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Safe Formation Control with Obstacle Avoidance

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Abstract: In this paper, we consider the general problem of guaranteeing safe interaction of humans and a group of mobile agents in an environment of interest. The way that we guarantee such a safe interaction is via proposing algorithms that prevent the agents from colliding with environmental obstacles with unknown sizes and locations, e.g. human operators. To do so, we first consider the problem of forcing a set of autonomous agents form a desired formation and follow a trajectory only known to one of them, namely the leader of the formation. Then we build upon the solution given to this problem and solve another problem of interest. This is to guarantee obstacle collision avoidance while the agents are following an unknown trajectory and keeping the desired shape of the formation. The stability of the control laws presented here is analytically established and some simulation results are presented to show the performance of the proposed control laws in different scenarios.

Keywords: Motion Control, Obstacle Avoidance, Formation Shape Preservation, Safe Human-robot Interaction

1. INTRODUCTION

Providing a safe environment for the simultaneous operation of humans and robots has been the subject of research for the last few years, see Najmaei and Kermani [2010] and references therein. Additionally, formation control of multi-agent systems continues to be an expanding area of research, see Moshtagh and Jadbabaie [2007], Justh and Krishnaprasad [2004], Basiri et al. [2010], and providing such safe environments for humans-agents system interactions has become an important research question. This is mainly due to the introduction of warehouse automated systems, e.g. see Wurman et al. [2008] where formations of mobile agents operate in the same environment as human operators. An important element that one needs to take into account while designing control laws to ensure the safety in such environments is to guarantee obstacle avoidance for the formation of mobile agents. This results in a renewed focus on obstacle avoidance in multi-agent systems. These obstacles can be human workers in the region of operation of the robots or can be natural barriers where an unmanned vehicle is operating, possibly even carrying human passengers.

A survey of collision avoidance methods has recently been published by Albaker and Rahim [2009]. There are two dominant general approaches to achieve obstacle collision avoidance in the literature *(i)* planning-based approaches, and *(ii)* behaviour-based approaches. Planning based approaches have their roots in classical control theory, and they are highly dependent on exact mathematical world models. Examples of such approaches towards the problem of obstacle avoidance can be found in Rimon and Koditschek [1992]. In those works the construction of a special potential function, a navigation function, is proposed whose most important property is the absence of undesired local minima. Implementation of these functions can be found in Ogren and Leonard [2002], Soukieh et al. [2009], Tedesco et al. [2010]. In these methods the agent follows a calculated path that guarantees that no collision occurs between itself and any obstacles in the environment. The most important assumption made in all the aforementioned works is the availability of an exact model for the environment, which may neither be practical nor realistic in most situations. In most of the behavior-based approaches, e.g. Balch and Arkin [1998], Dougherty et al. [2004], Egerstedt and Xiaoming [2001], Mudie et al. [2001], Balch and Hybinetre [2000], the motion of an agent is determined by taking into account different behaviours, such as avoid-moving-obstacles, avoid-stationary-obstacles, and move-to-goal. These approaches do not usually rely on a strong *a priori* knowledge about the environment, however, they might cause the agent to seek to follow contradicting goals, for example move-to-goal behaviour may force the agent to move on a collision course with an obstacle. Recently, due to much attention being invested in multi-agent systems, collision avoidance methods synthesizing the two approaches have been sought. It is necessary to guarantee the safety of the agents operating in an environment to achieve a goal cooperatively. Because of the challenging nature of multi-agent systems the classical approaches to deal with collision avoidance are rendered impractical, and researchers have moved to solve this problem by taking the virtues of both of the abovementioned approaches and rejecting their

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vices. For example, the approaches proposed in Hoffmann and Tomlin [2008], Mastellone et al. [2008], Dimarogonas et al. [2006], Shames et al. [2010] fall in this category; they have elements of both planning-based and reaction-based approaches.

The contributions of this paper are twofold. The first contribution is to propose a control law based on the one proposed in Cao and Ren [2010] to force the agents achieve a desired formation while following a trajectory only known to one of the agents, namely the leader. The second contribution of this paper is to present an obstacle avoidance method using the control law introduced earlier which guarantees obstacle collision avoidance in an obstacle field for the period of time where the formations are moving in a region of interest. A safe formation control with obstacle avoidance is presented by Zheng et al. [2008] for leaderless formations. It is shown that when obstacles are detected, the formation squeezes through the obstacles and then expands back to the original shape. In this paper, we consider a leader-follower case and use a different approach for formation control and obstacle avoidance.

The rest of this paper is organized as follows. In the next section, after introducing some preliminaries and assumptions we propose an algorithm to force the agents form a formation with a desired shape in finitetime and follow a trajectory which is unknown to all but one, the leader of the formation. In Section 3 we introduce an obstacle collision avoidance scheme to control and coordinate the motion of a formation of autonomous agents in an obstacle field. In Section 4 some simulation results are presented to show the applicability of the proposed control methods here. Some concluding remarks and future research directions are presented in Section 5.

2. FORMATION SHAPE PRESERVATION AND MOTION ALGORITHM

Consider a formation of n interconnected nodes and let $\mathcal{G}(\mathcal{V},\mathcal{E},\mathcal{A})$ be the underlying sensing graph of this formation, where $\mathcal{V} = \{i\}_1^n$ is the vertex set with $i \in \mathcal{V}$ corresponding to node i, $\mathcal{E}$ is the edge set of the graph, and $\mathcal{A} \in \mathbb{R}^{n\times n}$ is the adjacency matrix with nonnegative entries. In this paper we consider both the cases where the edges in the graph are undirected and directed. For the undirected case, the undirected edge $\{i,j\} \in \mathcal{E}$ is incident on vertices i and j if nodes i and j share a sensing link (allowing determination of relative position), in which case the corresponding entry in the adjacency matrix $\mathcal{A}_{ij} = \mathcal{A}_{ji}$ is one. For the directed case, the directed edge $(i,j) \in \mathcal{E}$ connects i to j if nodes i senses the relative position of j, where the corresponding entry in the adjacency matrix $\mathcal{A}_{ij}$ is one. The out-degree of node i is $\deg(i) = \sum_{j\in\mathcal{N}_i} \mathcal{A}_{ij}$, where $\mathcal{N}_i = \{j \in \mathcal{V} : \{i,j\} \in \mathcal{E}\}$ is the neighborhood set of i. The degree matrix $\Delta(\mathcal{G}) \in \mathbb{R}^{n\times n}$ is a diagonal matrix defined as

$$\Delta_{ij} = \begin{cases} \deg(i) & , \ i = j \\ 0 & , \ i \neq j \end{cases}$$

and the weighted Laplacian of $\mathcal{G}$ is defined as $\mathcal{L}(\mathcal{G}) = \Delta - \mathcal{A}$. Consider an n-agent formation of point agents. The problem of interest here is to force the agents of the formation to form a desired shape and orientation and move along a desired trajectory while keeping the shape and orientation intact. We assume that one of the agents in the formation is the leader and others are followers, and there is always a path from the leader to each of the agents. The followers can measure the relative positions of their neighbors which might be the leader or other followers, and update their states based on these measurements. Therefore, the control law proposed in this paper is distributed and each agent only uses local information. Without loss of generality, we take agent 1 to be the leader of the formation and other agents to be followers. First, we consider the case where the sensing and communication among the followers can be modeled by an undirected graph, i.e. $\mathcal{A}_{ij} = \mathcal{A}_{ji}$. Later, we state that a similar control method can be applied to the case where such sensing and communication is modeled by a directed graph where $\mathcal{A}_{ij}$ and $\mathcal{A}_{ji}$ are not necessarily equal. Let $s_i(t)$ denote the position of i in $\mathbb{R}^2$ at time t. Note that the results stated in this paper are presented for the two-dimensional case but they can be easily extended to a space with arbitrary number of dimensions. We make the following connection on the connectivity of $\mathcal{G}$.

Assumption 1. *For the undirected case, $\mathcal{G}$ is connected, and there is a directed path from the leader to all the followers for the directed case.*

Assume the single integrator kinematics for the motion of agent i:

$$\dot{s}_i(t) = v_i(t)$$

where $v_i(t)$ is the control input. Since the motions in each of the dimensions are decoupled for the rest of this paper we only introduce the control laws for the x-coordinate of the position of agent i. Hence, we have

$$\dot{x}_i(t) = \upsilon_{xi}(t)$$

where $\upsilon_{xi}(t)$ is the x-coordinate of $v_i(t)$. The desired formation is represented in terms of displacements from $x_1(t)$, i.e. the desired position of agent i is $x_i^\star(t) = x_1(t) + \delta_{xi}$ for all $i \in \{2, \cdots, n\}$ where δ_{xi} is a predefined constant scalar.

We propose the following control law to control the positions of the followers ($i \in \{2, \cdots, n\}$)

$$\begin{aligned} \dot{x}_i(t) = &-\alpha \left(\sum_{j=1}^{n} \mathcal{A}_{ij} \Big(x_i(t) - x_j(t) \Big) + u_{xi} \right) \\ &- \beta \mathrm{sign} \left(\sum_{j=1}^{n} \mathcal{A}_{ij} \Big(x_i(t) - x_j(t) \Big) + u_{xi} \right). \end{aligned} \quad (1)$$

where A_{i1} is one if the leader's position is available to follower i and is zero otherwise, $\alpha, \beta > 0$, and $\boldsymbol{u}_x = [u_{x2}, \cdots, u_{xn}]^\top$

$$u_{xi} = -\sum_{j=1}^{n} \mathcal{A}_{ij} \Big(\delta_{xi} - \delta_{xj} \Big). \quad (2)$$

Remark 1. *The evaluation of u_{xi} only depends on local information, that is the information about the desired relative distances of an agent from its neighbours.*

Moreover, assume that the leader's motion is governed by

$$\dot{x}_1(t) = f(t)$$

where $f(t) \in \mathbb{R}$ is an arbitrary continuous function.

Theorem 1. *Under Assumption 1, the control law (1) forces the agents to form the desired formation while* *moving along the trajectory defined by the motion of the leader in finite time for motions satisfying $|f(t)| < \beta$ for all t.*

Proof. By defining $\tilde{x}_i(t) = x_i(t) - x_1(t) - u_{xi}$, (1) can be written as

$$\dot{\tilde{x}}_i(t) = -\alpha\left(\sum_{j=1}^{n}\mathcal{A}_{ij}\Big(\tilde{x}_i(t) + u_{xi} - \tilde{x}_j(t) - u_{xj}\Big) + u_{xi}\right)$$
$$-\beta\text{sign}\left(\sum_{j=1}^{n}\mathcal{A}_{ij}\Big(\tilde{x}_i(t) + u_{xi} - \tilde{x}_j(t) - u_{xj}\Big) + u_{xi}\right) - \dot{x}_1(t). \tag{3}$$

Putting this in vector form we have

$$\begin{aligned}\dot{\tilde{\boldsymbol{x}}}(t) &= -\alpha\left(\boldsymbol{M}(\tilde{\boldsymbol{x}}(t) + \boldsymbol{u}_x) + \boldsymbol{u}_x\right)\\ &\quad - \beta\text{sign}\left(\boldsymbol{M}(\tilde{\boldsymbol{x}}(t) + \boldsymbol{u}_x) + \boldsymbol{u}_x\right) - \mathbf{1}_{n-1}\dot{x}_1(t).\end{aligned} \tag{4}$$

where $\boldsymbol{M} = \mathcal{L} + \text{diag}(\mathcal{A}_{21}, \cdots, \mathcal{A}_{n1})$ is a nonsingular matrix Barooah and Hespanha [2007], $\mathcal{L}$ is the Laplacian matrix of the graph induced by vertices $\mathcal{V}_f = \{2, \cdots, n\} \subset \mathcal{V}$, $\tilde{\boldsymbol{x}}(t) = [\tilde{x}_2(t), \cdots, \tilde{x}_n(t)]^\top$, $\boldsymbol{\delta}_x = [\delta_{x2}, \cdots, \delta_{xn}]^\top$, $\mathbf{1}_{n-1} \in \mathbb{R}^{n-1}$ is a column vector with all entries equal to 1, and

$$\boldsymbol{u}_x = -\boldsymbol{M}\boldsymbol{\delta}_x. \tag{5}$$

Define the column vector $\boldsymbol{\eta}$ as

$$\boldsymbol{M}\boldsymbol{\eta} = -\boldsymbol{M}\boldsymbol{u} - \boldsymbol{u}$$

we have

$$\dot{\tilde{\boldsymbol{x}}}(t) = -\alpha\boldsymbol{M}(\tilde{\boldsymbol{x}}(t) - \boldsymbol{\eta}) - \beta\text{sign}\left(\boldsymbol{M}(\tilde{\boldsymbol{x}}(t) - \boldsymbol{\eta})\right) - \mathbf{1_{n-1}}f(t). \tag{6}$$

Define $\hat{\boldsymbol{x}}(t) = \tilde{\boldsymbol{x}}(t) - \boldsymbol{\eta}$, thus

$$\dot{\hat{\boldsymbol{x}}}(t) = -\alpha\boldsymbol{M}\hat{\boldsymbol{x}}(t) - \beta\text{sign}\left(\boldsymbol{M}\hat{\boldsymbol{x}}(t)\right) - \mathbf{1_{n-1}}f(t). \tag{7}$$

It is known that the solution of this equation converges to zero in finite time Cao and Ren [2010]. Hence $x_i(t)$ converges to $x_1(t) + \delta_{xi}$. The convergence time can be computed to be

$$\bar{t} = \frac{\sqrt{\tilde{x}^\top(0)\boldsymbol{M}\tilde{x}(0)\lambda_{max}}}{(\beta - f_{max})\lambda_{min}}$$

where λ_{max} and λ_{min} are the maximum and the minimum eigenvalues of $\boldsymbol{M}$, and $f_{max} = \max_t |f(t)|$. □

Remark 2. *By modifying the vector u_i for each of the agents, we can also scale the formation or rotate it around the leader. If*

$$u_{xi} = -\frac{\sum_{j=1}^{n}\mathcal{A}_{ij}\Big(\delta_{xi} - \delta_{xj}\Big)}{\sigma} \tag{8}$$

where $\sigma \in \mathbb{R}^+$ is a constant scalar parameter, then the desired distances between the leader and followers would be divided by σ and therefore the formation would be scaled. Also, by rotating all the vectors δ_{xi} we can rotate the whole formation around the leader. For example in 2D space, we can rotate the followers by modifying (2) into

$$u_{xi} = -\sum_{j=1}^{n}\mathcal{A}_{ij}\Big(\delta_{xi_{Rot}} - \delta_{xj_{Rot}}\Big) \tag{9}$$

where

$$\begin{bmatrix}\delta_{xi_{Rot}}\\ \delta_{yi_{Rot}}\end{bmatrix} = -\begin{bmatrix}\cos(\theta) & -\sin(\theta)\\ \sin(\theta) & \cos(\theta)\end{bmatrix}\begin{bmatrix}\delta_{xi}\\ \delta_{yi}\end{bmatrix},$$

θ is the angle of rotation and $\{.\}_{Rot}$ denotes the rotated vector.

A possible usage of the abovementioned modifications is to change the scale or orientation of the formation temporarily to enable the agents to navigate in regions that would be impossible otherwise. An example of such situations is when the agents are supposed to go through a narrow passage while the existing size of formation does not allow this. A possible solution is to reduce the inter-agent distances, i.e. contract the formation and move through a narrow corridor. In this case we assume that each of the agents knows the diameter of the smallest circle that encircles the formation. The contraction works in a way that the each agent, i that detects the passage calculates the factor σ^i necessary to reduce the diameter by that factor to ensure a safe passage through the narrow corridor. These agents broadcast their corresponding scaling values to their neighbours. If an agent receives two unequal scaling factors from its neighbours, it discards the smaller one and retains the larger one, and consequently broadcasts this larger value. The value that all the agents agree on will be the scaling factor σ. All the agents modify their corresponding control laws to scale the formation by this desired factor σ. The algorithm used to select this scale is presented in Algorithm 1. An example for such a scenario is presented in Section 4.

Algorithm 1 Scaling Factor Selection at Agent ℓ

```
initiate: Let L_G be the diameter of graph G;
Set σ^ℓ;
k ← 1;
while k ≤ L_G do
   Broadcast σ^ℓ to all the agents in N_ℓ;
   Receive σ^i for all i ∈ N_ℓ;
   σ^ℓ ← max_{i∈N_ℓ∪{ℓ}} σ^i;
   k ← k + 1;
end while
```

Proposition 1. *After $L_{\mathcal{G}}$ steps σ^i, $\forall i \in \{2, \cdots, n\}$ are equal under Algorithm 1.*

Proof. Without loss of generality assume $\sigma^m = \max_{i\in\{2,\cdots,n} \sigma^i$. Each agent j receives the value σ^m and replaces the value of σ^j by this after $k = L_{jm}$ steps, where L_{jm} is the shortest path between j and m in $\mathcal{G}$. Furthermore, since $L_{jm} \le L_{\mathcal{G}}$, after at most $L_{\mathcal{G}}$ steps, σ^i are equal $\forall i \in \{2, \cdots, n\}$. □

One can propose a similar control law to (1) for the case where the communication/sensing among the agents is modeled by a directed graph. For this case the controller will have the same form as (1). However, for this case $\boldsymbol{u}$ is calculated by

$$\boldsymbol{u}_x = -\boldsymbol{M}_d\boldsymbol{\delta}_x \tag{10}$$

where $\boldsymbol{M}_d = \mathcal{L}_d + \text{diag}(\mathcal{A}_{21}, \cdots, \mathcal{A}_{n1})$, and $\mathcal{L}_d$ is the Laplacian matrix of the directed graph induced by vertices $\mathcal{V}_f = \{2, \cdots, n\} \subset \mathcal{V}$. Moreover, it is assumed that there is a directed path from each of the agents to the leader. For this case we have the following theorem.

Theorem 2. *Under Assumption 1, and the control law (1) and (10) the agents form the desired formation while moving along the trajectory defined by the motion of the leader in finite time for motions satisfying $|f(t)| < \beta$ for all t.*

Proof. The proof is the consequence of Cao et al. [2010] and very similar to that of Theorem 1 and is omitted for brevity □

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Remark 3. *It can be shown by a slight modification of the result in Cao et al. [2010] that the convergence time to the desired shape under control law (1) and (10) is given by*

$$\bar{t} = \frac{\max\limits_{i\in\{2,\cdots,n\}} |x_i(0) - x_1(0) - u_{xi}|}{(\beta - f_{max})}$$

where $f_{max} = \max\limits_{t} |f(t)|$.

3. OBSTACLE AVOIDANCE SCHEME

For the rest of this paper we assume that agent 1 is the leader of the formation and it visits a set of waypoints in $\mathbb{R}^2$ (the three-dimensional case is a trivial extension of this case.). Let $\mathbb{W} = \{w_i\}_{i=1}^{\bar{w}}$ with $w_i \in \mathbb{R}^2$ be the ordered set of the $\bar{w}$ waypoints that 1 should visit. Moreover, assume that 1 travels between any consecutive pair of waypoints, w_j and w_{j+1}, on a straight line. Furthermore, assume that the environment where the agents operate in contains $\bar{o}$ non-overlapping stationary circular obstacles with centers at c_i^o and radii r_i^o, and $\max\limits_{i=1,\cdots,\bar{o}} r_i^o = r_{max}$. Each obstacle is detected by agent i at time t if $\|s_i(t) - c_i^o\| \leq d$, where $d \in \mathbb{R}^+$. We formally define the problem that we study in this section here.

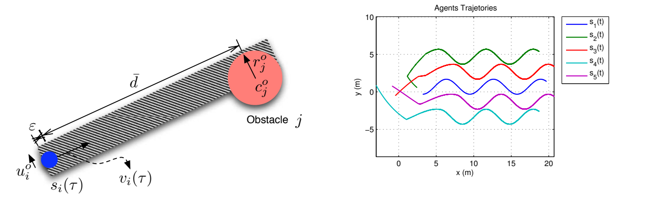
Problem 1. *How can one design a set of control laws to force the followers* $2\cdots,n$ *in the formation and the leader* 1 *to form a formation with a desired shape and avoid collision with external obstacles while the leader is moving between a pair of consecutive waypoints* w_j *and* w_{j+1} *on a straight line?*

To propose a solution to this problem, we introduce a control strategy for each of the agents that takes priority for avoiding collision with obstacles in the environment through temporary local modification of the shape of the formation. After a collision with an obstacle is avoided by an agent, this agent reverts to the shape preservation strategy, and returns the the formation to its desired shape. Inter-agent collisions are not actively avoided by the agents under the obstacle collision avoidance strategy introduced here. However, it can be shown that under an assumption, that comes later, such collisions would not occur under the proposed control strategy.

To answer this problem first we define a collision course flag $\Omega_i(t)$ for each i:

$$\Omega_i(t) = \begin{cases} 1 & \exists j \text{ such that } d_{ij}(t) \leq r_j^o + \varepsilon \,\&\, \|s_i(t) - c_i^o\| \leq \bar{d} \\ & \quad or \;\; \|s_i(t) - c_i^o\| \leq r_j^o\sqrt{2}/2 \\ 0 & otherwise \end{cases} \tag{11}$$

where $d_{ij}(t)$ is the distance of c_j^o to the ray originating from $x_i(t)$ and along the vector $v_{xi}(t)$ at time t, $\bar{d} < d$ is a positive scalar, and $0 < \varepsilon \ll 1$. Moreover, let the time where the value of $\Omega_i(t)$ switches from 0 to 1 be denoted by τ_i. Define $u_i^o = [u_{xi}^o, u_{yi}^o]^\top$ to be a perpendicular vector to $v_i(\tau_i)$ with a magnitude of $r_j^o - d_{ij}(\tau_i)$ and its direction parallel to a perpendicular vector departing from c_j^o towards the abovementioned ray. In Fig 1(a) an example for u_i^o and the region around an obstacle in which $\Omega_i(t) = 1$ is depicted. The reason for introducing $\Omega_i(t)$ comes from



(a) For all the points inside the dashed area Ω_i is equal to 1.

(b) Agents forming a fixed formation while following a sinusoidal path only known to the leader.

Fig. 1. Obstacles and agnets relations and agents trajectories.

the fact that the collision avoidance can be achieved by each of the agents through local manipulation of the shape of the formation. When $\Omega_i(t) = 1$, it means that if the agent continues its motion along its current trajectory it will be too close to the obstacle ahead and possibly collide with it.

Remark 4. *If the line connecting agent* i *to obstacle* j *and ray stemming from the agent* i *along* $v_i(t)$ *do not form an acute angle* $d_{ij}(t)$ *is set to be equal to* $\bar{D}$*, where* $\bar{D}$ *is a very large positive number. This is because it is impossible to have a collision with an obstacle in the opposite direction to the motion of the agent.*

If $\Omega_i(t) = 1$ we use the following control law to control the x-coordinate of i

$$\begin{aligned} \dot{x}_i(t) = &-\alpha\left(\sum_{j=1}^{n} \mathcal{A}_{ij}\Big(x_i(t) - x_j(t)\Big) + u_{xi} + \Delta_{ii}u_{xi}^o\right) \\ &- \beta\text{sign}\left(\sum_{j=1}^{n} \mathcal{A}_{ij}\Big(x_i(t) - x_j(t)\Big) + u_{xi} + \Delta_{ii}u_{xi}^o\right). \end{aligned} \tag{12}$$

The control law to control the y-coordinate of i is the same as (12) with the trivial change of notation from x to y. Additionally, for all $j \in \mathcal{N}_i$, $\mathcal{A}_{ji}$ is set to be zero and u_{xj} is modified accordingly for the period that $\Omega_i(t) = 1$. The interpretation of this action on the neighbours is that when agent i is undergoing an evasive action to avoid a collision with the obstacle the neighbours stop following it to prevent further deterioration of the formation shape. We summarize the approach presented here to solve Problem 1 in the Algorithm 2. However, first we make the following assumptions.

Assumption 2. *There is no time* t *such that there exists an agent* ℓ *such that* $\Omega_\ell(t) = 1$ *and* $\Omega_i(t) = 1$ *for all* $i \in \mathcal{N}_\ell$.

Assumption 3. *The desired inter-agent distances are more than* $2r_{max}$.

Assumption 2 ensures that there is at least a directed path to the leader from any of the agents at any given time. Assumption 3 guarantees that no collision occurs between the agents when they avoid colliding with the obstacles by changing their position in the formation relative to other agents.

Assumption 4. *The straight lines connecting any consecutive pairs of waypoints do not cross an obstacle.*

Note that this assumption can be relaxed if the leader re-plans the waypoints as it detects an impending collision with an obstacle on it path. The following corollary formally states the result of this section.

Algorithm 2 Motion Control with Obstacle Avoidance for Agent ℓ

initiate: $\overline{\mathcal{A}} \leftarrow \mathcal{A}$;
for $i \in \mathcal{N}_\ell$ **do**
  if $\Omega_i(t) = 1$ **then**
    $\mathcal{A}_{\ell i} \leftarrow 0$;
  else
    $\mathcal{A}_{\ell i} \leftarrow \overline{\mathcal{A}}_{\ell i}$
  end if
end for
if $\Omega_i(t) = 1$ **then**

$$\dot{x}_i(t) \leftarrow -\alpha\left(\sum_{j=1}^{n}\mathcal{A}_{ij}\left(x_i(t)-x_j(t)\right)+u_{xi}+\Delta_{ii}u_{xi}^o\right) - \beta\text{sign}\left(\sum_{j=1}^{n}\mathcal{A}_{ij}\left(x_i(t)-x_j(t)\right)+u_{xi}+\Delta_{ii}u_{xi}^o\right);$$

$$\dot{y}_i(t) \leftarrow -\alpha\left(\sum_{j=1}^{n}\mathcal{A}_{ij}\left(y_i(t)-y_j(t)\right)+u_{yi}+\Delta_{ii}u_{yi}^o\right) - \beta\text{sign}\left(\sum_{j=1}^{n}\mathcal{A}_{ij}\left(y_i(t)-y_j(t)\right)+u_{yi}+\Delta_{ii}u_{yi}^o\right);$$

else

$$\dot{x}_i(t) \leftarrow -\alpha\left(\sum_{j=1}^{n}\mathcal{A}_{ij}\left(x_i(t)-x_j(t)\right)+u_{xi}\right) - \beta\text{sign}\left(\sum_{j=1}^{n}\mathcal{A}_{ij}\left(x_i(t)-x_j(t)\right)+u_{xi}\right);$$

$$\dot{y}_i(t) \leftarrow -\alpha\left(\sum_{j=1}^{n}\mathcal{A}_{ij}\left(y_i(t)-y_j(t)\right)+u_{yi}\right) - \beta\text{sign}\left(\sum_{j=1}^{n}\mathcal{A}_{ij}\left(y_i(t)-y_j(t)\right)+u_{yi}\right);$$

end if

Corollary 1. *Algorithm 2 solves Problem 1 under Assumptions 2-4 in finite time.*

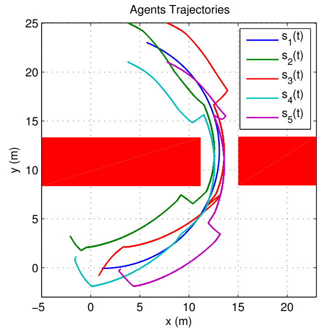
Proof. The proof is a direct consequence of Theorems 1 and 2. □

If we apply Algorithm 2 sequentially when the leader visits each of its waypoints we can solve the problem of moving the formation from a starting point to an end point while avoiding any collision with the obstacles along the way.

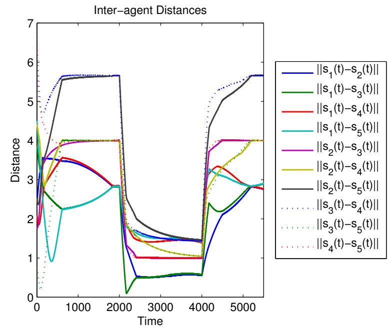
4. SIMULATIONS

In the first simulation we consider a formation with 5 agents that form a formation where 1 is the leader and moves on a sinusoidal trajectory. The agents trajectories are depicted in Fig 1(b).

In the next scenario we consider the case where the agents reach a consensus on the contraction factor necessary to reduce the size of the formation. Each agents i adjusts its u_i along the lines of (10) where $\sigma = 4$. This results in the formation going through a narrow passage. The agents trajectories in this scenario are presented in Fig. 2(a). Additionally, the inter-agent distances during this scenario are shown in Fig. 2(b).



(a) Agents Trajectories.



(b) Inter-agent Distances in the Second Scenario.

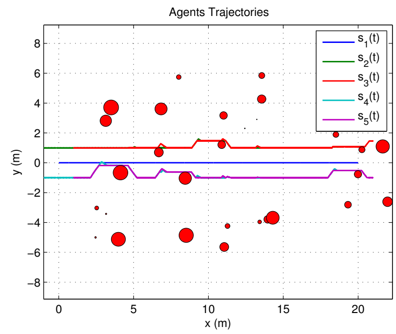
Fig. 2. Formation contraction to allow motion through a narrow passage.

In the third scenario, we consider the case where a formation of five agents is moving on a straight line in an obstacle field using Algorithm 2. The agents trajectories in this case are depicted in Fig. 3(a), and the inter-agent distances are depicted in Fig. 3(b).

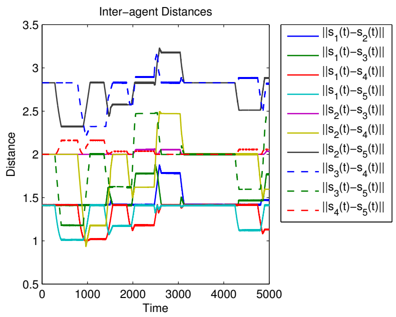
5. CONCLUSIONS AND FUTURE DIRECTIONS

In this paper, first, we introduced a method to control the shape of the formations of mobile autonomous agents with provable finite-time stability properties. We further introduced a way to modify the shape of the formations should the need arises during the course of motion of the formation. The information needed to implement these control methods are local to the agents and hence can be implemented locally. Later, we introduced an obstacle avoidance method whose performance can be analytically established and guarantees that no collision occurs between the agents and the obstacles in the environment as well as other agents as the agents travel on a path segmented into straight lines.

A possible future direction is to consider more realistic agent kinematic models and provide similar guarantees for such agent models. Another possible research problem for future consideration is to study the challenging case where the obstacles move. Another problem to be considered is developing an algorithm to choose among different obstacle avoidance strategies, e.g. scaling, orientation rotation, temporarily changing the interagent distances, in different situations.



(a) Agents Trajectories in an Obstacle Field.



(b) Inter-agent Distances in the Third Scenario.

Fig. 3. Formation Motion in an Obstacle Field

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