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Adaptive Output Feedback Control of Nonlinear Systems with Nonlinear Parameterization: A Dwell-time-switching Based Multiple Model Adaptive Control Approach

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Abstract— **A dwell-time-switching based MMAC scheme is proposed for the adaptive output feedback control problem of nonlinear systems with nonlinear parameterization. As in [1], the novel idea of combining the monitoring of the adequacy of candidate models (in terms of their estimation performances) in most MMAC schemes with the monitoring of the performance of the active candidate controller is employed and emphasis has been put on the design of candidate controllers, multiple estimators and monitoring signals so that they possess desirable properties. With the candidate controllers, multiple estimators and monitoring signals being carefully designed, a finite time switching result has been obtained, a characterization on the maximum number of switchings is provided, and sufficient conditions are derived to guarantee closed-loop stability. As an application of the dwell-time-switching based MMAC scheme, a constructive design based on back-stepping is provided for the adaptive output feedback control problem of a special class of nonlinearly parameterized systems, which can satisfy all those sufficient conditions to ensure closed-loop stability.**

I. Introduction

Nonlinear adaptive output feedback control has made a great progress since 1990s after the introduction of the back-stepping design approach [2], [3]. This has been achieved under a common assumption, that is, the unknown parameters enter linearly into nonlinear systems. However, if the unknown parameters enter nonlinearly into nonlinear systems, the related adaptive output feedback control problem proves to be much more difficult to tackle and this can be evidenced by only few results in this area that are available up to now.

One way of dealing with nonlinear parameterization is to use robust control methods [4], [3]. A potential drawback of such methods is that large control effort may be required because a conservative bound for all parameters has to be used.

Another way of dealing with nonlinear parameterization is to try to develop new adaptive control methods. Concave- or convex-parameterization based works can be found in [5] and [6]. Adaptive controllers based on robust control methods were proposed in [4], [7] for output-feedback form nonlinear systems. A new method based on the synthesis of adaptive internal models was proposed in [8], where the unknown parameters were treated as part of the states of the augmented exosystem. In [1], a multiple model adaptive control approach was proposed to deal with nonlinear parameterization, where only state feedback control was considered.

Despite the effort that has been made in dealing with nonlinear parameterization, for general nonlinear systems with nonlinear parameterization, the associated adaptive output feedback control problem remains challenging and largely open. This is one major motivation for the research.

Since we will use an MMAC approach to tackle the challenge caused by nonlinear parameterization, we shall provide a brief review on MMAC. One motivation for using MMAC is that one single model is often not sufficient to approximate a complicated system which has several operating points ([9]). Also, when the system under consideration, whether linear or not, changes suddenly or abruptly due to sensor or actuator faults ([10], [11]), fault accommodation/tolerant control based on multiple possible models seems to be a natural choice. Moreover, MMAC can be applied either to relax the conditions under which linear systems with unknown parameters can be adaptively stabilized ([12], [13]) or to improve control performance [14].

In the MMAC literature, the schemes based on monitoring the adequacy of the candidate models through the design of multiple estimators are perhaps dominant, see for example, [10], [15], [16], [17], [18], [12], [13], [20], [22]). For nonlinear systems, it is possible that two different parameters, say p_1 and p_2, will produce the same output and the corresponding controller for p_1 will destabilize the closed-loop system if the true system parameter is p_2. In such a case, the MMAC schemes based on monitoring the adequacy of the candidate models will run into trouble if the wrong candidate controller corresponding to p_1 is active for all the time. This means that it is not always sufficient to monitor only the adequacy of the candidate models.

In unfalsified control theory, the idea of monitoring the performance of the candidate controller has been proposed and has been researched extensively more recently, see [23], [24], [25], [26] and related references. Besides the possibility that the unfalsified adaptive control algorithms may end up connecting a destabilizing controller in the closed-loop for a long period of time [27], describing how to generate those fictitious reference signals and how to define appropriately those performance indices are also very challenging tasks,

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especially for nonlinear systems.

Aware of the strength and weakness of both ideas, in [1], a novel idea, which combines the monitoring of the adequacy of the candidate models and the monitoring of the performance of the candidate controller, was proposed to design a dwell-time-switching based MMAC scheme for the adaptive control of nonlinear systems with nonlinear parameterization. Compared with the idea of monitoring the adequacy of the candidate models alone, this new idea can be expected to enhance the closed-loop stability through monitoring additional the performance of the active controller. Unlike in unfalsified control algorithms, we monitor only the performance of the active candidate controller rather than the performances of all candidate controllers and thus do not need to generate fictitious reference signals as in e.g. [23], [24], [25], [26].

The purpose of this paper is to extend the results in [1] from state feedback case to output feedback case.

II. Nonlinear system description and problem formulation

In this paper, we are interested in nonlinear systems taking the following form

$$\begin{aligned} \dot{x}(t) &= f(x(t), \theta^*, u(t), d(t)) \\ y(t) &= h(x) \end{aligned} \quad (1)$$

where $x(t)$, $y(t)$ and $u(t)$ are the system state, output and control input, respectively. $\theta^* \in R^q$ is an unknown parameter vector and enters the systems nonlinearly. The vector $d(t)$ consists of disturbances and modeling errors. The function f is continuous with respect to its arguments, and $h(x)$ is continuous with respect to x.

We make the following assumption regarding the unknown parameter vector θ^*.

- A.1 $\theta^* \in \Theta = \cup_{j=1}^{M} S_j$, where M is a known finite positive integer, and $S_j, j = 1, 2, \cdots, M$ are known and bounded sets and $S_j \cap S_l = \phi$ for $j \neq l$ with ϕ being an empty set. Here, we do not know to which set θ^* belongs although $\theta^* \in \Theta$ is known.

Define $B_r = \{x \| \|x\| \leq r\}$ with r a positive constant. In this paper, we are interested in the adaptive output feedback control problem of (1) formulated as below.

Adaptive Output Feedback Stabilization Problem For (1), design an adaptive output feedback controller to ensure that, for any $x_0 \in B_r$, all the closed-loop system signals are bounded, and there exists a positive constant δ such that $x(t)$ enters B_δ asymptotically.

When r is fixed and finite, the above formulated problem is a local stabilization problem. If the problem is required to be solvable for any finite $r > 0$, it becomes a semi-global stabilization problem. Finally, when $r = \infty$, it is a global stabilization problem.

III. Output feedback control using an MMAC approach

In this section, we shall present a general dwell-time-switching based MMAC scheme for the adaptive output feedback control of (1), and establish a general theory for closed-loop stability by providing sufficient conditions.

Since $S_j, j = 1, 2, \cdots, M$ are known and bounded under the assumption A.1, a representative parameter point denoted as θ_j can be chosen for each S_j such that θ_j is known and $\theta_j \in S_j$. For each θ_j, we can have the following candidate model

$$\begin{aligned} \dot{x}(t) &= f(x(t), \theta_j, u(t), d(t)) \\ y(t) &= h(x) \end{aligned} \quad (2)$$

A. Multiple estimators

For (1), based on (2), we shall design M estimators of the following form.

$$\begin{aligned} \dot{\mu}_{\theta_j} &= \Upsilon(\mu_{\theta_j}, \theta_j, y, u) \\ y_{\theta_j} &= \nu(\mu_{\theta_j}, \theta_j, y, u), j = 1, \cdots, M \end{aligned} \quad (3)$$

where Υ and ν are design functions.

Define output estimation error signals as $e_\theta = y_\theta - y, \theta = \theta_1, \cdots, \theta_M$. Then, the multiple estimators of the form (3) are required to possess the following property.

- Assumption A.2 The estimators given by (3) are designed such that $\dot{V}(e_{\theta_j}) \leq -K_{1,e}(e_{\theta_j}) + K_{2,e}(\mu_{\theta_j}, e_{\theta_j}) + \kappa_1 K_{3,e}(\mu_{\theta_j}, e_{\theta_j})$ when both θ_j and θ^* are in the same set, where V and $K_{1,e}$ are known positive definite continuous functions with $V(0) = 0$ and $K_{1,e}(0) = 0$, which satisfy $lim_{\|e_{\theta_j}\| \to \infty} V(e_{\theta_j}) = \infty$ and $lim_{\|e_{\theta_j}\| \to \infty} K_{1,e}(e_{\theta_j}) = \infty$, κ_1 is an unknown positive constant, $K_{2,e}, K_{3,e}$ are known continuous functions and $K_{2,e} + \kappa_1 K_{3,e}$ is bounded above.

B. Candidate output feedback controllers

Based on each candidate model given by (2), a candidate controller will be designed. The designed controller is required to satisfy the following assumption.

- Assumption A.3 For each $j \in \{1, 2, \cdots, M\}$, there exists a dynamic candidate controller defined as

$$\begin{aligned} \dot{\psi}_{\theta_j} &= \Gamma(\psi_{\theta_j}, \theta_j, u_{\theta_j}, y) \\ u_{\theta_j} &= \omega(\psi_{\theta_j}, \theta_j, y) \end{aligned} \quad (4)$$

  such that, for $x(0) \in B_r$, $\psi_{\theta_j}(0)$, the solution of the following equation

$$\begin{aligned} \dot{x} &= f(x, \theta^*, \omega(\psi_{\theta_j}, \theta_j, y), d) \\ \dot{\psi}_{\theta_j} &= \Gamma(\psi_{\theta_j}, \omega(\psi_{\theta_j}, \theta_j, y), y) \end{aligned} \quad (5)$$

  exists and is unique, x and ψ_{θ_j} are bounded and x enters a neighborhood of zero asymptotically if u_{θ_j} and y are bounded.
- Assumption A.4 For each θ_j and the corresponding candidate controller defined by (4), there exists a known positive definite function $V_{\theta_j}(\chi_{\theta_j})$, where $\chi_{\theta_j} = (\psi_{\theta_j}, y)$ is bounded if and only if ψ_{θ_j} is bounded given that y is bounded (see e.g. Section IV) and $V_{\theta_j}(0) = 0$, such that a) u_{θ_j} is bounded if χ_{θ_j} and y are bounded; b) if $\theta^* \in S_j$, along the solution of (5), $\dot{V}_{\theta_j}(\chi_{\theta_j}) \leq -K_{1,\theta_j}(\chi_{\theta_j}) + K_{2,\theta_j}(\psi_{\theta_j}, y) +$

$\kappa_2 K_{3,\theta_j}(\psi_{\theta_j}, y)$, where $V_{\theta_j}, K_{1,\theta_j}$ are known positive definite continuous functions with $V_{\theta_j}(0) = 0$ and $K_{1,\theta_j}(0) = 0$ and satisfy $lim_{\|\chi_{\theta_j}\|\to\infty} V_{\theta_j}(\chi_{\theta_j}) = \infty$ and $lim_{\|\chi_{\theta_j}\|\to\infty} K_{1,\theta_j}(\chi_{\theta_j}) = \infty$, κ_2 is an unknown positive constant, $K_{2,\theta_j}(\psi_{\theta_j}, y), K_{3,\theta_j}(\psi_{\theta_j}, y)$ are known continuous functions and $K_{2,\theta_j}(\psi_{\theta_j}, y) + \kappa_2 K_{3,\theta_j}(\psi_{\theta_j}, y)$ is upper bounded for all ψ_{θ_j}, y and all finite κ_2.

We have the following result.

Lemma 1: For system (1), suppose assumptions A.1, A.3, and A.4 are satisfied. Then, there exists a candidate controller $u_{\theta_{j_0}}$ defined by (4) with $j = j_0$ and $\theta_{j_0}, \theta^* \in S_{j_0}$ such that for any $x_0 \in B_r$, $x(.)$ and $u(.)$ are bounded, and there exists a positive constant δ such that x enters B_δ asymptotically.

C. Monitoring signals

In switching based MMAC, an important task is to design the monitoring signals properly to possess desirable properties. In most switching MMAC schemes, monitoring signals are generated from the output estimation error e_{θ_j} in order to check the adequacy of the candidate models given by (2). In unfalsified control, the monitoring signals are generated to check the performance of the candidate controllers. In [1], a novel idea was proposed to generate monitoring signals to monitor *simultaneously* both the adequacy of the candidate models and the performance of the active candidate controller.

In dwell-time-switching, since any active candidate controller will stay in the control loop for a period of time larger than the dwell-time constant, we must have a countable switching time sequence denoted as $t_k, k = 0, 1, 2, \cdots$. Note that t_0 is always set to zero, which is not really a switching time instant but is introduced for notational convenience. When only a finite number, k_0 say, of switchings occur, we define $t_{k_0+1} = \infty$.

The group of monitoring signals for checking the model adequacy are defined as

$$\begin{aligned} W_\theta(t_k, t) &= \int_{t_k}^{t} e^{-\lambda(t-\tau)}(1 - sgn[V(e_\theta(t_k)) \\ &\quad + \int_{t_k}^{\tau} (-K_{1,e} + K_{2,e} + \hat{\kappa}_1 K_{3,e}) d\varsigma \\ &\quad - V(e_\theta(\tau))]) d\tau, t \geq \tau \geq t_k \\ \dot{\hat{\kappa}}_1 &= \begin{cases} k_{c,1} & t_k \leq t \leq t_k + \tau_D \\ 0 & t_k + \tau_D < t < t_{k+1} \end{cases} \end{aligned} \tag{6}$$

where $\theta = \theta_1, \cdots, \theta_M$, τ_D is the dwell-time constant in the switching control mechanism that will be introduced later, and $k_{c,1}$ is a positive design constant, the "sgn" function is defined as follows: $sgn(x) = 1$ if $x \geq 0$; $sgn(x) = -1$ if $x < 0$. We choose $\hat{\kappa}_1(0) = 0$.

The following two results, which will be used later to prove the finite time switching result, are immediate.

Lemma 2: For system (1), suppose assumptions A.2 is satisfied and the multiple estimators are given by (3). Assume also the switching time sequence is denoted as $t_k, k = 0, 1, 2, \cdots$ and the monitoring signals are defined by (6). Then, $\hat{\kappa}_1(t_k) = k_{c,1}\tau_D k$ for any $t_k < \infty$.

Lemma 3: For system (1), suppose assumptions A.2 is satisfied and the multiple estimators are given by (3). Assume also the switching time sequence is denoted as $t_k, k = 0, 1, 2, \cdots$ and the monitoring signals are defined by (6). If there exists k_0 such that $\hat{\kappa}_1(t_{k_0}) \geq \kappa_1$, then there exists at least one $W-$ signal that is identical to zero for all $t \geq t_{k_0}$.

The group of monitoring signals for checking performance of the candidate controllers are defined as

$$\begin{aligned} \bar{W}_{\theta_j}(t_k, t) &= \int_{t_k}^{t} e^{-\lambda(t-\tau)}(1 - sgn[V_{\theta_j}(t_k) \\ &\quad + \int_{t_k}^{\tau} (-K_{1,\theta_j} + K_{2,\theta_j} + \hat{\kappa}_2 K_{3,\theta_j}) d\varsigma \\ &\quad - V_{\theta_j}(\tau)]) d\tau, t \geq \tau \geq t_k \\ \dot{\hat{\kappa}}_2 &= \begin{cases} k_{c,2} & t_k \leq t \leq t_k + \tau_D \\ 0 & t_k + \tau_D < t < t_{k+1} \end{cases} \end{aligned} \tag{7}$$

where $j = 1, \cdots, M$ and $k_{c,2}$ is a positive design constant, and we also choose $\hat{\kappa}_2(0) = 0$.

Remark 1: The results in Lemma 2 and Lemma 3 also hold for monitoring signals defined by (7).

D. Output feedback switching control scheme

Through (6) and (7), we have defined $2M$ monitoring signals, that is, $W_\theta, \theta = \theta_1, \cdots, \theta_M$ and $\bar{W}_\theta, \theta = \theta_1, \cdots, \theta_M$, which will be used in our switching control mechanism. The online monitoring of these signals via a well defined switching logic will generate a piecewise constant switching signal denoted as $\sigma(t) \in \{1, 2, \cdots, M\}$, which is used to determine a controller amongst M candidate controllers ($u_\theta = \omega(\psi_\theta, \theta, y), \theta = \theta_1, \cdots, \theta_M$) to be switched on. We let $u = u_{\theta_\sigma} = \omega(\psi_{\theta_\sigma}, \theta_\sigma, y)$. For example, if $\sigma(t) = j$, then $u = u_{\theta_j} = \omega(\psi_{\theta_j}, \theta_j, y)$ is switched on.

In the following, let $t_k = t_0$ for any $k < 0$.

The specific switching control mechanism we use in this paper is stated in the following algorithm.

Output feedback switching control mechanism (OFSCM):

- Step 1. Choose a dwell-time constant τ_D.
- Step 2. Let $t_0 = 0$ and $k = 0$, and pick a candidate controller in the candidate controller family.
- Step 3. For $t_k \leq t < t_k + \tau_D$, let $\sigma(t) = \sigma(t_k)$ and $u = u_{\theta_{\sigma(t)}}$.
- Step 4. For $t \geq t_k + \tau_D$, monitor all $W-$ signals and only one $\bar{W}-$ signal, viz, $\bar{W}_{\theta_{\sigma(t_k)}}(t_k, t)$, and let $S_t = \{i | W_{\theta_i}(t_k, t) = 0\}$.

  If S_t is empty, and
  - if $\bar{W}_{\theta_{\sigma(t_k)}}(t_k, t) = 0$, no new controller is switched on and let $\sigma(t) = \sigma(t_k)$ and $u = u_{\theta_{\sigma(t)}}$.
  - if $\bar{W}_{\theta_{\sigma(t_k)}}(t_k, t) \neq 0$, then increment k by 1, let $t_k = t$, let $\sigma(t_k) = \sigma(t_{k-1}) - 1$ when $\sigma(t_{k-1}) > 1$ and $\sigma(t_k) = \sigma(t_{k-1}) + 1$ when $\sigma(t_{k-1}) = 1$, and let a new controller $u = u_{\theta_{\sigma(t_k)}}$ be switched on at t. Go back to Step 3.

  If S_t is not empty, and

– if $\sigma(t_k) \in S_t$ and $\bar{W}_{\theta_{\sigma(t_k)}}(t_k, t) = 0$, no new controller is switched on and let $\sigma(t) = \sigma(t_k)$ and $u = u_{\theta_{\sigma(t)}}$.
– if $\sigma(t_k)$ does not belong to S_t or if $\sigma(t_k) \in S_t$ but $\bar{W}_{\theta_{\sigma(t_k)}}(t_k, t) \neq 0$, then increment k by 1, let $t_k = t$ and define $S_{t,M} = \{\sigma(t_{k-M+1}), \cdots, \sigma(t_{k-1})\}$. If $S_t \setminus S_{t,M}$ is not empty, pick $\sigma(t)$ as any element in $S_t \setminus S_{t,M}$. If $S_t \setminus S_{t,M}$ is empty, pick $\sigma(t)$ as any element in $\{1, \cdots, M\} \setminus S_{t,M}$. Now, let a new controller $u = u_{\theta_{\sigma(t_k)}}$ be switched on at t. Go back to Step 3.

E. Analysis of the switching property of the OFSCM

Regarding the number of switchings, we have the following result.

Lemma 4: For system (1), suppose the multiple output estimators are given by (3) and the monitoring signals are defined through (6) to (7). If **OFSCM** is employed under the assumptions $A1 - A4$, then switching stops in finite time and the maximum number of switchings is strictly less than

$$max\{\frac{\kappa_1}{k_{c,1}\tau_D}, \frac{\kappa_2}{k_{c,2}\tau_D}\} + M + 1.$$

Proof: Omitted due to lack of space.¶

F. Stability analysis of output feedback controller design using OFSCM

In this subsection, we shall analyze the stability of the closed-loop system resulting from the output feedback controller designed using **OFSCM**. We need the following assumption.

- Assumption A.5 For any time instant t_a, and for any u_{θ_j} given by the feedback law $\omega(\psi_{\theta_j}, \theta_j, y)$ defined in assumption A.3, the solution of the following equation

$$\begin{aligned} \dot{x} &= f(x, \theta^*, \omega(\psi_{\theta_j}, \theta_j, y), d) \\ \dot{\psi}_{\theta_j} &= \Gamma(\psi_{\theta_j}, \omega(\psi_{\theta_j}, \theta_j, y), y) \end{aligned} \quad (8)$$

will be bounded on $[t_a, t_a + \tau_D]$, that is, will not escape in a time period of τ_D if $x(t_a)$, $\psi(t_a)$ and $u_{\theta_j}(t_a)$ are finite.

Theorem 1: Suppose that the proposed **OFSCM** is applied to system (1) and assumptions $A1 - A5$ hold. Then, for any $x_0 \in B_r$ and for any χ_0, all the closed-loop signals are bounded and there exists a constant $\delta > 0$ such that x enters B_δ asymptotically.

Proof: According to Lemma 4, switching will stop in finite time and thus κ_1, κ_2 are bounded. Let the switching time instants be $t_k, 1 \leq k \leq k_0$, with k_0 be the number of switchings.

According to **OFSCM**, for any $1 \leq k \leq k_0$, we must have either $t_k - t_{k-1} = \tau_D$ or $t_k - t_{k-1} > \tau_D$.

If $t_k - t_{k-1} = \tau_D$, then χ_{θ_j}, y, and $u_{\sigma(t_{k-1})}$ are bounded on $[t_{k-1}, t_k]$ if $\chi(t_{k-1})$, $y(t_{k-1})$, and $u_{\sigma(t_{k-1})}(t_{k-1})$ are finite according to assumption A5.

If $t_k - t_{k-1} > \tau_D$, according to **OFSCM**, we must have $W_{\sigma(t_{k-1})}(t_{k-1}, t) \equiv 0$ and $\bar{W}_{\theta_{\sigma(t_{k-1})}}(t_{k-1}, t) \equiv 0$ for all $t_{k-1} \leq t < t_k$. It follows from the design of the monitoring signals that $\dot{V}_{\theta_{\sigma(t_{k-1})}} \leq -K_{1,\theta_{\sigma(t_{k-1})}} + K_{2,\theta_{\sigma(t_{k-1})}} + \kappa_2 K_{3,\theta_{\sigma(t_{k-1})}}$ for all $t_{k-1} \leq t < t_k$. This inequality guarantees that $\chi_{\theta_{\sigma(t_{k-1})}}, y$ are bounded for all $t_{k-1} \leq t < t_k$. This together Assumption A.4 a) implies that $u_{\theta_{\sigma(t_{k-1})}}$ is also bounded for all $t_{k-1} \leq t < t_k$. Now, because of Assumption A.3, x is bounded on $[t_{k-1}, t_k)$.

Because of continuity, we have proved that all closed-loop signals are bounded on $[0, t_{k_0}]$. Now, we consider, $t \in [t_{k_0}, \infty)$. Since there is no switching, we have $\dot{V}_{\theta_{\sigma(t_{k_0})}} \leq -K_{1,\theta_{\sigma(t_{k_0})}} + K_{2,\theta_{\sigma(t_{k_0})}} + \kappa_2 K_{3,\theta_{\sigma(t_{k_0})}}$ for all $T \geq t_{k_0}$. By definitions of these $K-$ functions, we know that $\chi_{\theta_{\sigma(t_{k_0})}}(t), y(t)$ are bounded on $[t_{k_0}, \infty)$. Using Assumption A.4 a) again, we conclude $u_{\theta_{\sigma(t_{k_0})}}(t)$ is bounded on $[t_{k_0}, \infty)$ too. Since $\chi_{\theta_{\sigma(t_{k_0})}}(t)$, $u_{\theta_{\sigma(t_{k_0})}}(t)$, and $y(t)$ are bounded, it follows from Assumption A.3 that the conclusions of the theorem are true. ¶

IV. CONSTRUCTIVE DESIGN OF OFSCM FOR A SPECIAL CLASS OF NONLINEAR SYSTEMS

In this section, we are going to demonstrate how to apply the general **OFSCM** to a special class of nonlinear systems.

A. A special class of nonlinear systems

$$\begin{aligned} \dot{x}_i &= x_{i+1} \\ &+ f_i(y, \theta^*) + g_i(x, d), 1 \leq i \leq n-1 \\ \dot{x}_n &= f_n(y, \theta^*) + g_n(x, d) + u \\ y &= x_1 \end{aligned} \quad (9)$$

where $x = (x_1 \ \cdots \ x_n)^T$, y, and u are the state, the output, and the input respectively. $f_i(y, \theta^*), i = 1, \cdots, n$ are known smooth functions of y and θ^*, and $g_i(x, d), i = 1, \cdots, n$ are unknown smooth functions with d representing bounded disturbances and/or modeling errors. $\theta^* \in R^q$ is an unknown parameter vector and enters the systems nonlinearly, which belongs to a known compact set Θ. Similar type of systems have been a subject of study in [4], [2], [3], [7].

To make the output feedback controller design using **OFSCM** tractable, we make the following assumptions.

- Assumption B.1 There exists a known nonnegative function $\rho(y, \theta)$ which is smooth with respect to its arguments and a continuous function $\gamma(.)$ which is strictly increasing, positive and satisfies $\gamma(0) = 0$ and $lim_{\eta \to \infty} \gamma(\eta) = \infty$ such that , for any $\theta \in \Theta$ and $\bar{\theta} \in \Theta$, $f(0, \theta) = 0$ and $\|f(y, \theta) - f(y, \bar{\theta})\| \leq \rho(y, \theta)\gamma(\|\theta - \bar{\theta}\|)$.
- Assumption B.2 For $g(x, d) = (g_1(x, d) \ \cdots \ g_n(x, d))^T$, there exist a unknown positive constant k_g and a known nonnegative function $\varrho(y)$ smooth with respect to its argument such that $\|g(x, d)\| \leq k_g \varrho(y)$.

Because of the properties of the function $\gamma(r)$, for any given $\epsilon > 0$, Θ can be partitioned into M subsets $S_j, j = 1, \cdots, M$ such that Assumption A.1 is satisfied and $\gamma(\|\theta - \bar{\theta}\|) \leq \epsilon$ for any $\theta, \bar{\theta} \in S_j$

B. Multiple estimator design

We need to design the following observers.

$$\begin{aligned}\dot{\xi}_\theta &= A_0\xi_\theta + L_0 y \\ &\quad + f(y,\theta) + Bu, \theta = \theta_1, \cdots, \theta_M \qquad (10)\end{aligned}$$

where $\xi_\theta = (\xi_{\theta,1} \cdots \xi_{\theta,n})^T$, and A_0, L_0 and B are defined as

$$A_0 = \begin{pmatrix} -l_{0,1} & 1 & 0 & \cdots & 0 \\ -l_{0,2} & 0 & 1 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ -l_{0,n-1} & 0 & \ddots & \ddots & 1 \\ -l_{0,n} & 0 & 0 & \cdots & 0 \end{pmatrix}, \qquad (11)$$

and

$$L_0 = \begin{pmatrix} l_{0,1} \\ l_{0,2} \\ \vdots \\ l_{0,n} \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}, \qquad (12)$$

where L_0 can be chosen freely and is chosen such that A_0 is Hurwitz.

Define $\varepsilon_\theta = \xi_\theta - x$ and compute two known positive constants K_o and λ_0 such that

$$\|e^{A_0(t-\tau)}\| \le K_0 e^{-\frac{\lambda_0}{2}(t-\tau)}. \qquad (13)$$

For the sake of simplicity, we introduce following signals:

$$\begin{aligned}\dot{m}_{\theta,1} &= -\frac{\lambda_0}{2}m_{\theta,1} + \rho(y,\theta), m_{\theta,1}(0) = 0, \\ \dot{m}_2 &= -\frac{\lambda_0}{2}m_2 + \varrho(y), m_2(0) = 0. \qquad (14)\end{aligned}$$

The multiple estimators take the following form

$$\begin{aligned}\dot{y}_\theta &= -(\frac{\lambda_1}{2}+1)(y_\theta - y) + \xi_{\theta,2} + f_1(y,\theta) \\ &- \lambda_2(y_\theta - y)(\rho^2(y,\theta) + \varrho^2(y)) \\ &- \lambda_2(y_\theta - y)(K_0^2 m_{\theta,1}^2 + K_0^2 m_2^2) \qquad (15)\end{aligned}$$

where $\theta = \theta_1, \cdots, \theta_M$, $y_\theta(0) = y(0)$, $\lambda_i, i = 1,2$ are positive design constants.

Let $e_\theta = y_\theta - y$, and we have the following result.

Lemma 5: For system (9), suppose that assumptions B1 and B2 are satisfied. Then the multiple estimators defined by (15) can ensure that assumption A.2 is satisfied.

C. Candidate controller design

The candidate controllers can be designed using back-stepping design based on (**??**) and (10). Since the back-stepping approach is now quite standard [2], we present the designed candidate controllers directly below without providing the detailed back-stepping procedure.

$$\begin{aligned}u_\theta &= -c_n z_{\theta,n} - z_{\theta,n-1} + l_{0,n}\xi_{\theta,1} - l_{0,n}y - f_n(y,\theta) \\ &+ \sum_{j=1}^{n-1}\frac{\partial\alpha_{\theta,n-1}}{\partial\xi_{\theta,j}}(-l_{0,j}\xi_{\theta,1} + \xi_{\theta,j+1} + l_{0,j}y + f_j(y,\theta)) \\ &+ \frac{\partial\alpha_{\theta,n-1}}{\partial m_{\theta,1}}(-\frac{\lambda_0}{2}m_{\theta,1} + \rho(y,\theta)) \\ &+ \frac{\partial\alpha_{\theta,n-1}}{\partial m_2}(-\frac{\lambda_0}{2}m_2 + \varrho(y)) \\ &+ \frac{\partial\alpha_{\theta,n-1}}{\partial y}(\xi_{\theta,2} + f_1(y,\theta)) \\ &- l_n(\frac{\partial\alpha_{\theta,n-1}}{\partial y})^2[(\rho(y,\theta)\epsilon)^2 + \varrho^2(y)]z_{\theta,n} \\ &- k_n(\frac{\partial\alpha_{\theta,n-1}}{\partial y})^2(K_0\epsilon m_{\theta,1})^2 z_{\theta,n} \\ &- k_n(\frac{\partial\alpha_{\theta,n-1}}{\partial y})^2(K_0 m_2)^2 z_{\theta,n} - (\frac{\partial\alpha_{\theta,n-1}}{\partial y})^2 z_{\theta,n} \quad (16)\end{aligned}$$

where the $\xi-$ and the $m-$ signals are gvien by (10) and (14), the $z-$ signals are defined as $z_1 = z_{\theta,1} = y$, $z_{\theta,2} = \xi_{\theta,2} - \alpha_{\theta,1}(y, m_{\theta,1}, m_2)$, and $z_{\theta,i} = \xi_{\theta,i} - \alpha_{\theta,i-1}(\xi_{\theta,1}, \cdots, \xi_{\theta,i-1}, y, m_{\theta,1}, m_2), 3 \le i \le n$, and the $\alpha-$ signals are defined as follows

$$\begin{aligned}\alpha_{\theta,1} &= -(c_1+1)z_1 - f_1(y,\theta) \\ &- l_1[(\rho(y,\theta)\epsilon)^2 + \varrho^2(y)]z_1 \\ &- k_1(K_0\epsilon m_{\theta,1})^2 z_1 - k_1(K_0 m_2)^2 z_1, \\ \alpha_{\theta,2} &= -c_2 z_{\theta,2} - z_1 \\ &+ l_{0,2}\xi_{\theta,1} - l_{0,2}y - f_2(y,\theta) \\ &+ \frac{\partial\alpha_{\theta,1}}{\partial y}(\xi_{\theta,2} + f_1(y,\theta)) \\ &+ \frac{\partial\alpha_{\theta,1}}{\partial m_{\theta,1}}(-\frac{\lambda_0}{2}m_{\theta,1} + \rho(y,\theta)) \\ &+ \frac{\partial\alpha_{\theta,1}}{\partial m_2}(-\frac{\lambda_0}{2}m_2 + \varrho(y)) \\ &- l_2(\frac{\partial\alpha_{\theta,1}}{\partial y})^2[(\rho(y,\theta)\epsilon)^2 + \varrho^2(y)]z_{\theta,2} \\ &- k_2(\frac{\partial\alpha_{\theta,1}}{\partial y})^2(K_0\epsilon m_{\theta,1})^2 z_{\theta,2} \\ &- k_2(\frac{\partial\alpha_{\theta,1}}{\partial y})^2(K_0 m_2)^2 z_{\theta,2} - (\frac{\partial\alpha_{\theta,1}}{\partial y})^2 z_{\theta,2} \\ \alpha_{\theta,i} &= -c_i z_{\theta,i} - z_{\theta,i-1} \\ &+ l_{0,i}\xi_{\theta,1} - l_{0,i}y - f_i(y,\theta) \\ &+ \sum_{j=1}^{i-1}\frac{\partial\alpha_{\theta,i-1}}{\partial\xi_{\theta,j}}(-l_{0,j}\xi_{\theta,1} + \xi_{\theta,j+1} + l_{0,j}y + f_j) \\ &+ \frac{\partial\alpha_{\theta,i-1}}{\partial m_{\theta,1}}(-\frac{\lambda_0}{2}m_{\theta,1} + \rho(y,\theta)) \\ &+ \frac{\partial\alpha_{\theta,i-1}}{\partial m_2}(-\frac{\lambda_0}{2}m_2 + \varrho(y)) \\ &+ \frac{\partial\alpha_{\theta,i-1}}{\partial y}(\xi_{\theta,2} + f_1(y,\theta)) \\ &- l_i(\frac{\partial\alpha_{\theta,i-1}}{\partial y})^2[(\rho(y,\theta)\epsilon)^2 + \varrho^2(y)]z_{\theta,i}\end{aligned}$$

$$
\begin{aligned}
&- \quad k_i(\frac{\partial \alpha_{\theta,i-1}}{\partial y})^2 (K_0 \epsilon m_{\theta,1})^2 z_{\theta,i} \\
&- \quad k_i(\frac{\partial \alpha_{\theta,i-1}}{\partial y})^2 (K_0 m_2)^2 z_{\theta,i} \\
&- \quad (\frac{\partial \alpha_{\theta,i-1}}{\partial y})^2 z_{\theta,i}, i = 3, \cdots, n-1 \qquad (17)
\end{aligned}
$$

and $c_i, k_i, l_i, i = 1, \cdots, n$ are positive design constants.

Denote $\chi_\theta = (z_{\theta,1} \ \cdots \ z_{\theta,n})^T$ and choose $V_\theta(\chi_\theta) = \frac{1}{2}\sum_{i=1}^{n} z_{\theta,i}^2$. Then, we have the following result.

Lemma 6: For system (9), suppose that assumptions B1 and B2 are satisfied. Then the M candidate controllers of the form (16) and (17) can ensure that Assumptions $A3 - A5$ are satisfied.

Proof: Omitted due to lack of space.¶

D. Stability analysis

Because of Lemma 5 and Lemma 6 and Theorem 1, we have the following global stability result.

Corollary 1: For system (9), suppose that Assumptions $B.1$ and $B.2$ hold. Given multiple estimators (15), candidate controllers (16) and (17) with $\theta = \theta_1, \cdots, \theta_M$, and the monitoring signals designed as (6) and (7), if the control law generated by **OFSCM** is applied to system (9), then, for any x_0 and $\xi_\theta(0), \theta = \theta_1, \cdots, \theta_M$, all the closed-loop signals are bounded and x enters asymptotically to a small neighborhood of the origin.

Proof: Omitted due to lack of space.¶

V. Conclusions

In this paper, a dwell-time-switching based MMAC scheme called **OFSCM** was proposed for the adaptive output feedback control problem of a class of nonlinear systems with nonlinear parameterization. The major contribution is that the novel idea of monitoring simultaneously the adequacy of candidate models and the performance of the active candidate controller proposed in [1] has been extended from state feedback control to output feedback control.

Our results do not address yet the question of where to select the nominal parameters θ_j, or how many there should be. It is also interesting to study how to extend the obtained results to systems with unknown parameters that are time-varying and enter the system nonlinearly.

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