Range-only sensing for formation shape control and easy sensor network localization

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Abstract— **This paper is concerned with two related problems using graph theoretic methods for their solution. The central contribution to formation control is to demonstrate that if each agent senses a modest number of additional distances beyond those which are actively being controlled, and with limited message passing between neighboring agents, each agent can infer the relative positions of its neighbors in its own coordinate basis. Additional effort to related this to the basis used for viewing its controls may be required. Each agent can thus apply the known algorithms for formation shape control based on distance preservation,** *without needing to actually sense bearings*. **The contribution to sensor network localization is to identify circumstances in which the complexity is effectively** *linear* **in the number of nodes.**

Index Terms— **Formation Control, Sensor Network Localization, Graph Rigidity**

I. Introduction

This paper is concerned with two related problems using graph theoretic methods for their solution. These are, first, the control of collections of robotic agents, in such a way that the collection maintains a prescribed formation shape, of course under a series of assumptions; and second, the localization of a sensor network via a procedure using a series of local localizations.

Formation shape control problems are motivated by problems such as localization of objects of interest through sensors carried on the agents (in which case it is typical that there are optimum geometries minimizing localization error in the presence of noisy measurements), use of collections of satellites for remote sensing, and transportation of massive objects by a collection of robots.

As with many studies, [1], [2], [3] we assume in this paper that point-agent modelling is adequate, with single integrator control. Further, we restrict attention to two-dimensional formations, with no essential loss of generality.

In order to maintain the shape of a formation, certain variables must be sensed and certain variables must be controlled, and various possibilities exist for what these are. The contribution of this paper is to show that, for a broad class of problems, control and sensing of sets of inter-agent distances alone will allow formation control. The set of controlled distances in general is a subset of the set of sensed distances.

There are different approaches to the formation shape control problem, depending on what is controlled and what is sensed. See e.g. [4]. At one extreme, one can envisage a central controller, and desired locations for every single agent; each agent knows the global coordinate basis and navigates to its designated location. As an intermediate position, one can envisage that each agent knows the desired shape of the formation, including its orientation, but the formation center is left unspecified. Distributed control through each agent, which has knowledge of the desired relative positions of neighbor agents, plus operation of a consensus algorithm, aims to achieve the formation shape. In this case too, agents need knowledge of a global coordinate basis. However, a number of treatments of the formation shape control problem have relied on controlling distances alone, in distinction to controlling relative positions [1], [2], [3], [5]. The associated problem statements leave unspecified both the center and orientation of the formation. Almost all of the associated algorithms however rely also on richer sensed information than simply distances, viz. typically the relative positions of neighbor agents. The implicit assumption is that each agent can sense the range and bearing of its neighbors, at least in a coordinate basis particular to the agent, if not a global coordinate basis shared by all agents, even though it only needs to control the distances to its neighbors. [The separate question of how many and which interagent distances should be controlled in order to control a whole formation shape, without regard to its absolute position or orientation, is bound up with graph rigidity theory; for an introduction to the application of this theory to formation control, see [6]. Graph rigidity issues are also explored in most of the cited papers dealing with distance-based formation shape control].

The central contribution of this paper in relation to formation control is to demonstrate that if each agent senses a modest number of additional distances beyond those which are actively being controlled, and with limited message passing between neighboring agents, each agent can infer the relative positions of its neighbors in its own coordinate basis. Some additional effort to relat this basis to the basis used for viewing its controls may be required. Each agent can thus apply the known algorithms for formation shape control based on distance preservation, *without needing to actually sense bearings*.

We note that a different approach to the problem of dispensing with bearing measurements and relying just on range measurements is contained in other work involving the current authors, [7]; broadly speaking, this alternative approach involves collecting multiple range measurements at isolated time instants or in continuous time, and then inferring relative positions at a particular point in time. This

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is like running an observer. In contrast, the approach of this paper can provide relative position information sometimes instantaneously, using the additional range measurements.

When the variables actively controlled are inter-agent distances, alternative possibilities exist of requiring directed control or undirected control. In directed control, each agent pair between which a distance is controlled forms a leader-follower pair, and the task of distance maintenance is given to the follower agent. In undirected control, both agents seek to maintain the distance between them. The method of this paper is equally applicable to both approaches. Where necessary to fix ideas, we will assume that undirected control is used.

The second contribution of the paper deals with the problem of sensor network localization. In this problem, one presupposes there is a planar network of point sensors. The positions of certain of these sensors, termed anchors, are known. The distances between a number of sensor pairs are also known. The task is to determine the position of all the sensors. Key questions include: what conditions must be satisfied in order to secure localizability, and how complex is a localization algorithm? An answer to the first question is now well understood, and can be cast in graph theoretic terms. See [8]. It is also known that the complexity of localization in general will be exponential in the number of nodes, but in certain circumstances, may be only polynomial in the number of nodes. The contribution here is to identify circumstances in which the complexity is effectively *linear* in the number of nodes.

While the two problems of the paper are apparently different, the tool we use for them, graph rigidity theory, and indeed the way we use it, is similar. In fact, it was the study and resolution of one of the problems that led us in the direction of the other.

The structure of the paper is as follows. In the next section, we review concepts of rigidity that underpin the problems and methods of the paper. Section 3 contains a statement of the formation problem of interest. In section 4, we describe the notion of the square of a graph, and establish a property of subgraphs of this graph. Section 5 explains how this property can be exploited to obtain for each agent the relative positions of all its neighbors, so that distance based control of the shape can then be implemented. Section 6 deals with the sensor network question, and the final section contains concluding remarks.

II. A brief digression on rigidity

This section serves to review several aspects of the rigidity concept. For more details, see e.g. [6]. Formally, a formation is a collection of agents with distance constraints applying between nominated pairs of agents. The formation is termed rigid if and only if the only continuous motions possible which preserve the distance constraints are translation and rotation of the whole formation. A graph can always be associated with a formation, and one graph obviously corresponds to an infinite number of formations, obtained by varying the actual values of the constrained distances. It turns out that if a formation is rigid, almost all formations with the same graph are rigid. As a consequence, the graph itself is termed rigid. Combinatorial and linear algebra tests exist for checking rigidity.

If distance constraints are successively removed from a formation, at some point the formation will be able to sustain flexing motions (i.e. motions which are not pure translation or rotation); it is then not rigid. If a formation has the property that it is rigid but the removal of any single distance constraint destroys rigidity, the formation is termed minimally rigid. If the formation has $|V|$ agents, lies in a two-dimensional ambient space and is minimally rigid, it necessarily has $2|V| - 3$ distance constraints. Given a set of distance constraints, any two formations satisfying the constraints are termed equivalent. They may or may not be congruent, and in fact for a minimally rigid formation in two dimensions with 4 or more agents, there are always noncongruent formations satisfying the distance constraints. When there are sufficient distance constraints that all formations satisfying them are always congruent, the formations and the associated graph are termed globally rigid. Global rigidity can be checked using combinatorial or linear algebra methods.

Global rigidity is a property that also arises in considering the problem of sensor network localization [8]. Consider a sensor network in a two-dimensional ambient space, in which a number of sensors, the anchor sensors, are located at known positions, and a number of inter-sensor distances are known. The task of sensor network localization is to determine the position of all the non-anchor sensors. The specification of inter-sensor distances means that the network can be regarded as a formation. If it is globally rigid, then it is effectively determined up to a translation or rotation. If there are three or more noncollinear anchors, this nonuniqueness can be removed, and the network can be localized. Indeed, a two-dimensional sensor network is localizable if and only if there are three or more noncollinear anchors and the associated graph is globally rigid. [8] Note though that the task of executing calculations to localize all sensors may be very demanding.

III. Formation shape problem of interest

We assume that a formation is specified by a (simple) graph $G = (V, E)$ together with a set of inter-agent desired distances, $D = \{d^*_{ij}, ij \in E\}$. Each agent of the formation is associated with a vertex, each distance which should be controlled is associated with an edge of G, and the associated desired inter-agent distance is defined by the corresponding member of D.

It is assumed that the set of prescribed distance values is in fact attainable by a real formation, and the graph G is assumed to be rigid [6]. It may or may not be minimally rigid, and it may or may not be globally rigid [6]. If it is not globally rigid, then there will be noncongruent formations whose inter-agent distances attain the values specified by D. If it is globally rigid, then all formations attaining the values specified by D will be congruent. The minimum number of edges for rigidity is $2|V|-3$ and the minimum number of edges for global rigidity is $2|V|-2$. Note that neither the location of the center of the formation nor its angular orientation are part of the specified data.

Let p_i denote the position coordinate of agent i. Let $\mathcal{N}_i$ denote the neighbor set for agent i, i.e. $\mathcal{N}_i = \{j|ij \in E\}$. Let $d_{ij} = ||p_i - p_j||$. Distance-based formation control algorithms are almost all of the form

$$\dot{p}_i = \sum_{j \in \mathcal{N}_i} \phi(d_{ij}^2 - d_{ij}^{*2})(p_j - p_i) \tag{1}$$

where ϕ is a smooth strictly monotone increasing function on the interval $(-d_{ij}^{*2}, \infty)$ with $\phi(0) = 0$. The case $\phi = k$ for some positive constant k is often considered. The nonlinear ϕ notion was dealt with in [3]. Such control laws are steepest descent laws associated with an error function. For the case of constant k, the error function is $V = \frac{k}{2}\sum_{ij \in E}[d_{ij}^2 - d_{ij}^{*2}]^2$.

As is immediately apparent from the above equation, to implement a control law of the above type, each agent needs to know the relative position of its neighbors as well as the difference between the actual and desired distance.

Now there is a very important but not novel observation that should be recalled in relation to the control law for each agent. In order to implement the above law, it is enough for each agent to have a local coordinate basis, and not to have to know a global coordinate basis This means that implementation of the control law using local coordinates causes each agent to follow the same trajectory as if global coordinates were used.

It is not our purpose here to study the convergence properties of the above control law, which are dealt with in e.g. [1], [3]. Suffice it to say that formation rigidity, reviewed briefly in the next section, guarantees the local convergence to an equilibrium corresponding to the correct distances. On the other hand, global convergence properties are less well understood. If a formation has a minimally rigid graph, then the equation set has equilibria which do not correspond to the correct distances, though these may all be saddle points or unstable equilibria. Likewise, at least for some globally rigid formations, there are equilibria which do not correspond to the correct distances; whether or not they are necessarily saddles or unstable equilibria has not been determined.

The problem of interest now is: by measuring distances from an agent to additional agent than just its neighbors, can each agent learn the relative positions of its neighbors in its own coordinate basis? In the following sections, we will provide an affirmative answer to this question.

IV. The Graph G^2 and a Completeness Property for Certain Subgraphs

Let $G = (V, E)$ be an arbitrary graph. Define the graph $G^2 = (V, E \cup E^2)$ where $e_{ik} \in E^2$ if and only if $e_{ik} \notin E$ and for some $j, e_{ij}, e_{jk} \in E$. Thus G^2 is obtained from G by adding edges between all the two-hop neighbors of G. We shall prove the following.

***Theorem** 1:* Let G be a rigid graph. Then every node v is a node of a complete subgraph of G^2 with vertex set $v \cup \mathcal{N}_v$, where the order of the subgraph is $d(v) + 1 \geq 3$.

Proof: All vertices v of G necessarily have degree at least 2, in view of the rigidity of G. The $d(v)$ neighbors of v will be linked by edges in G^2, and each is linked with v. Thus the vertex set of v and its (one-hop) neighbors in G induces a complete subgraph of G^2 with $d(v) + 1 \geq 3$ vertices.

■

Two comments can be added. First, with minor modification the theorem is valid for a connected G which is not rigid; however, we are interested in using the theorem for rigid graphs. Second, with v a degree 2 vertex in a rigid G, v is also a vertex of a complete subgraph of G^2 with at least 4 vertices, i.e. with more vertices than $v \cup \mathcal{N}_v$. This last observation requires a little more work to see.

For the purposes of formation control, we shall suppose henceforth that all agents can sense the distance to their two-hop neighbors. Equivalently, all distance lengths are known in the formation regarded as having the graph G^2. Notice that in practice, there will often be an upper bound on the distances that can be sensed. This will be determined by the hardware and signal processing used for distance sensing. If a certain upper bound just allows sensing of the distances in the original graph G, i.e. the upper bound corresponds to the longest distance in the associated formation, then a doubling of the bound will ensure that all distances of edges in G^2 can be sensed.

V. Realization of Complete Graphs and Application to Formation Shape Control

The main result of the previous section indicated that when G is a rigid graph corresponding to a formation, each vertex in the graph defines a complete subgraph of G^2 whose vertices are the original vertex and its neighbors. We will now argue that knowledge of the associated distances in the corresponding formation allows determination of relative positions.

The following Lemma is based on a result of [9], which has been quoted in a quite accessible form in a much more recent reference, see [10]. The reformulation of the Lemma was communicated to us by S Dasgupta, and is in fact slightly generalized here.

***Lemma** 1:* Consider a set of $M > 2$ points $p_1, p_2, \ldots, p_M$ in $\mathcal{R}^2$.Let $d_{ij} = ||p_i - p_j||$. Define the $2 \times (M-1)$ matrix A_1 by

$$A_1 = [p_2 - p_1 \;\; p_3 - p_1 \;\; p_4 - p_1 \ldots p_M - p_1] \tag{2}$$

Then there holds $A_1^T A_1 =$

$$\begin{bmatrix} d_{12}^2 & \frac{d_{12}^2 + d_{13}^2 - d_{23}^2}{2} & \cdots & \frac{d_{12}^2 + d_{1M}^2 - d_{2M}^2}{2} \\ \frac{d_{12}^2 + d_{13}^2 - d_{23}^2}{2} & d_{13}^2 & \cdots & \frac{d_{13}^2 + d_{1M}^2 - d_{3M}^2}{2} \\ \vdots & & & \vdots \\ \frac{d_{12}^2 + d_{1M}^2 - d_{2M}^2}{2} & \frac{d_{13}^2 + d_{1M}^2 - d_{3M}^2}{2} & \cdots & d_{1M}^2 \end{bmatrix} \tag{3}$$

Proof:

That the diagonal entries of $A_1^T A_1$ are as indicated is trivial to check. Consider the off-diagonal entry in position $i-1, j-1$ for some $i \neq j$. It is given by

$$(p_i - p_1)^T (p_j - p_1) = p_i^T p_j - p_1^T p_j - p_1^T p_i + p_1^T p_1 \quad (4)$$

while

$$\begin{aligned} d_{1i}^2 + d_{1j}^2 - d_{ij}^2 &= ||p_1 - p_i||^2 + ||p_1 - p_j||^2 - ||p_i - p_j||^2 \\ &= (p_1^T p_1 - p_1^T p_i - p_i^T p_1 + p_i^T p_i) \\ &\quad + (p_1^T p_1 - p_1^T p_j - p_j^T p_1 + p_j^T p_j) \\ &\quad - (p_i^T p_i - p_i^T p_j - p_i^T p_j + p_j^T p_j) \\ &= 2[p_i^T p_j - p_1^T p_j - p_1^T p_i + p_1^T p_1] \end{aligned}$$

The claim is then proved. ■

Denote the matrix on the right of equation (3) by D_1. It is an immediate consequence of the above result that D_1 has rank equal to that of A_1, which is evidently 2 provided the M points p_i are not collinear. Moreover, knowing D_1, the matrix A_1 is determined up to left multiplication by an orthogonal matrix.

In terms of agent 1 of the formation, the matrix D_1 formed just from distance data allows determination of the vectors $p_2 - p_1, p_3 - p_1, \ldots, p_M - p_1$ up to an orthogonal multiplier. Equivalently, the relative positions as seen from agent 1 of the other agents are all determinable up to an orthogonal multiplier, i.e. up to choice of coordinate basis. It is necessary that agent 1 be able to express its control in the same coordinate basis as the relative positions, even if this is not a global basis. This can be done in several ways. (a) Ensure each agent can sense three landmarks (fixed points); this allows everything to be expressed in a global basis. (b) Provide each agent with a knowledge of north, and measure two bearings in the whole formation; using connectedness of the formation graph, all relative positions can then be determined in the global coordinate basis. (c) Arrange for agent 1 to move for a brief period in one direction and then a different direction, and infer from the resulting measured distance changes the relation between the coordinate basis used to view neighbors and that used for implementing control action.

Putting this result together with that of the previous section, we have:

Theorem 2: Consider a formation with an associated rigid graph G. Assume that all distances corresponding to edges in the graph G^2 can be sensed, and that each node of the formation knows the desired distances to its neighbors (as defined by edges of G), the actual distances to those neighbors, and the actual distances between each pair of its neighbors (i.e. certain distances associated with edges of G^2). Then given an ability to perform a Cholesky factorization, each node can determine the relative positions of all its neighbors in G. Given additionally an ability to harmonize the coordinate basis each agent uses to view its neighbors and the basis it uses to implement a control, a distributed formation shape control law based on maintaining defined distances between agent pairs defined by the edges of the graph G can be implemented.

VI. Application of ideas to sensor networks

In this section, we shall show how some of the preceding ideas can be applied to sensor network localization. Sensor network localization for a globally rigid network (given an appropriate number of anchors) is in general an NP hard problem, see [8]. Our interest here is to specify circumstances in which localization is computationally simpler. Typically, the inter-sensor distances in a sensor network are all less than some value termed the sensing radius, and the sensing radius is adjustable by adjusting the transmission power of the sensors; accordingly, the question arises as to whether through adjusting the sensor power, a sensor network can be made localizable in the first place, or, with a higher power, possibly localizable with relative simple computations. Actually, there is an immediate trivial answer to this question: if the transmission power is high enough that the associated network has a complete graph, i.e. every sensor pair are neighbors of one another, then localization is straightforward. We are however interested in conditions for which do not require larger and larger transmission powers as the diameter of the physical network goes to infinity.

By way of background, let us recall a result which appeared in [11]. The result uses the concept of 'edge 2-connectedness'. A graph is termed edge 2-connected if it remains connected with the removal of any single edge; by contrast, it is termed vertex 2-connected or simply 2-connected if it remains connected with the removal of any single vertex. It is known that vertex 2-connectedness implies edge 2-connectedness.

Theorem 3: Let $G = (V, E)$ be an edge 2-connected graph in a two-dimensional ambient space. Then G^2 is globally rigid.

This theorem in effect says that if the sensing radius is such that the associated graph is edge 2-connected, then doubling the sensing radius will ensure that the graph is globally rigid, and if there are three or more noncollinear anchors, then the network is localizable. Note though that the theorem says nothing about the ease or otherwise of localization of the network with graph G^2, and indeed there is a very simple (though not completely obvious) example described in [11] which exhibits exponential complexity in the memory and calculations: simply take G as a simple cycle with $|V|$ vertices. In this section, our goal is to obtain a refinement of this result which will provide more acceptable complexity.

Now a rigid graph can be easily shown to be edge 2-connected. Hence the construction provided earlier in the context of the formation shape stabilization problem with underlying (rigid) graph G and which relied on sensing distances in a formation or network with a graph G^2 was actually introducing a globally rigid graph. The earlier argument in fact associated a complete subgraph of G^2 with each node of G, and solved a localization problem for the sub-network associated with each complete subgraph separately, in a coordinate basis associated with the relevant defining node. It is natural then to ask whether one can somehow glue together these separate localizations, or equivalently find a common coordinate basis, and thereby localize the whole network (given three or more noncollinear anchors).

Let us now study how one might combine 'adjacent' localizations of subnetworks corresponding to complete subgraphs of G^2. Consider a rigid graph G with two adjacent vertices a, d of degree 3. The edges incident on a are ab, ac, ad and the edges incident on d are ad, de, df. There are two cases to consider.

Case 1. Assume there is no edge in G linking either of b, c to any of d, e, f, no edges in G linking either of e, f to a and no edge in G^2 linking either of b, c to either of e, f. Figure 1(a) displays the relevant part of G; the particular vertex degrees for b, c, e, f are irrelevant (though rigidity requires them to be at least 2). When one constructs G^2, new edges appear, see Figure 1(b). This figure displays no vertices of G^2 apart from a through f, and it only displays edges of G^2 which are incident on two vertices in the set a through f. Evidently, the complete subnetworks with vertex sets a, b, c, d and a, d, e, f can both be individually localized. Notice that edge ad is common to both.

Let p_i, q_i denote the coordinates of sensor i in the local coordinate bases associated with sensors a, d respectively. There are two descriptions of edge ad, viz. $p_d - p_a$ and $q_d - q_a$ in the two coordinate bases. There must then be a 2×2 orthogonal matrix Q_d such that

$$q_d - q_a = Q_d(p_d - p_a) \tag{5}$$

Since Q_d is necessarily of the form

$$\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \text{ or } \begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{bmatrix} \tag{6}$$

it is evident that there are precisely two possibilities for Q_d. This implies that the subgraph with vertices a through f is not globally rigid; indeed, it is evident that a subnetwork with the same edge lengths as the subnetwork depicted in Figure 1(b) could be obtained by rotating the triangle with vertices a, e, f out of the plane about the axis ad until e, f move to reflections of their original positions in the original plane.

Case 2. Assume there is at least one edge in G which links one of b, c to one of d, e, f, or one edge in G linking one of e, f to a. The argument of Case 1 can be followed to conclude that there are two possibilities for Q_d before taking into account the extra edge. Now if there is an edge in G linking, say b to d, then there is an edge in G^2 linking b to e. Hence the assumption in Case 2 means that there is at least one edge of G^2 linking one of b, c and e, f; with knowledge of the length of this edge, the binary ambiguity in Q_d can be resolved, and a single coordinate basis found for the vertices a through f. [The relation between the two coordinate bases is of the form $q_i = Q_d p_i + b_d$ for all i. With Q_d determined, b_d follows immediately as $q_d - Q_d p_d$].

In the event that Case 1 holds, the binary ambiguity in the two possibilities for Q_d can only be resolved by considering further parts of G^2 than the subgraph induced by the nodes a through f. Given that G^2 is known to be globally rigid, the ambiguity is indeed resolvable; the computational complexity however is not clear.

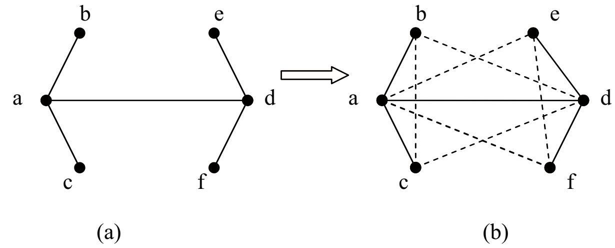

Fig. 1. Squaring a graph (a) G to (b) G^2

Based on the above remarks, it is evident that the following conclusion can be drawn.

Theorem 4: Let G be a rigid graph. Suppose a sensor network has as its associated graph G^2. Suppose that a, d are two neighboring vertices of G, and let G_a, G_d be the complete subgraphs of G^2 induced by a and its neighbors in G, and d and its neighbors in G. Let $G_{a,d}$ be the subgraph of G^2 induced by the vertices of G_a, G_d. Then this subgraph is globally rigid and a single coordinate basis for the nodes of the associated subnetwork can be readily determined using realizations of the subnetworks associated with G_a, G_d if and only if one or both of the following conditions holds:

1) there is an edge of G linking a to any neighbor of d or linking d to any neighbor of a
2) There is an edge of G or G^2 linking any neighbor of a to any neighbor of d

To sum up, if G is rigid, G^2 while certainly globally rigid may not have sufficient connectivity as to allow easy localization of a sensor network with graph G^2, where in using the term easy localization, we contemplate a series of separate localizations of complete subgraphs followed by the recursive determination of a single coordinate basis, at each step of the recursion working with pairs of the subgraphs with a common edge to incorporate an additional subgraph in the description using the single coordinate basis.

Two crucial points should be made: local message passing can be used to check the satisfaction or otherwise of the conditions listed in the theorem, and in the event that the conditions are not met, a power increase for one of the nodes in one of the complete subgraphs to increase its sensing radius will ensure that the condition is indeed met. If the sensing radius giving rise to G is r, then a sensing radius $2r$ for all nodes will ensure that the graph of the sensor network is G^2, and an increase to $3r$ at one node will ensure satisfaction of the condition of the theorem.

The overall computational complexity of the localization will be linear in the number of vertices.

There is some similarity here with localization using trilateration, see [11]. If G is simply a connected graph, then the graph G^3 is a trilateration graph and a sensor network whose graph is G^3 is localizable in linear time. Thus if power levels in a sensor network are initially set as to guarantee connectivity, increasing them to treble the sensing radius will ensure the network with its new connections is localizable in linear time.

VII. Concluding remarks

In this section, we discuss possible extensions to the ideas. The main result of this paper is to the effect that decentralized control of the shape of a multi-agent formation can be effected by controlling distances and measuring these distances and indeed further ones, together with pinning down a coordinate basis using landmarks, agent motions or limited bearing information. This is in contrast to most other procedures which require measurement of distances and bearings or angles of all neighbors, even though the information is just used to control distances.

It is of interest to consider whether other restrictions on the type of sensed variables could lead to a similar conclusion. Could one for example using angle- or bearing-only sensing, and possibly angle- or bearing-only control, control the shape of a formation? Immediately, one can see there is a problem of scale; without some distance entering somewhere in the problem formulation, the scale of the formation shape will be unspecified. However, one way such information can reasonably enter the formulation however is to postulate that in addition to the formation, there are landmarks, perhaps as few as two, located in the vicinity of the formation. Landmarks by definition are at known locations and so the distance between them is fixed. If the underlying graph is defined to include among its vertices both the formation agents and the landmarks, conceivably control of formation shape, with the necessary sensing, could be achieved through bearing/angle sensors alone on the formation agents, together with knowledge of the landmark positions, or at least inter-landmark distances.

The broad approach of the paper was cast in a two-dimensional ambient space framework; the changes required to tackle three dimensions are however trivial. The control graph G must be rigid, and the sensing graph is again G^2, where each node defines a complete subgraph whose vertices are the node itself and its neighbors. Parenthetically, we note that no necessary and sufficient combinatorial test is known to check whether the control graph associated with a three-dimensional formation is rigid, though a linear algebra test is available. Despite this hole in general graph rigidity theory, the main result is unaffected.

A further refinement to contemplate would be one in which agents had to adaptively sense their neighbors, and indeed sufficient numbers of them, to ensure that distances in G^2 were known to the agents. The problem data, viz. the required distances between agent pairs, in effect also provides G. Separately from that, each agent would have to properly sense the actual distance to its neighbors (as defined by G) and its two-hop neighbors (to provide the extra edges appearing in G^2.

The final unexamined issue we record is to understand the effect of noise. In particular, it might be that the determination of relative positions for some of the subformations corresponding to complete subgraphs of G^2 is very numerically sensitive to errors in the distance measurements. Defining ‘bad’ and ‘good’ geometries (assuming geometries to have such qualities) would be helpful, particularly if qualitative descriptions were possible. Could it be for example that if the area of a subformation was very small (so that agents were nearly collinear), it is harder to determine relative positions?

In connection with the sensor network localization problem, it would be very desirable to have a better characterization than that of the paper as to what extra edges are needed to allow a localization algorithm that has complexity linear in the number of nodes.

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