2010 8th IEEE International Conference on
Control and Automation
Xiamen, China, June 9-11, 2010

Geometric Dilution of Localization and Bias-Correction Methods

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***Abstract*— A particular geometric problem–the collinearity problem–which may prevent effective use of localization algorithms is described in detail in this paper. Further analysis illustrates the methods for improving the estimate for localization algorithms also can be affected by the collinearity problem. In this paper, we propose a novel approach to deal with the collinearity problem for a localization improvement method–the bias-correction method [1, 2, 3]. Compare to earlier work such as [4], the main feature of the proposed approach is that it takes the level of the measurement noise into consideration as a variable. Monte Carlo simulation results demonstrate the performance of the proposed method. Further simulation illustrates the influence of two factors on the effect of the bias-correct method: the distance between sensors and the level of noise. Though it mainly aims to the bias-correction method, the proposed approach is also valid for localization algorithms because of the consistent performance of localization algorithms and the bias-correction method.**

I. INTRODUCTION

The localization problem has been widely investigated in various areas for a long time and many localization algorithms have been proposed [5-8]. Further a lot of techniques also have been presented to enhance the estimate accuracy of existed localization algorithms such as bias-correction methods [1, 2, 3, 9]. However in much literature on both localization algorithms and techniques to improve the localization estimation accuracy, it is a common assertion that when a particular geometric problem happens not only localization algorithms but also improvement techniques will become less effective or cannot work completely. This is the so called collinearity problem which will be described in detail in Section III. In the sequel, we will first briefly review the literature dealing with the collinearity problem in localization algorithms, categorized by distance-based and bearing-only localization approaches.

In distance-based localization algorithms, the study of the collinearity problem mainly focuses on the sensors collinearity which means all sensors are exactly or approximately on a straight line. C. G. Chang et al. [10] addressed the collinearity problem in distance-based localization of sensor networks. To overcome the collinearity case, they presented MAP (Maximum A Posterior) and MMSE (Minimum Mean Square Error) methods to estimate a target location. C. Poggi et al. [11] also addressed the collinearity problem. They introduced a new parameter called normalized collinearity into the localization estimation mechanism in order to flag the collinearity situation. In bearing-only localization algorithms, the collinearity problem concerns the collinearity between all sensors and a target. In other words, when a target and all sensors are exactly or approximately on a straight line localization algorithms will fail which means a target position estimate cannot be obtained. In [4], Fidan et al. analyzed the collinearity problem in DF (direction-finding) based localization algorithms. A concept of "problematic area" is used in the paper. If a target is in the problematic area, the collinearity problem is likely to be encountered when noise exists in the measurements. At that time it is very possible localization algorithms cannot locate a target. Three different methods are proposed to calculate a problematic area. However the level of noise in the measurements which is a very important factor influencing the estimated target position is not considered.

Although the study on the collinearity problem in localization algorithms has begun, few works concern the collinearity problem in localization improvement techniques. In this paper, we focus on when will the collinearity problem affect the bias-correction method proposed in [1, 2, 3]. While the bias-correction method itself works in both distance-based and bearing-only localization algorithms, here bearing-only algorithms are selected as example. To begin with, let us define the concept of "effectiveness area": given a set of sensors, if a target falls in the effectiveness area, then collinearity problem will not present and therefore the bias-correction method can indeed correct the bias. A novel approach is proposed in this paper to calculate such an effectiveness area for the bias-correction method analytically. In order to calculate a boundary of the effectiveness area, the determinant of the MSE (mean square error) matrix is selected as a criterion to be compared with a predetermined threshold. An important and distinct feature of the proposed approach is that, compare to earlier work such as [4], the level of the measurement noise is incorporated into our formula. This allows us to obtain Monte Carlo simulation results to study the relationship among the sensor geometry, the measurement noise level and the size of the effectiveness area. In addition, the proposed approach is also valid for localization algorithms because earlier simulation results illustrated that the area in which the bias-correction method can correct the bias and the area in which localization algorithms can be applied is consistent [1].

The rest of this paper is organized as follows. In Section

This work is supported by National ICT Ltd. Australia-NICTA and the Australian Research Council under DP-0877562. C. Yu is the recipient of an ARC Australian Postdoctoral Fellowship.

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II a review of localization and the bias correction will be presented. The collinearity problem will be described in detail in Section III. A novel approach to calculate an effectiveness area for the bias-correction method will also be proposed in Section III. In Section IV, Monte Carlo simulation results will be provided. Section V summarizes the paper.

II. Review of Localization and Bias-Correction

In this section we will review the bearing localization problem and the bias-correction method proposed in [1, 2, 3] (the correction method is not specifically analyzed based on bearing measurements). All the analysis is restricted to two dimensional space. The results however will be similar in three dimensional space [3].

A. Review of Localization

In two-dimensional space, the localization problem can be addressed as follows. Suppose there is an emitter or a target whose coordinate is (x, y) and a set of bearing measurements $(\theta_1, \theta_2, ..., \theta_i)$ $(i = 1, 2, ..., N$, where N denotes the number of sensors) can be obtained. In the noiseless case we have

$$\theta_i = f_i(x, y) \tag{1}$$

where f_i denotes the mapping from the target position to measurements for sensor i. Generally, the function f_i can be obtained analytically.

However, in practice the noise is inevitable in the measurements which means the true value of θ_i cannot be obtained. In fact the measurement has the following form:

$$\tilde{\theta}_i = \quad f_i(x, y) + \delta\theta_i \tag{2}$$

where $\tilde{\theta}_i$ denotes the noisy measurement and $\delta\theta_i$ is the noise in measurements which is usually assumed to be an i.i.d Gaussian random variable with zero mean and known variance σ_i^2.

When the number of sensors is greater than or equal to three, equation (2) will become overdetermined. Except for the noiseless situation, there will generally be no solution to the equation. There are many methods proposed, such as maximum likelihood, least squares, etc [12, 13], to solve the no-solution problem. The main idea of these approaches is to convert the localization problem to an optimization problem. Here we take least squares for example. In least squares the optimization problem is formulated as follows.

$$(x, y) = \arg\min_{x,y} \sum_{i=1}^{N} \delta\theta_i^2 \tag{3}$$

where

$$\delta\theta_i = \tilde{\theta}_i - f_i(x, y) \tag{4}$$

If the noises have different variances, the inverse of the variances will be introduced as weights into the equation (3). By solving the equation (3) and (4) we can finally obtain the estimated position; of course this may not be a straightforward task.

B. Review of Bias-Correction

The main factor which causes the estimation bias in a localization problem is that the function mapping from measurements to target position estimates is nonlinear, and at the same time, the measurements are contaminated by noise [1]. The bias-correction method proposed in [1, 2, 3] aims to deal with the nonlinear problem by using Taylor series and Jacobian matrices.

Consider a scenario with two sensors and one target first. As described above, we have:

$$\tilde{\theta}_i = f_i(x, y) + \delta\theta_i \qquad i = 1, 2 \tag{5}$$

where $\tilde{\theta}_i, f_i$ and $\delta\theta_i$ have the same definitions as above.

Assume the inverse function of $\mathbf{f} = [f_1, f_2]^T$ is $\mathbf{g} = [g_1, g_2]^T$ which is in fact a localization mapping delivering a position estimate from the measurements. Here we have:

$$\begin{aligned} \tilde{x} &= x + \delta x \\ &= g_1(\tilde{\theta}_1, \tilde{\theta}_2) \end{aligned} \tag{6}$$

$$\begin{aligned} \tilde{y} &= y + \delta y \\ &= g_2(\tilde{\theta}_1, \tilde{\theta}_2) \end{aligned} \tag{7}$$

Now we can expand g_1 and g_2 about the point (θ_1, θ_2) by Taylor series. Suppose the Taylor series is truncated to second order. Here we take g_1 as example.

$$\begin{aligned} x + \delta x &= g_1(\tilde{\theta}_1, \tilde{\theta}_2) \\ &= g_1(\theta_1 + \delta\theta_1, \theta_2 + \delta\theta_2) \\ &= g_1(\theta_1, \theta_2) + (\tfrac{\partial g_1}{\partial \theta_1}\delta\theta_1 + \tfrac{\partial g_1}{\partial \theta_2}\delta\theta_2) \\ &\quad + \tfrac{1}{2!}(\delta\theta_1^2 \tfrac{\partial^2 g_1}{\partial \theta_1^2} + \delta\theta_1\delta\theta_2 \tfrac{\partial^2 g_1}{\partial \theta_1 \partial \theta_2} + \delta\theta_2^2 \tfrac{\partial^2 g_1}{\partial \theta_2^2}) \end{aligned} \tag{8}$$

By taking the expected value of the equation (8) we can obtain an approximation for the expected value of δx as follows in terms of derivatives of g_1 and the measurement noise variances.

$$E(\delta x) = \frac{1}{2!}(\sigma_{\theta_1}^2 \frac{\partial^2 g_1}{\partial \theta_1^2} + \sigma_{\theta_2}^2 \frac{\partial^2 g_1}{\partial \theta_2^2}) \tag{9}$$

$E(\delta y)$ can be obtained in the same way. Here $E(\delta x)$ and $E(\delta y)$ are considered as the bias.

For two sensors in a 2D situation, to compute the derivatives of the localization mappings (g_1 and g_2) analytically is not very difficult. However, when one considers for example a three dimensional problem involving more sensors, the calculation of the localization mappings and their derivatives would be much harder. In fact, in almost all cases, it is much easier to obtain the derivatives of forward mappings (f_1 and f_2). Therefore we consider how to use f_1, f_2 and their derivatives to calculate the derivatives of g_1 and g_2 so we can compute the bias in an easy way. To achieve this, we introduce the Jacobian matrix. The following equation is derived from one property of the Jacobian matrix:

$$\begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\partial g_1}{\partial \theta_1} & \frac{\partial g_1}{\partial \theta_2} \\ \frac{\partial g_2}{\partial \theta_1} & \frac{\partial g_2}{\partial \theta_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{10}$$

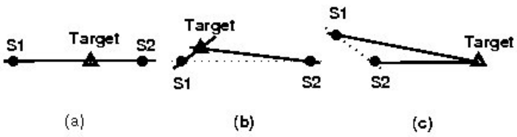

Fig. 1: (a) Exact Collinearity (b) Near Collinearity (c) Near Collinearity

By solving the equation (10) $\frac{\partial g_i}{\partial \theta_j}$ ($i = 1, 2$ and $j = 1, 2$) can be expressed by formulas including only the derivatives of f_1 and f_2. Here we use $g^i_{\theta_j}$ to denote the expressions of $\frac{\partial g_i}{\partial \theta_j}$ as functions of x and y. We take $\frac{\partial g_1}{\partial \theta_1}$ for instance. Now we have the following equation.

$$\frac{\partial g_1}{\partial \theta_1} = g^1_{\theta_1} \tag{11}$$

Differentiating the above equation in respect to x, y respectively we can obtain an equation set as follows.

$$\begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_2}{\partial x} \\ \frac{\partial f_1}{\partial y} & \frac{\partial f_2}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\partial^2 g_1}{\partial \theta_1^2} \\ \frac{\partial^2 g_1}{\partial \theta_1 \partial \theta_2} \end{bmatrix} = \begin{bmatrix} \frac{\partial g^1_{\theta_1}}{\partial x} \\ \frac{\partial g^1_{\theta_1}}{\partial y} \end{bmatrix} \tag{12}$$

By solving the above equation set, $\frac{\partial^2 g_1}{\partial \theta_1^2}$ can be expressed only by the derivatives of f_1 and f_2. The formulas for $\frac{\partial g_2}{\partial \theta_2}$ can be obtained in the same way. Substituting the formulas into $E(\delta x)$ and $E(\delta y)$ we can finally obtain the easily-calculated equations.

When the number of sensors is more than two, an analytical expression for the bias can be obtained in the same way. More details are presented in [1, 2, 3].

III. Collinearity Problem in Localization

In this section a detailed description of the collinearity problem in bearing-only localization will be presented in subsection A. In subsection B we will propose a novel method to calculate an effectiveness area for the bias-correction method. Unless flagged, we shall limit our analysis to two-dimensional space with two sensors and one target.

A. Problem Statement

The bias-correction method will become less effective or noneffective when the collinearity problem arises. To grasp the idea, consider first the notion of exact collinearity which means all the sensors and a target are exactly one a straight line. Fig. 1. (a) depicts the exact collinearity situation with two sensors and one target. In bearing-only localization algorithms, an angle measurement from each sensor ($S1$ and $S2$) to the target can be obtained. In other words, for each sensor we have a ray. The intersection of the two rays is the position of the target. However, when the two sensors and the target are exactly collinear, the two rays coincide with each other and no well-defined single point of intersection exists. In this case the bias-correction method will obviously be noneffective because no unique position of the target can be obtained.

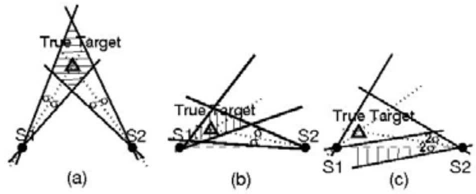

Fig. 2: (a) Noisy Model (b) With the Low Level of Noise (c) With the High Level of Noise

The exact collinearity is not generic. However, in the presence of noise, we have to worry about near collinearity problem. Generally there are two types of geometries of the sensors and a target may cause the nearly collinear problem: Fig. 1. (b) illustrates the situation where a target is very close to the line joining two sensors; Fig. 1. (c) depicts the other situation where a target is in the far field. Here we focus on Fig. 1 (b) to give a detailed description. In practice, the measurement noise is inevitable. Because of the noise the obtained angle measurements from $S1$ and $S2$ to the target have a small excursion from the true value related to the probability distribution of the measurement noise. Assume the noise in the measurements is i.i.d. Gaussian with zero mean and variance σ^2. The bearing localization model can be described as shown in Fig. 2 (a). In this noisy situation, the variation of the small excursion can be considered as [-σ, σ]. Therefore the single ray is replaced by a 2σ-cone (the dashed line denotes the noiseless bearing measurement) in which the noisy measurements can be obtained. Further, the single intersection point of two rays in noiseless case is replaced by an intersection area (the dashed area in the figure) of two 2σ-cones from $S1$ and $S2$. The intersection area indicates a high-probability region in which the target estimate (only the triangle denotes the true position of the target) would be [4].

Now the situation shown in Fig. 1. (b) can be redrawn as depicted in Fig. 2. (b) and Fig. 2. (c) with the different level of noise in the measurements. First in Fig. 2. (b) the standard deviation of the noise in measurements is σ. We can see that though the target is very close to the line joining the two sensors the nearly collinearity problem will not happen. Because there will always be an intersection of the two measurements from $S1$ and $S2$. Therefore the bias-correction method can correct the bias in the situation shown in Fig. 2. (b). However, once the measurement noise level increases the situation will change completely. In Fig. 2. (c) the standard deviation of the measurement noise increases to 2σ. An ill-conditioned case will happen if the noisy measurement ray from $S2$ is in the dashed region. At that time, there will be no intersection point existing which means no estimated position of the target can be obtained. The bias-correction method can be considered as a device for improving the estimate for localization algorithms. Therefore when no estimated position exists the bias-correction method is obviously noneffective.

From the analysis above, we can see that in the noisy case, not only the exact collinearity but also the near collinearity will cause a problem in localization. In addition, the collinearity problem must be viewed in relation to the measurement noise level. The larger the level of the measurement noise is, the more likely the collinearity problem will be encountered. Therefore it is very important to take the measurement noise level into consideration when we analyze the collinearity problem.

B. A Novel Method

The collinearity problem has been described in the previous subsection. We have noted that when the exact or just near collinearity happens, to calculate the estimated position of the target will become a problem. When the estimated position cannot be obtained, the bias-correction method will be noneffective. If we can calculate an effectiveness area for the bias-correction method, many unnecessary calculations can be avoided. This is especially useful in wireless sensor networks because of the limited resource of each sensor.

In this section, we will propose a novel approach which can calculate an effectiveness area analytically for the bias-correction method. The specific feature of our approach is that we take the level of noise in the measurements into consideration.

Assume f_1 and f_2 are mappings from the target position (x, y) to the noiseless bearing measurements θ_1, θ_2, which are relative to a global direction (i.e. North), of the sensors $S1 = (x_1, y_1)$ and $S2 = (x_2, y_2)$. According to the simple geometry relationship shown in Fig. 3, we can obtain the following equations.

$$\begin{aligned} \theta_1 &= f_1(x, y) = \pi + \arctan(\tfrac{x-x_1}{y-y_1}) \\ \theta_2 &= f_2(x, y) = \arctan(\tfrac{x-x_2}{y-y_2}) \end{aligned} \tag{13}$$

The corresponding Jacobian matrix is given:

$$J_f = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} \tag{14}$$

In [4], the determinant of Jacobian matrix is considered as a criterion to calculate a problematic area analytically. However, the level of the measurement noise is not considered. In our method we use the determinant of the MSE (mean square error) matrix which includes the variance of measurement noise as a criterion to compute the effectiveness area. The motivation for doing this is that a small value for the determinant means that one or both of x, y will be very sensitive to the variance of the noise in θ_1 and θ_2. The MSE matrix can be obtained as follows.

$$MSE_f = J_f^{-1} \begin{bmatrix} \sigma_{\theta_1}^2 & 0 \\ 0 & \sigma_{\theta_2}^2 \end{bmatrix} (J_f^{-1})^T \tag{15}$$

Once a threshold for being collinearity-problem is fixed, the effectiveness area can be defined as follows.

$$\text{Effectiveness Area} = \{x, y | |\det(MSE_f)| \le \text{threshold}\} \tag{16}$$

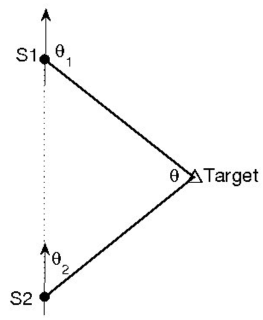

Fig. 3: Geometry of the Sensors and the Target

From the analysis in III. A, we can obtain that two factors may cause the collinearity problem when the positions of sensors are fixed: 1. The true position of a target. 2. The level of the measurement noise. From the equation (15) and (16) we can see that an effectiveness area is also dependant on the coordinates of the target (x, y) and the variance of the measurement noise. As described in III. A, we know that when the target is very close to or very far away from the link line of the two sensors (shown in Fig. 1. (b) and Fig. 1. (c)), the near collinearity problem is likely to be encountered in the noisy case. Correspondingly, in equation (15) when the target is either very close to the line joining the two sensors or in the far field, the Jacobian matrix J_f will become nearly singular. At that time the determinant of the MSE will be very large, which means it is very likely to be greater than the fixed threshold. Once the determinant of the MSE is larger than the fixed threshold we can consider the collinearity problem is encountered. Furthermore we know when the level of the measurement noise increases (while the position of target is fixed), it becomes more likely to encounter the collinearity problem (shown in Fig. 2). Again, in equation (15), when the level of the measurement noise (here is the variance σ^2) increases, the determinant of the MSE will also increase. The larger the variance of the measurement noise is, the greater the determinant of the MSE will be. Thus the determinant of the MSE is more likely to be greater than the predefined threshold. In other words, the collinearity problem is more likely to happen.

Therefore, we can conclude that the proposed approach is consistent with the analysis in III. A. We will verify the performance of our method by using Monte Carlo simulation in the section IV. In addition, this method also can be used for localization algorithms, because the area in which the bias-correction method can correct bias is consistent with the area in which localization algorithms can be applied [1].

C. More than Two Sensors

With more than two sensors, any collinearity of the target with two or more sensors will be deleterious, but not fatal, unless all sensor and the target are collinear. Therefore we should take more than two sensors situation into consideration. The method established in the last subsection can also be used in analyzing the situations involving a network of more than two sensors with respect to the effectiveness

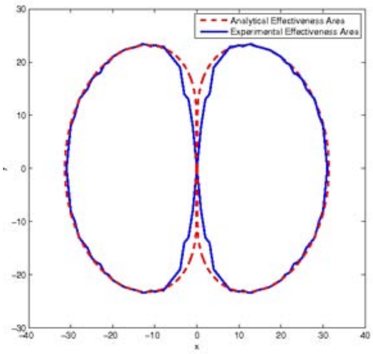

Fig. 4: Comparison of Analytical Effectiveness Area and Experimental Results (Sensors at (0,8) and (0,-8))

area. For such situations, considering only the cooperative localization between pairs of sensors, the effectiveness area for the whole network can be obtained by intersecting the effectiveness area of each pair of sensors. The effectiveness area of each pair of sensors can be determined as explained in the previous section.

D. Simulation Assumption

- All the simulations are done with two sensors in two-dimensional space.
- The measurement errors are produced by independent Gaussian distributions with zero mean and variance σ^2. In the simulation we assume the measurement noise for the two sensors have the same variance.
- All the simulation results are obtained from 5000 Monte Carlo experiments.
- Analytical effectiveness area denotes the area obtained by using analytical formula (16) provided in III. B. Here we set the threshold as 5 for all simulations.
- Experimental effectiveness area denotes the effectiveness area obtained by using simulations. For a particular target position (x, y), if the simulation results illustrate the bias-correction method can correct the bias we say the position is in the effectiveness area.

E. Comparison

In this subsection, we will compare the analytical effectiveness area with the one obtained from simulations. Here we only present two examples. The other situations have similar results.

First we fix the two sensors at (0,8) and (0,-8) with the standard deviation $\sigma = 0.05$. Fig. 4 illustrates the comparison between the analytical effectiveness area and the experimental one. From the figure we can obtain that the analytical effectiveness area enclosed by dash lines is almost consistent with the experimental results which is enclosed by solid lines. This simulation result verifies the performance of the proposed approach.

In the second example, we adjust the position of two sensors to (0,5) and (0,-5) resulting in a shorter distance between them. In addition, we increase the noise level in measurements by raising the standard deviation σ up to 0.1.

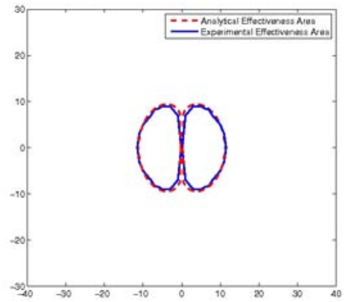

Fig. 5: Comparison of Analytical Effectiveness Area and Experimental Results (Sensors at (0,5) and (0,-5))

The simulation result is depicted in Fig. 5. In the figure the analytical effectiveness area still matches the experimental one which demonstrates the capability of the proposed approach again. However the effectiveness area becomes smaller. There are two factors which will influence the size of an effectiveness area. We will describe them in detail in the next two subsections. In addition, from the comparison results we obtain that no matter how the distance between two sensors or the variance of measurement noise changes, the analytical effectiveness always matches the experimental one very well. Therefore, in the next subsections, we only use the analytical effectiveness area to analyze the influence made by the two factors.

F. Distance Between Sensors

In this subsection, the relationship between the distance of two sensors and the effectiveness area will be provided. Different distance between sensors results in different size of the effectiveness areas. In the simulation we set $\sigma = 0.05$. The distance between the two sensors varies from 4 to 20, with steps of 2.

Fig. 6 illustrates the relation between the effectiveness area and the distance between sensors. From the figure, we can conclude that when the distance between the two sensors increases the effectiveness area will become larger. However the changes in the effectiveness area is nonlinear. The longer the distance is, the smaller the incremental effectiveness area will be for a fixed step size. There is in fact an upper bound [1] for the effectiveness area which means that when the distance between two sensors increases to a certain distance the effectiveness area will not become larger. We also learn that when the two sensors' distance goes to zero, the effectiveness area also goes to zero.

G. Different Level of Noise

As already noted, the level of the measurement noise will affect the effectiveness area. In this subsection, the simulation results will be presented to demonstrate the influence

[1] Further investigation of the simulations shows that the bound is closely relative to the standard deviation σ. When the σ increases, the bound will become smaller. In contrast, the bound will increase by decreasing σ.

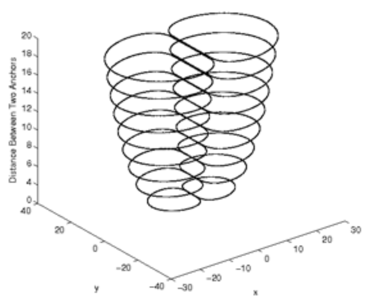

Fig. 6: Influence Made by the Distance between Two Sensors

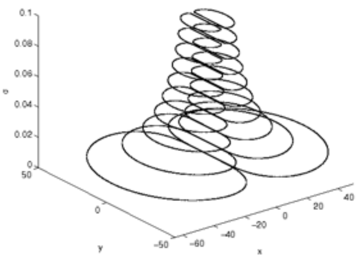

Fig. 7: Influence Made by the Level of Measurement Noise

of the level of the measurement noise. Here we adjust the level of the measurement noise by changing the standard derivation σ. The variation of σ is adjusted from 0.01 to 0.1 with steps of 0.01. In addition, the two sensors are fixed at (0,8) and (0,-8).

Fig. 7 depicts the relation between the effectiveness area and the level of the measurement noise. From the figure we can conclude that the effectiveness area becomes smaller when the standard deviation σ increases. When σ is large enough, the effectiveness area will goes to zero. In contrast, when σ goes to zero the effectiveness area will become infinite.

From the above three types of simulation we can obtain that the analytical effectiveness area is consistent with the experimental one which demonstrates the performance of the proposed approach. In addition the effectiveness area becomes larger when the distance between two sensors increases. However there is a maximum area which means when the distance between two sensors is long enough the effectiveness area will not increase further. Further simulations illustrate the relationship between the effectiveness area and the level of noise in the measurements. The larger the standard deviation is, the smaller the effectiveness area will be.

IV. CONCLUSION

In this paper, we first briefly review the bearing-only localization problem in two-dimensional space. The bias-correction method based on bearing-only localization algorithms is also briefly presented. Further we address an important problem in the bias-correction method that is how to obtain an effectiveness area analytically in order to avoid unnecessary calculation saving time and resources of sensors. A novel method is proposed: we use the determinant of the MSE (mean square error) matrix, which includes a Jacobian matrix and the variance of the measurement noise σ^2, as a criterion to obtain an effectiveness area for the bias-correction method analytically. The key feature of our method is that we introduce the variance of measurement noise σ^2 into the analytical expression. The level of measurement noise is an essential factor which will influence the effect of the bias-correction method. Monte Carlo simulation results demonstrate the performance of the proposed approach and illustrate the influence of changing the distance between two sensors and the level of the measurement noise. From the simulation results we conclude that the greater the distance between two sensors is, the larger the effectiveness area will be. Moreover the effectiveness area will become smaller while the level of the measurement noise increases. In addition, because of the consistence between the area in which we can apply bias-correction method and localization algorithms [1], the proposed approach is also valid for determining when localization algorithms are effective.

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