demonstrate the performance of the proposed bias-correction method in scan-based localization problems. All the simulated data is provided by the Defence Science Technology Organization (DSTO).

The simulations were done using DSTO's synthetic integration lab (SIL). The still accurately simulates existing radar and receiver systems, it includes accurate, senor, emitter, terrain and propagation models. The fidelity of the SIL means that there is no significant difference between its simulation results and those generated by field tests. Simulation results are provided in 2-dimensional space with two scenarios: (1) three receivers and one emitter (2) four receivers and one emitter. Different emitter positions are considered.

The simulation set-up is as follows:

- The measurement error δt_i for each sensor is produced by an independent Gaussian distribution with zero mean and known variance σ^2. The level of noise (the standard deviation σ) is adjusted in the simulation set as 0.05, 0.1 or 0.15 seconds for TOA measurements t_i.
- All the simulation results are obtained from 1000 Monte Carlo experiments.
- In the simulations the bias is considered as the average absolute distance (average of 1000 experimental results) between the true emitter position and the estimated emitter position. In the simulation figures it is termed the 'average absolute distance error'.[2]

[2]In practice, the bias is a vector whose entries can be negative or positive. Here we only focus on how large the bias is. Therefore the absolute distance between the estimated target position and the true position is used to evaluate the bias.

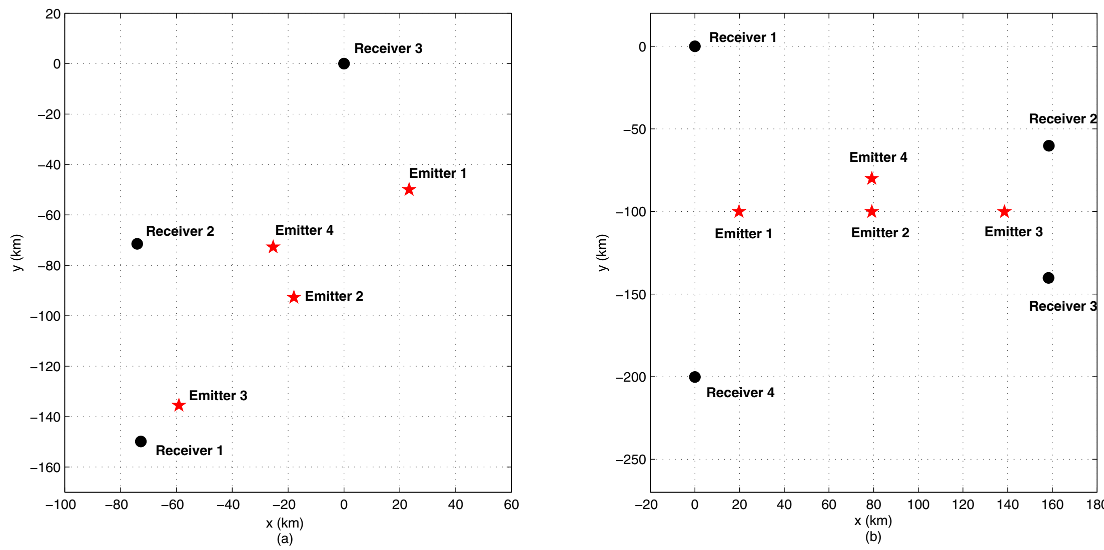

Fig. 3. Positions of emitters and receivers (a) Three receivers situation (b) Four receivers situation

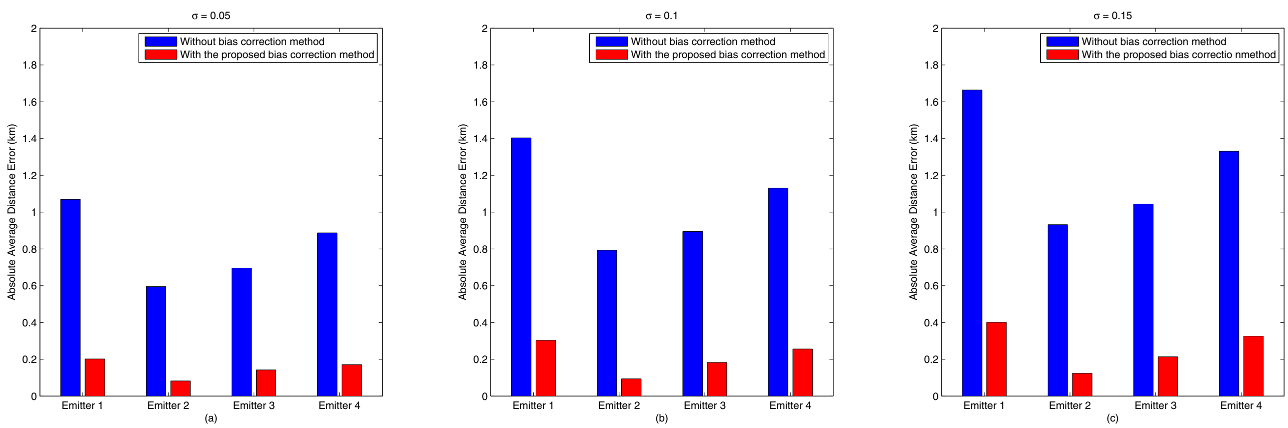

Fig. 4. Three receivers scenario with different noise level (a) σ=0.05 (b) σ=0.10 (c) σ=0.15

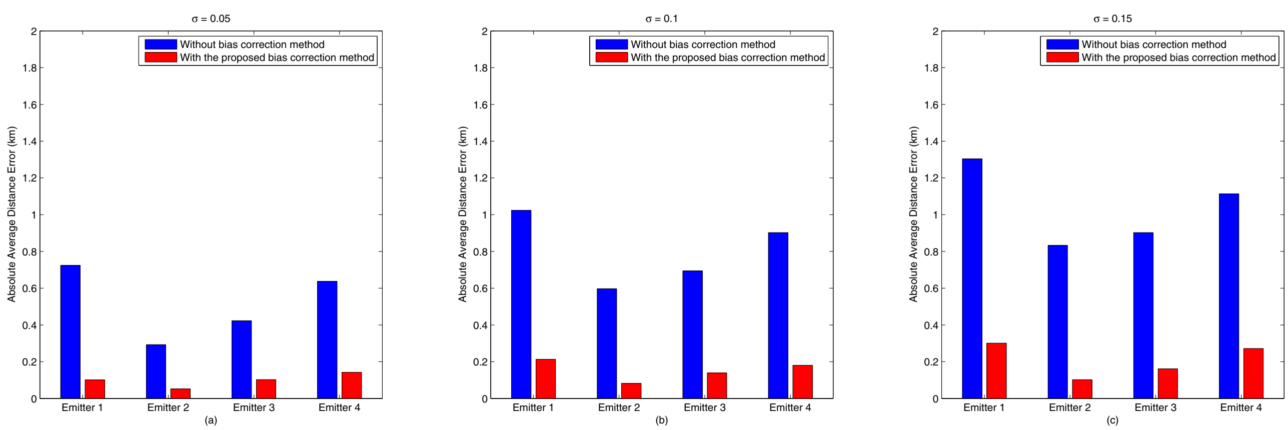

Fig. 5. Four receivers scenario with different noise level (a) σ=0.05 (b) σ=0.10 (c) σ=0.15

- 'Analytical bias' denotes the bias obtained by using the analytical expression derived from the proposed bias-correction method.
- 'Experimental bias' denotes the bias obtained by using simulation.
- The scan rate is $\frac{4\pi}{5}$ radians per second. Thus one period is 2.5 seconds, and so the standard deviation of TOA measurements is between 2% and 6% of a period.
- The distance unit used in the simulations is kilometers (km).
- The time unit and the angle unit used in the simulations are seconds and radians.

1) Three Receivers Scenario

In this situation, three receivers give rise to two quasi-measurements (t_{12} and t_{23}). Therefore the ambient space dimension n is equal to the number of measurements N.

Figure 3(a) depicts the three receivers and the three different emitter positions. The three receivers are located as follows:

- Receiver 1: (-72.74, -149.86)
- Receiver 2: (-74.05, -71.46)
- Receiver 3: (0, 0)

The four different emitter positions are:

- Emitter 1: (23.32,-49.98)
- Emitter 2: (-17.937,-92.719)
- Emitter 3: (-59.096,-135.554)
- Emitter 4: (-25.364, -72.736)

Figure 4(a) illustrates the comparison of the bias in two different situations: without any bias-correction method (blue bar) and with the proposed bias-correction method (red bar). The standard deviation of the measurement error is 0.05 seconds ($\sigma = 0.05$). Evidently, the proposed method can reduce the localization bias by typically between 70% and 80% for any of the four emitter positions.

The effect of different adjusting the level of noise to 0.1 seconds ($\sigma = 0.1$) and 0.15 seconds ($\sigma = 0.15$) is depicted in figures 4(b) and 4(c) respectively, from which we can conclude that, though the bias is enlarged when the level of noise increases, the proposed bias correction method still performs very well (after reduction, the bias is no more than 0.4 km). The simulation results, demonstrate that the proposed bias correction method is robust to the noise level.

2) Four Receivers Scenario

In this situation, there are four receivers, and we obtain three independent quasi-measurements. We denote these quasi-measurements as t_{12}, t_{23} and t_{34}. The ambient space dimension $n = 2$ is less than the number of quasi-measurements, which corresponds to the $N > n$ situation for the proposed bias-correction, therefore we use the method outlined in Section III to introduce an extra variable which makes $N = n$ again.

Figure 3(b) shows the location of the four receivers and the four emitters positions. The four receivers are positioned as follows:

- Receiver 1: (0, 0)
- Receiver 2: (158.41,-60.21)
- Receiver 3: (158.29,-140.26)
- Receiver 4: (0,-200.24)

There are four different emitter positions:

- Emitter 1: (19.77,-100.12)
- Emitter 2: (79.17,-100.15)
- Emitter 3: (138.58,-100.21)
- Emitter 4: (79.19,-80.08)

Figure 5(a) shows the simulation results in 2-dimensional space with three quasi-measurements. Again, from the figure we can see that the proposed method (the red bars) reduces the bias very effectively (reducing it by up to 75%). Furthermore, by comparing to the simulation results for the three receivers scenario (Figure 4(a)) we can see that introducing an extra sensor, unsurprisingly, improves the accuracy of the localization.

Figures 5(b) and 5(c) illustrate the performance of the proposed bias correction method with different levels of noise ($\sigma = 0.1$ and $\sigma = 0.15$). Similarly to the three receivers scenario, the proposed method is effective in reducing bias even with a high level of noise. Again, comparing to the three receiver scenario (Figure 4(b) and Figure 4(c)) the accuracy of the localization is enhanced by introducing one more sensor.

V. Conclusions

In previous work [5, 6, 7], we proposed a method to determine and thus correct the bias in localization algorithms using range measurements or bearing measurements. The bias-correction method combines Taylor series expansions and Jacobian matrices to express the bias analytically. In this paper we present the bias-correction method in a more generic way. To demonstrate the validity of our bias correction method, we have applied the proposed bias-correction method to the scan-based localization problem in which the original method needs an adjustment, due to the way noise enters the quasi-measurements. Monte Carlo simulation results based on the simulated data provided by DSTO Australia's synthetic integration laboratory (SIL) demonstrated the performance of the proposed bias correction method. Our future work is aimed at improving the performance of the proposed method by using higher order terms of the Taylor series; this may be important in high noise situations.

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