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Properties of 1-D Infrastructure-based Wireless Multi-hop Networks

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***Abstract*—Many real wireless multi-hop networks are deployed with some infrastructure support, where the results on ad-hoc networks cannot be readily extended to understand the properties of those networks. In this paper, we study those networks in 1-D. Specifically, we consider two types of nodes in the networks: ordinary nodes and powerful nodes, where ordinary nodes are i.i.d and Poissonly distributed in a unit interval and powerful nodes are arbitrarily distributed within the same unit interval. These powerful nodes are inter-connected via some backbone infrastructure. The network is said to be connected, i.e. any two nodes can communicate with each other, if each ordinary node is connected to *at least one* of the powerful nodes. We call this type of connectivity *type-II connectivity*. Exact and simplified asymptotic formulas for type-II connectivity probability and the average hop count between two arbitrary nodes are obtained. Further we prove that equi-distant powerful nodes deployment delivers the optimum performance which maximizes the type-II connectivity probability. These results are important for the design and deployment of 1-D infrastructure-based networks and provide useful insights into the analysis of higher dimensional networks.**

***Index Terms*—wireless networks, 1-D networks, type-II connectivity, random geometric graph, hop counts**

I. Introduction

Extensive results have been obtained on the connectivity of wireless ad-hoc networks in recent years. A wireless ad-hoc network is said to be connected if there is a (multi-hop) path between *any* pair of distinct nodes. One of the best known results on the connectivity of ad-hoc networks is by Gupta and Kumar [1], where they investigated the critical transmission range required for a connected network in $\Re^2$ with a total of n nodes independently, identically (i.i.d.) and uniformly distributed in a unit disk area and the communication between nodes follows the unit disk communication model as n goes to infinity. In addition to the above result, there are many other studies on the connectivity of one or higher dimensional networks (e.g. see [2], [3], [4], [5]).

Nevertheless, those studies are restricted in that they only apply to ad-hoc networks without any embedded infrastructure. Real networks are often deployed with some infrastructure support [6] and many not be modelable by homogeneous random geometric graphs. Several examples can be found in the fields of wireless sensor networks and vehicular ad-hoc networks (see [7] for more examples). In wireless sensor networks, sensors are often deployed to collect useful information which is then delivered to one of the data sinks in the sensing region. An example is the deployment of a sensor network on Great Duck Island for habitat monitoring [8]. The sensor nodes perform sensing tasks, and transmit their data in a multi-hop fashion to the sensor network gateways. These gateways then aggregate and transmit the data via a local transit network to a remote control base station. In vehicular networks, wireless nodes deployed at the roadside and with a possibly larger transmission range than those mounted on vehicles play an important role in the reliable and timely distribution of important information [9]. The active research projects in this field include IntelliDrive [10] in USA and a number of projects under CAR 2 CAR Communication Consortium [11] in Europe. These projects target improvement of road safety and traffic flow, as well as providing other value added services, which involve collaborative communication among vehicles on the road and the roadside infrastructure. In the above examples, the communication between nodes involve some infrastructure inter-connected by either wired or wireless links. Existing studies on the ad-hoc networks are inadequate to understand the properties of these networks. Indeed, a novel conceptual framework is required to investigate the properties of these networks properly.

In this paper, we propose a new concept of connectivity for infrastructure-based wireless multi-hop networks, which we term *type-II connectivity* to distinguish it from the conventionally studied connectivity in ad-hoc networks. A network is said to be type-II connected if every node in the network is connected (via one-hop or multi-hop paths) to at least one of a small subset of "powerful" nodes. The powerful nodes are inter-connected by wired or wireless links. In this paper, our analysis focuses on characterizing type-II connectivity for *one-dimensional (1-D) networks*. It is expected that the results will be useful for many real world applications which can be modeled by 1-D networks. For example, a vehicular network built along a highway, a sensor network deployed along the body of a bridge to monitor the vibration and stress of the bridge, or a group of sensors deployed along the border of a defined region for intrusion detection. Apart from these examples, it is evident that the results of type-II connectivity of 1-D networks will also provide useful insights into the analysis of higher dimensional networks. In addition to obtaining an analytical result on the type-II connectivity, the average hop

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count between two arbitrary nodes, which is closely related to the end-to-end delay, is analyzed. Finally, we obtain the optimum powerful node distribution that maximizes the type-II connectivity probability.

The rest of this paper is organized as follows: In Section II we introduce related work on type-II connectivity. In Section III we define the network model. We obtain the type-II connectivity probability in Section IV and the average hop count in Section V. In Section VI, we discuss the optimal distribution of powerful nodes. In Section VII we compare analytical and simulation data. Finally, Section VIII concludes this paper and discusses future work.

II. Related work

In [4], Miorandi and Altman assumed that there is a predetermined node located at the origin. Other nodes are arbitrarily or uniformly distributed along a semi-infinite line. They investigated the probability of other nodes being connected to the node at the origin, following unit disk communication model and other random channel models. This scenario can be considered as a special case of type-II connectivity in 1-D networks with only one powerful node placed at the origin. We consider multiple powerful nodes in this paper.

Another related problem has been studied in the context of *Multihop Cellular Network* (MCN) [12]. Unlike conventional Cellular Networks, MCN allows mobile stations to reach the base stations in multiple hops. Ojha et al. [13] considered a network of n uniformly distributed node in a circular area of unit radius and obtained a lower bound on the transmission range required for all nodes in the network to be asymptotically connected to the base station at the center of the area as $n \to \infty$. Under a more generic assumption of having both base stations and subscriber stations Poissonly distributed in $\Re^2$ and the log-normal channel model, Mukherjee et al. [14] obtained a lower bound on the probability that an arbitrary subscriber station cannot reach any base station in less than or equal to t hops. In this paper, we consider multiple powerful nodes and do not limit the maximum hop count between two connected nodes, which add further complexity to the study.

A closely related study on type-II connectivity is that Dousse et al. [15]. In their work on 1-D networks, the nodes are assumed to be Poissonly distributed on a line of length L, and communicate following the unit disk model. The authors then obtained analytically the probability (say $p(x)$) of a node located at distance x from the left end of the line being connected to at least one of the two base stations placed at each end of the line. The authors considered this line segment as a "reduced" version of a more generic network where an infinite number of base stations are placed every L units distance on an infinite line.

The work of Dousse et al. is different from ours in two ways. First, we consider not only two, but generally any given numbers of base stations *arbitrarily* distributed within a finite interval which adds considerable analytical complexity and practical significance to the problem. Second, it is not trivial to derive the probability that a network is type-II connected using $p(x)$. The difficulty lies in the fact that the probability that two distinct nodes are connected to a base station are not independent, but correlated in a complicated way. Therefore, a different technique is used in this paper to analyze the network type-II connectivity probability. Using our results, $p(x)$ can be readily derived but to derive our results using $p(x)$ appears rather complicated.

III. Network model and notations

Consider a wireless multi-hop network with two types of nodes: ordinary nodes and powerful nodes. Ordinary nodes are i.i.d and Poissonly distributed with a known density λ in a unit interval [0,1]. There are $n_a \geq 2$ powerful nodes in the network, where two of them are placed at both ends of the interval and the others are arbitrarily distributed in the interval. Then, the powerful nodes divide the unit interval into $n_a - 1$ sub-intervals. Each sub-interval i has length w_i for $1 \leq i \leq n_a - 1$.

Let r be the transmission range of both type of nodes (powerful nodes also being connected by infrastructure). Any two nodes are directly connected if their distance is less than or equal to r. We say that *the network is type-II connected if each ordinary node is connected (either directly or via a multi-hop path) to at least one of the powerful nodes.* The same transmission range for both types of nodes is considered to avoid technicalities which may obscure the main idea of solving type-II connectivity problem. In Section IV-B, we explain how the result can be extended to the situation where powerful nodes have a larger transmission range.

IV. Type-II connectivity probability

A network is type-II connected if and only if each sub-interval is type-II connected. Let $\mathcal{A}_i(m_i, w_i)$ be the event that sub-interval i with length w_i is type-II connected given there are m_i ordinary nodes in the sub-interval. The derivation of $\Pr(\mathcal{A}_i(m_i, w_i))$, the probability that $\mathcal{A}_i(m_i, w_i)$ occurs, relies on the following lemma by Godehardt and Jaworski [16].

Lemma 1. *Let $[x, x+y]$ be a sub-interval of length y within $[0,1]$. Assume two of k given vertices have been placed at the borders of this sub-interval. Define two vertices to be neighbors if and only if they are at distance r or less apart, let $\mathcal{Z}_{k,y,r}$ be the event that $k-2$ remaining vertices, which are uniformly placed in $[0,1]$, are inside $[x, x+y]$ and "join" the borders, that is, the k vertices form a connected subgraph of length y; and let $P(k,y,r) = \Pr(\mathcal{Z}_{k,y,r})$. Then*

$$P(k,y,r) = \sum_{j=0}^{\min(k-1, \lfloor y/r \rfloor)} \binom{k-1}{j} (-1)^j (y - jr)^{k-2}, \quad k \geq 2 \tag{1}$$

A sub-interval is type-II connected if all ordinary nodes within the sub-interval are connected to at least one of the two powerful nodes located at both ends of the sub-interval.

The probability that this event happens is

$$\Pr(\mathcal{A}_i(m_i, w_i)) = P(m_i + 2, 1, \hat{r}) + m_i(m_i + 1) \int_0^{1-\hat{r}} P(m_i + 1, \hat{x}, \hat{r}) \, \mathrm{d}\hat{x} \quad (2)$$

where $\hat{r} = \frac{r}{w_i}$ is the normalized transmission range, and $\hat{x} = \frac{x}{w_i}$ is the normalized distance of x. The two terms on the right hand side of Eq. (2) represent the two possible cases of the event respectively. They are illustrated in Fig. 1.

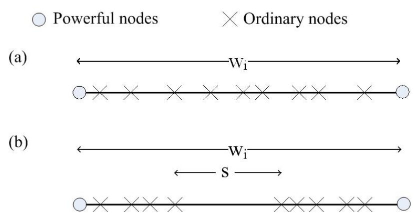

Fig. 1. An illustration of possible situations for a type-II connected sub-interval. Subgraph (a) shows that all ordinary nodes are connected to both powerful nodes. Subgraph (b) shows an example that there is a big spacing with length s in the sub-interval, and $s > r$.

Fig. 1(a) shows the possible case where all m_i ordinary nodes within sub-interval i are connected to both powerful nodes. That is, $m_i + 2$ nodes (m_i ordinary nodes and two powerful nodes) form a connected "subgraph" of length w_i. From Lemma 1 this probability is $P(m_i + 2, 1, \hat{r})$.

Fig. 1(b) shows the other possible case where the m_i ordinary nodes inside sub-interval i are connected to either one of the two powerful nodes but not both. Then among the $m_i + 1$ spacings between the adjacent nodes, there is exactly one spacing with length $s > r$. Suppose temporarily that the big spacing and the ordinary node attached to the left end of the big spacing are removed from sub-interval i, as illustrated in Fig. 2; then the $m_i - 1$ remaining ordinary nodes and the two powerful powerful nodes form a connected "subgraph" of length $x = w_i - s$, and the probability is given by $P(m_i+1, \hat{x}, \hat{r})$. A special case occurs when the big spacing

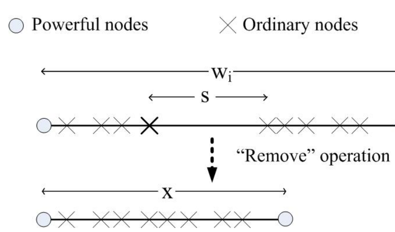

Fig. 2. An illustration of the big spacing with length s and the ordinary node attached to the left end of the big spacing are removed (the "remove" operation) from the sub-interval i.

is the left most spacing in the sub-interval. If this is the case, then we remove the ordinary node attached to the right end of the big spacing instead. Following the convention of [16] that nodes are treated as distinguishable, any of the m_i ordinary nodes can be attached to the left end of the big spacing (or attached to the right end for the special case), and the big spacing can be any of the $m_i + 1$ spacings in sub-interval i. As a result, the probability that events like Fig. 1(b) occur is then $m_i(m_i + 1)P(m_i + 1, \hat{x}, \hat{r})$, for $\hat{x}$ ranging from zero to $1 - \hat{r}$. So we obtain the second term in Eq. (2).

After inserting Eq. (1) into the second term of Eq. (2), we can get rid of the integral in the second term by moving the inner sum outside the integral. With changes to the range of summation and integral we obtain

$$m_i(m_i + 1) \int_0^{1-\hat{r}} P(m_i + 1, \hat{x}, \hat{r}) \, \mathrm{d}\hat{x} = (m_i + 1) \sum_{j=0}^{\min(m_i, \lfloor 1/\hat{r} \rfloor - 1)} \binom{m_i}{j} (-1)^j \left(1 - (j+1)\hat{r}\right)^{m_i} \quad (3)$$

Using Eq. (3), and replacing the first term in Eq. (2) by Eq. (1), we can simplify Eq. (2):

$$\Pr(\mathcal{A}_i(m_i, w_i)) = \sum_{j=0}^{\min(m_i+1, \lfloor w_i/r \rfloor)} (1 - j) \binom{m_i + 1}{j} (-1)^j (1 - j\frac{r}{w_i})^{m_i} \quad (4)$$

Since all sub-intervals bounded by powerful nodes are non-overlapping segments with length w_i and ordinary nodes are Poissonly distributed, m_i is an *independent* Poisson random variables with mean $w_i\lambda$. Let $\mathcal{A}_i(w_i)$ be the event that sub-interval i with length w_i is type-II connected. Then,

$$\Pr(\mathcal{A}_i(w_i)) = \sum_{m_i=0}^{\infty} \Pr(\mathcal{A}_i(m_i, w_i)) \frac{(w_i\lambda)^{m_i}}{m_i!} \exp(-w_i\lambda) \quad (5)$$

$$= \sum_{j=0}^{\lfloor w_i/r \rfloor} (1 - j)(-1)^j \frac{1}{j!} (j + w_i\lambda - jr\lambda) \times (w_i\lambda - jr\lambda)^{j-1} \exp(-jr\lambda) \quad (6)$$

where from Eq. (5) we first exchange the order of the inner sum and the outer sum after we replace $\Pr(\mathcal{A}_i(m_i, w_i))$ with Eq. (4). Then we substitute $\binom{m_i+1}{j} = \frac{1}{j!} \frac{\mathrm{d}^j}{\mathrm{d}t^j} t^{m_i+1} \Big|_{t=1}$, move the derivative outside the inner sum, and with some arithmetic steps we obtain Eq. (6). With Eq. (6), the probability that a network with each sub-interval i having different length w_i is type-II connected (say, event $\mathcal{B}_\lambda(w_1, \cdots, w_{n_a-1})$) is

$$\Pr(\mathcal{B}_\lambda(w_1, \cdots, w_{n_a-1})) = \prod_{i=1}^{n_a-1} \Pr(\mathcal{A}_i(w_i)) \quad (7)$$

Finally, for a network with n_a powerful nodes *arbitrarily* distributed in the unit interval, it is type-II connected (say, event $\mathcal{B}_\lambda(n_a)$) with probability

$$\Pr(\mathcal{B}_\lambda(n_a)) = \int_D \left(\prod_{i=1}^{n_a-1} \Pr(\mathcal{A}_i(w_i)) \right) f(\mathbf{w}) \mathrm{d}\mathbf{w} \quad (8)$$

where $D = \{(w_1, \cdots, w_{n_a-1}) : \sum_i w_i = 1\}$, $f(\mathbf{w}) = f(w_1, \cdots, w_{n_a-1})$ is the joint distribution of the distances between adjacent powerful nodes. If the powerful nodes are uniformly distributed, then $f(\mathbf{w}) = (n_a - 2)!$ [17]. if the powerful nodes are placed in an equi-distant fashion, then $\Pr(\mathcal{B}_\lambda(n_a)) = [\Pr(\mathcal{A}_i(w))]^{n_a-1}$ where $w = \frac{1}{n_a-1}$. In the

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following sub-sections, we consider two variations to the current model and results.

A. Asymptotic probability that a sub-interval is type-II connected

Eq. (6) is in a very complicated form, which may prevent us from obtaining in-depth understanding on the relations among parameters that determine $\Pr(\mathcal{A}_i(w_i))$. In the following we derive a simplified asymptotic approximation for $\Pr(\mathcal{A}_i(w_i))$.

Let $\hat{w} = w_i/r$ be the normalized length of sub-interval i by r; let $u = 2r\lambda$ be the average node degree; let $f(\hat{w}) = \Pr(\mathcal{A}_i(\hat{w}r))$. It can be shown that Eq. (6) implies the following difference-differential equation.

$$\frac{\mathrm{d}^2}{\mathrm{d}\hat{w}^2} f(\hat{w}) + 2\beta \frac{\mathrm{d}}{\mathrm{d}\hat{w}} f(\hat{w}-1) + \beta^2 f(\hat{w}-2) = 0 \tag{9}$$

where $\beta = \frac{u}{2}\exp(-\frac{u}{2})$. The Laplace transform of $f(\hat{w})$ is

$$F(s) = \frac{1}{s+\beta\exp(-s)} + \frac{\beta\exp(-s)}{(s+\beta\exp(-s))^2} \tag{10}$$

As $s \to 0$, $\exp(-s) \approx 1-s$. Substitute the approximation into Eq. (10); the inverse Laplace transform of $F(s)$ is then

$$f(\hat{w}) \approx \left[\frac{1-2\beta}{(1-\beta)^2} + \frac{\beta\hat{w}}{(1-\beta)^3}\right]\exp(\frac{-\beta\hat{w}}{1-\beta}) \tag{11}$$

Fig. 3 shows that Eq. (11) serves as a good approximation for the exact result in Eq. (6) provided $u \geq 6$, and virtually all values of $\hat{w} \geq 2$, not just large values of $\hat{w}$. Solving Eq. (11)

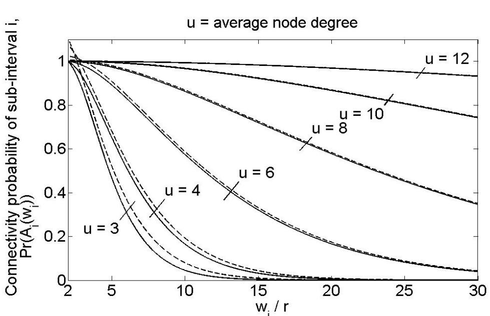

Fig. 3. The type-II connectivity probability of a sub-interval. The solid lines are the exact results, and the dashed lines are the asymptotic results.

for $\hat{w}$ leads to

$$\begin{aligned}\hat{w} = \{ & -\mathrm{W}_{-1}[-(1-\beta)^2\exp(-(1-2\beta))f(\hat{w})] \\ & -(1-2\beta)\}\frac{1-\beta}{\beta}\end{aligned} \tag{12}$$

where $\mathrm{W}_{-1}[.]$ denotes the real-valued, non-principal branch of the LambertW function [18]. Eq. (12) provides a guideline of how far away two powerful nodes can be placed to meet the required connectivity requirement.

B. Powerful nodes with a larger transmission range

Let r_p be the transmission range of the powerful nodes where now $r_p > r$. To differentiate from the previous assumption that $r_p = r$ we call the situation where $r_p > r$ as a *heterogeneous setup* and the situation where $r_p = r$ a *homogeneous setup*. In the heterogeneous setup, any ordinary node is directly connected to a powerful node, i.e. they can communicate with each other in both directions, if the distance between them is less than or equal to r_p. It is trivial that sub-interval i is type-II connected if $w_i \leq 2r_p$. For $w_i > 2r_p$, a realization of a type-II connected sub-interval i under the heterogeneous setup is type-II connected under the homogeneous setup after we remove an interval of length $r_p - r$ (and the ordinary nodes therein) from the left end and right end of sub-interval i respectively. Hence for $w_i > 2r_p$, the probability that sub-interval i with length w_i is type-II connected under the heterogeneous setup equals to the probability that sub-interval i with length $w_i - 2(r_p - r)$ is type-II connected under the homogeneous setup.

In the next section, formulas will be derived under the assumption that powerful nodes are placed in an equi-distant fashion, and $r_p = r$. The discussion in this section has provided the conceptual basis to generalize the results.

V. AVERAGE HOP COUNT

In this section, the average hop count between two arbitrary nodes in a network is studied, given equi-distant spacing of powerful nodes. Before proceeding to calculate the average hop count, we note the following result of [19].

Assume that a source node is placed at the origin of a semi-infinite line starting at the origin and extending to $+\infty$. Other nodes are Poissonly distributed (with known density λ) on the right hand side of the source node. Let ξ_h be the position of the furthest node separated from the source node by h hops. Dulman et al. has shown in [19] that the joint probability density function of $\xi_h, \xi_{h-1}, \cdots, \xi_1$ is

$$\begin{aligned} f(\xi_h, \xi_{h-1}, \cdots, \xi_1) &= \lambda^h \exp(-\lambda(hr - \xi_h)) \\ \times\ & \Gamma_{\xi_{h-2}+r}^{\xi_{h-1}+r}(\xi_h)\Gamma_{\xi_{h-3}+r}^{\xi_{h-2}+r}(\xi_{h-1})\cdots\Gamma_r^{\xi_1+r}(\xi_2)\Gamma_0^r(\xi_1) \end{aligned} \tag{13}$$

where $\Gamma_{l_1}^{l_2}(x) = 1$ for $l_1 < x \leq l_2$, and zero otherwise. Let $H(x)$ be the number of hops between two nodes separated by Euclidean distance x. Then the destination node is h hops away from the source node ($H(x) = h$) if $x - r \leq \xi_{h-1} \leq x$, and $\xi_i < x - r$ for $1 \leq i \leq h-2$. Hence, the probability that $H(x) = h$ is

$$\begin{aligned}\Pr(H(x) = h) = \int_0^{x-r} \cdots \int_0^{x-r} \int_{x-r}^{x} \\ f(\xi_{h-1}, \xi_{h-2}, \cdots, \xi_1)\mathrm{d}\xi_{h-1}\mathrm{d}\xi_{h-2}\cdots\mathrm{d}\xi_1\end{aligned} \tag{14}$$

To complete the definition of $H(x)$, we define that $H(x) = \infty$ if two nodes separated by a known distance x are *disconnected*. Then, $\Pr(H(x) = \infty) = 1 - \sum_{h=1}^{\infty}\Pr(H(x) = h)$.

With Eq. (14), we can derive the average hop count between two arbitrary ordinary nodes. Let $\hat{H}(x, y)$ be the smallest number of hops between two ordinary nodes, at x (i.e. x Euclidean distance away from the leftmost point of the unit interval) and y respectively. Then,

$$\hat{H}(x, y) = \min(H(|y - x|), \bar{H}(x) + \bar{H}(y)) \tag{15}$$

where $\bar{H}(x)$ is the minimum number of hops of an ordinary node at x to its nearest powerful node either on its left or on its right, and $H(.)$ is given in Eq. (14). That is, $\hat{H}(x,y)$ equals to the minimum of the number of hops for the ordinary nodes at x and y to reach each other via the powerful nodes: $\bar{H}(x)+\bar{H}(y)$, and not via the powerful nodes: $H(|y-x|)$. Let $i(x)$ be the sub-interval at which the ordinary node at x is located. Without loss of generality, we assume the unit interval being considered is $[0,1]$. Then $i(x)=\lceil x/w \rceil$, the smallest integer larger than x/w. It is clear that

$$\bar{H}(x)=\min(H(x-(i(x)-1)w), H(i(x)w-x))$$

Note that in Eq. (15), $H(y-x)$ and $\bar{H}(x)+\bar{H}(y)$ are not independent on each other. For example, when $y>x$, the ordinary nodes that lie between the ordinary node at x and the nearest powerful node on its right will contribute to $H(y-x)$, as well as $\bar{H}(x)+\bar{H}(y)$ in some ways. Nevertheless, we assume they are independent which greatly simplifies the numerical computation of the distribution of $\hat{H}(x,y)$. As will be shown later in Section VII, the above approximation is however accurate.

Finally, we can obtain the average hop count between two arbitrary nodes in a network (say $E[\hat{H}]$). To facilitate the numerical computation of $E[\hat{H}]$, for every pair of disconnected ordinary nodes, we assign it a large hop count of $2\left\lceil \frac{1}{r} \right\rceil$, which is an upper bound on the number of hops between any two connected nodes within the unit interval [19]. Then,

$$E[\hat{H}]=\sum_{h=1}^{2\lceil 1/r \rceil} hP_c(h)+2\left\lceil \frac{1}{r} \right\rceil \left[1-\sum_{h=1}^{2\lceil 1/r \rceil} P_c(h)\right] \qquad (16)$$

where $P_c(h)=\int_0^1\int_0^1 \Pr(\hat{H}(x,y)=h)\mathrm{d}x\mathrm{d}y$ is the probability that two arbitrary ordinary nodes are h hops away. Eq. (15) relates $P_c(h)$ to $\Pr(H(x)=h)$ and hence it suggests how Eq. (16) can be calculated numerically.

VI. Optimal distribution of powerful nodes

In this section, we prove that the equi-distant placement of powerful nodes will maximize the asymptotic type-II connectivity probability under the constraint of a fixed number of powerful nodes. Finding the optimal powerful node placement to maximize the probability can be formulated as a constrained optimization problem:

$$\begin{aligned} \text{maximize} \quad & \Pr(\mathcal{B}_\lambda(w_1,\cdots,w_{n_a-1})) \\ \text{subject to} \quad & \sum_{i=1}^{n_a-1} w_i = 1 \end{aligned}$$

In the following we prove that $\Pr(\mathcal{B}_\lambda(w_1,\cdots,w_{n_a-1}))$ is a log-concave function of w_i, for $1\le i\le n_a-1$.

Proof: Substitute Eq. (11) into Eq. (7) we have

$$\begin{aligned} &\Pr(\mathcal{B}_\lambda(w_1,\cdots,w_{n_a-1})) \\ &=\prod_{i=1}^{n_a-1}\left[\frac{1-2\beta}{(1-\beta)^2}+\frac{\beta\hat{w}_i}{(1-\beta)^3}\right]\exp(\frac{-\beta/r}{1-\beta}) \qquad (17) \end{aligned}$$

Since $0\le\beta\le\exp(-1)$, $\frac{1-2\beta}{(1-\beta)^2}+\frac{\beta\hat{w}_i}{(1-\beta)^3}\ge 0$ for all $\hat{w}_i\ge 0$. This implies that $\frac{1-2\beta}{(1-\beta)^2}+\frac{\beta\hat{w}_i}{(1-\beta)^3}$ is log-concave on $\hat{w}_i\ge 0$ [20]. Given that the product of log-concave functions is log-concave, Eq. (17) is log-concave w.r.t all w_i's. ■

Hence the optimization problem is a concave optimization problem. It is then straightforward to show that a maximum of $\Pr(\mathcal{B}_\lambda(w_1,\cdots,w_{n_a-1}))$ is achieved when $w_1=\cdots=w_{n_a-1}=\frac{1}{n_a-1}$ and by concavity it is a global maximum.

VII. Discussion

In this section, we investigate the impact of the density of ordinary nodes λ, the number of powerful nodes n_a, and the transmission radius r on the type-II connectivity of a network. All figures are plotted under the condition that powerful nodes are placed in an equi-distant fashion.

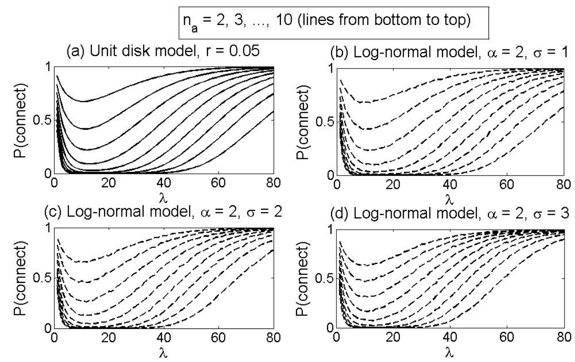

Fig. 4. The type-II connectivity probability. Subgraph (a) is plotted using Eq. (7). Subgraph (b)-(d) are plotted using simulation results obtained from 10000 randomly generated network topologies.

First, Fig. 4(a) shows the probability that the network is connected given different values of λ, n_a, and assuming that $r=0.05$. The number of powerful nodes has been varied from 2 to 10. With $r=0.05$, the network will be fully covered by the powerful nodes for $n_a>10$. It is shown that increase in n_a significantly improves network connectivity. The impact of λ on connectivity is rather interesting. When λ is small, the network connectivity probability drops as λ increases. That is because when the number of ordinary nodes is small, the probability that an ordinary node is connected to a powerful node via a multi-hop connection is small and can be almost neglected. Therefore, one ordinary node has to be close to a powerful node in order to be connected. Thus, when the number of ordinary nodes is small, an increase in the number of ordinary nodes causes a drop in the probability that all ordinary nodes are connected to at least one nearby powerful node. As the number of ordinary nodes further increases, the probability that a ordinary node far away from a powerful node can establish a multi-hop connection to the powerful node increases, which consequently causes an increase in the probability of having a (type-II) connected network.

Similar properties are observed when a log-normal shadowing model is considered. Fig. 4(b)-(d) are plotted using simulation results obtained from 10000 randomly generated network topologies, and following the log-normal model given by $p_{rx} = p_0 - 10\alpha \log_{10} \frac{d}{d_0} + g$ [21] where p_{rx} is the received power at the destination node; p_0 is the power at a reference distance d_0; α is the path loss exponent; g is a Gaussian random variable with zero mean and variance σ^2 reflecting the shadowing effect; d is the Euclidean distance between source node and destination node. Two nodes are directly connected if p_{rx} is greater than or equal to the threshold power p_{th}. In the simulations, we assign $p_{th} = p_0 - 10\alpha \log_{10} \frac{r}{d_0}$ so that the results can be compared with the results under unit disk model. Fig. 4(b)-(d) show that the log-normal shadowing model only changes quantitatively, but not qualitatively the impact of the powerful nodes on the connectivity probability. Further, increase in shadowing variance σ^2 will improve the connectivity probability.

Next, the average hop count between two arbitrary ordinary nodes is plotted in Fig. 5. The analytical formula (Eq. (16)) to

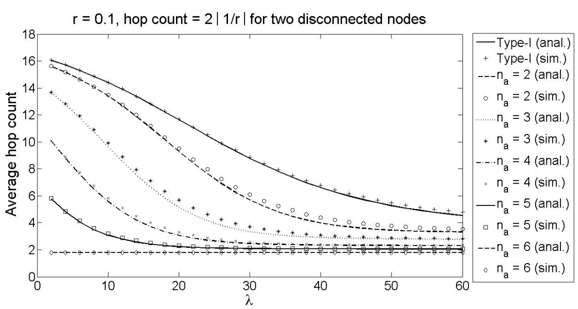

Fig. 5. The average hop count between two arbitrary ordinary nodes. The lines are plotted using Eq. (16), verified by simulation results obtained from 40000 randomly generated network topologies. As the number of instances of random networks used in the simulation is very large, the confidence interval is too small to be distinguishable and hence ignored in this plot. The result for networks without powerful nodes are labeled as "Type-I".

calculate the average hop count between two arbitrary ordinary nodes has been verified by simulations: for each network configuration, 40000 network topologies are generated and the hop count between every pair of ordinary nodes is averaged. The discrepancies between the analytical and simulation results are caused by the dependency problem mentioned in Section V. The result for ad-hoc networks (without any powerful node) is also added to the same plot, and labeled as "Type-I". The figure shows that powerful nodes in a network will greatly improves the average hop count, especially when the ordinary node density is low.

VIII. Conclusions and Future Work

In this paper, we proposed a concept termed *type-II connectivity* to investigate the connectivity problem of infrastructure-based wireless multi-hop networks. Analytical results were obtained for the connectivity probability and the average hop count of 1-D networks with arbitrarily distributed powerful nodes and Poissonly distributed ordinary nodes. First, the result proves that an increase in the number of powerful nodes in a network always has a positive impact on the network connectivity and the average hop count. On the other hand, an increase in the number of ordinary nodes first degrades the network connectivity. Once the number of ordinary nodes increases beyond a certain value, the network connectivity improves as the number of ordinary nodes further increases. Second, the deployment of powerful nodes will greatly reduce the average hop count, which makes the network more scalable. Finally, we proved that equi-distant placement of powerful nodes will maximize the type-II connectivity probability.

Several issues remain for future work. In this paper we obtained the analytical results under the unit disk communication model and the simulation results under the log-normal shadowing model. In future, we may replace these models with a more realistic/generic random connection model. In addition, the impact of node mobility and work/sleep cycle on the type-II connectivity may be studied. The existing work should also help us in considering the type-II connectivity problem of 2-D networks in the future.

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