Joint 48th IEEE Conference on Decision and Control and
28th Chinese Control Conference
Shanghai, P.R. China, December 16-18, 2009

Multiple Model Adaptive Control (MMAC) for Nonlinear Systems with Nonlinear Parameterization

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Abstract— **A novel dwell-time-switching based multiple model adaptive control (MMAC) scheme is proposed for the state feedback stabilization problem of a class of general nonlinear systems with nonlinear parameterization. One major contribution is that it has advanced Morse's dwell-time-switching from linear systems to a class of nonlinear systems. Another significant contribution is that it combines the idea of monitoring the adequacy of candidate models (in terms of their estimation performances) in most MMAC schemes with the idea of monitoring the performance of candidate controllers in unfalsified control. Moreover, sufficient conditions for closed-loop stability are established for the proposed dwell-time-switching based MMAC scheme when applied to the considered class of nonlinear systems. To fulfil those sufficient conditions, emphasis has been put on the design of multiple estimators, candidate controllers and monitoring signals. The carefully designed estimators, candidate controllers, and monitoring signals enable us to derive a finite time switching result and provide a characterization on the maximum number of switchings for the dwell-time-switching algorithm proposed. In order to show how the general design of our dwell-time-switching based MMAC scheme can be applied to a particular nonlinear system, a constructive design based on back-stepping is provided for the adaptive control problem for a special class of nonlinearly parameterized systems, which can satisfy all the sufficient conditions to ensure closed-loop stability.**

I. INTRODUCTION

Multiple model adaptive control (MMAC) has attracted a great deal of interest since the late 1980s. There are several motivations for the use of MMAC to control complex systems. The first motivation comes from the fact that it is often not sufficient to use a single linear system to approximate a nonlinear system which has several operating points. In such a situation, multiple linear models are needed if one prefers to use linear models to solve the nonlinear control problem at hand ([1]).

The second motivation is that the system under consideration, whether linear or not, may change suddenly or abruptly due to sensor or actuator faults ([2]). In such a situation, fault accommodation/tolerant control based on multiple possible models seems to be a natural choice.

This work has been supported by ARC Discovery Project Grant DP0664427.

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The third motivation grows from the desire to relax the conditions under which linear systems with unknown parameters can be adaptively stabilized, which led to the use of MMAC in linear adaptive control ([3], [4]).

The last motivation we would like to mention is the desire to use MMAC to improve control performance, see [5], [6], [7] and the references therein.

Since the MMAC scheme in this paper is based on monitoring simultaneously both the adequacy of the candidate model through multiple state estimator design and the performance of the related candidate controller, we will focus our discussions only on those results, which are based on monitoring either the adequacy of the candidate model through multiple estimator design or the performance of the candidate controller.

In the MMAC literature, the schemes based on multiple estimator design are perhaps dominant. This type of MMAC schemes can be classified into two groups according to how the control law is constructed. The first group of schemes do not use switching to determine the control law. Instead, they construct their control law by blending all candidate controllers ([2], [8], [9], [10], [11]). In order to take advantage of the ability of the well-developed linear robust controllers in 1980s to deal with uncertainties, a robust MMAC scheme was proposed in [8], where the control law was constructed through blending a finite number of robust controllers by the use of pseudo-probabilities or weights. A similar idea was later used in [9], [10], [11] to propose a robust MMAC scheme called RMMAC, where robust candidate controllers were first designed by the use of $\mu-$synthesis and then the overall controller was obtained through blending a finite number of $\mu-$synthesized candidate controllers with probabilistic weighting. Although the simulation results are quite promising, solid theoretical analysis is lacking because it is extremely difficult as pointed out in [11].

Instead of blending all candidate controllers, the other group of MMAC schemes make use of a switching logic to determine the appropriate controller amongst a finite number of candidate controllers ([3], [4], [5], [6], [12], [13], [14], [15], [16], [17]). The MMAC schemes in [5], [6] allow switching to happen at every time instant. While this might be a good thing to do in order to catch anything changing in the system in a timely manner, switching too quickly might also lead to the possibility of unbounded chatter ([4]). A hysteresis switching algorithm was proposed in [3] to prevent switching from happening too fast. It was also used in the MMAC scheme proposed in [4], where it was proved switching stops after a finite time. This line of research led to the so-called scale-independent hysteresis switching algorithms, which were used to control linear uncertain systems ([15], [17]) and nonlinear uncertain systems based on the concept of integral-input-to-state stability ([16]). One interesting result obtained in [17] is the derivation of a bound on the number of switchings on an arbitrary time interval, where the bound is a function of time. Another way to prevent switching from happening too fast is to use so-called dwell time switching, which was proposed in [12] for linear systems. It was proved in [13] that the resulting dwell time switching controller possesses robustness against bounded disturbances and noises and certain unmodeled dynamics.

In unfalsified control theory, instead of monitoring the adequacy of the candidate model, the idea of monitoring the performance of the candidate controller has been proposed and has been researched extensively recently, see [18], [19], [20], [21] and related references.

Since the target application area of our MMAC scheme is nonlinear adaptive control with nonlinear parameterization, a very brief review is provided here for nonlinear adaptive control. Nonlinear adaptive control has made great progress since the 1990s after the introduction of the backstepping design approach [22], [23]. This has been achieved under a common assumption, that is, the unknown parameters enter linearly into nonlinear systems. However, if the unknown parameters enter nonlinearly into nonlinear systems, the related adaptive control problem proves to be much more difficult to tackle and this can be evidenced by the sparsity of results in this area available up to now. By assuming the nonlinear functions are concave or convex with respect to the unknown parameters, an adaptive tracking controller was proposed for a class of nonlinear systems with unknown nonlinear parameters in [25]. The result was extended in [26] to the case where the nonlinear functions are not required to be concave or convex with respect to unknown parameters but rather their envelopes are concave or convex with respect to unknown parameters. The state feedback adaptive control problems for nonlinearly parameterized nonlinear systems were considered later in [27], [28] for triangular nonlinear systems. The output feedback adaptive control problem was studied in [29] for output-feedback form nonlinear systems. For general nonlinear parameterized nonlinear systems, their adaptive control problems remain challenging and largely open.

The primary purpose of this paper is to advance Morse's dwell-time-switching from linear to nonlinear systems. To this end, a novel dwell-time-switching based MMAC scheme for the state feedback stabilization problem of a class of nonlinearly parameterized nonlinear systems has been proposed. One major contribution is that it has advanced Morse's dwell-time-switching from linear systems to a class of nonlinear systems. Another significant contribution is that it combines the idea of monitoring the adequacy of candidate model in most MMAC schemes with the idea of monitoring the performance of candidate controller in unfalsified control. Moreover, sufficient conditions for closed-loop stability are established for the considered class of nonlinear systems with nonlinear parameterization. To fulfil those sufficient conditions, emphasis has been put on the design of multiple estimators, candidate controllers and monitoring signals. The carefully designed estimators, candidate controllers, and monitoring signals enable us to derive a finite time switching result and provide a characterization on the maximum number of switchings for the dwell-time-switching algorithm proposed. In order to show how the general design of our dwell-time-switching based MMAC scheme can be applied to a particular nonlinear system, a constructive design is provided for the adaptive control problem for a special class of nonlinearly parameterized systems, which can satisfy all the sufficient conditions to ensure closed-loop stability.

II. Nonlinear systems of interest and problem formulation

In this paper, we are interested in nonlinear systems of the following form.

$$\dot{x}(t) \quad = \quad f(x(t), \theta^*, u(t), d(t)) \tag{1}$$

where $x(t)$,$u(t)$ and $d(t)$ are the system state, control input, and disturbances, respectively. $\theta^* \in R^q$ is an unknown parameter vector. $f(x, \theta^*, u, d)$ is continuous with respect to its arguments.

We make the following assumption regarding the unknown parameter vector θ^*.

- A.1 $\theta^* \in \Theta = \cup_{j=1}^{M} S_j$, where M is a known finite positive integer, and $S_j, j = 1, 2, \cdots, M$ are known and bounded sets and $S_j \cap S_l = \phi$ for $j \neq l$ with ϕ being an empty set. Here, we do not know to which set θ^* belongs although $\theta^* \in \Theta$ is known.

In this paper, we are interested in the following state feedback control problem:

Multiple Model Adaptive Control (MMAC) Problem For (1), design a state feedback multiple model adaptive controller to ensure that, for any x_0, all the closed-loop system signals are bounded, and to make $x(t)$ enter asymptotically a neighborhood of zero.

III. A general dwell-time-switching based MMAC scheme

In this section, we shall present a general dwell-time-switching based MMAC scheme for system (1), and establish a general theory for closed-loop stability by providing sufficient conditions.

Since the nonlinear system under consideration is given in a general form, to make the MMAC problem tractable, we need the following assumption.

- A.2 For each set $S_j \subset \Theta$, where $j = 1, \cdots, M$, there exist a parameter vector $\theta_j \in S_j$ and a corresponding continuous control law $u_{\theta_j}(t) = \phi(x(t), \theta_j)$ such that, for any x_0, a) the solution $x(t)$ of $\dot{x} = f(x, \theta, \phi(x, \theta_j), d(t))$ exists and is unique for any $\theta \in \Theta$; b) along the solution $x(t)$ of $\dot{x} = f(x, \theta, \phi(x, \theta_j), d(t))$, $\dot{V}_{\theta_j}(x(t)) \leq -K_{1,\theta_j}(x(t)) + K_{2,\theta_j}(x(t))$ if $\theta \in S_j$, where V_{θ_j} and K_{1,θ_j} are known positive definite continuous functions of $x(t)$ with $V_{\theta_j}(0) = 0$ and $K_{1,\theta_j}(0) = 0$, which satisfy $lim_{\|x\|\to\infty} V_{\theta_j}(x) = \infty$ and $lim_{\|x\|\to\infty} K_{1,\theta_j}(x) = \infty$, and K_{2,θ_j} is a nonnegative function of $x(t)$ and is bounded for all $x(t)$ and each θ_j.

Remark 1: It should be pointed out that $K_{2,\theta_j}(x(t))$ in Assumption A.2 and $K_{4,\theta_j}(x, e_{\theta_j})$ appeared later in Assumption A.3 usually depend on the knowledge of an upper bound of $\|d\|$. By requiring $K_{2,\theta_j}(x(t))$ and $K_{4,\theta_j}(x, e_{\theta_j})$ to be known and bounded, we implicitly require that $\|d\|$ is bounded and an upper bound of $\|d\|$ is available. For this reason, we do not provide explicit assumptions on $\|d\|$ in addition to these two assumptions.

Under assumption A.2, we have the following result.

Lemma 1: For system (1), suppose assumption A.2 is satisfied. Then, there exists a candidate controller $u_{\theta_{j_0}}(t) = \phi(x(t), \theta_{j_0})$ with $\theta_{j_0}, \theta^* \in S_{j_0}$ such that for any x_0, $x(.)$ and $u(.)$ are bounded and $x(t)$ enters asymptotically a neighborhood of zero.

Remark 2: Since $K_{2,\theta_j}(x(t))$ usually depends on $\|d\|$ and the way that $S_j, j = 1, 2, \cdots, M$ are defined, the size of the neighborhood of zero mentioned in Lemma 1 will depend on $\|d\|$ and the way that $S_j, j = 1, 2, \cdots, M$ are defined as well.

Remark 3: Given the generality of the considered nonlinear systems, Assumption A.2 is a reasonable assumption because many Lyapunov function based controllers for nonlinear systems can satisfy it, see [22], [23], [24] and the related references in them. It simply means that there exists at least one candidate controller that can stabilize the considered nonlinear systems, which is a necessary requirement for implementing the MMAC strategy.

In the remainder of this paper, the dependence of variables on the time t will drop out whenever it is appropriate. For example, we will write $u_\theta(t) = \phi(x(t), \theta)$ as $u_\theta = \phi(x, \theta)$.

According to assumption A.2, we have a family of M candidate controllers, that is, $u_\theta = \phi(x, \theta), \theta = \theta_1, \cdots, \theta_M$.

To deal with the problem formulated, we are going to use a dwell-time-switching mechanism based on the idea of multiple estimator design for the considered system. Therefore, before we introduce the switching mechanism, multiple estimators for system (1) are designed first.

$$\begin{aligned} \dot{\hat{x}}_{\theta_j} &= H(\hat{x}_{\theta_j} - x) + f_m(x, \theta_j, u) \\ &\quad + g(x, \hat{x}_{\theta_j}, \theta_j, u), j = 1, \cdots, M \end{aligned} \tag{2}$$

where H is a Hurwitz matrix that can be chosen freely, and $\hat{x}_{\theta_1}(0) = \cdots = \hat{x}_{\theta_M}(0)$. $f_m(x, \theta_j, u)$ is the modeled part of $f(x, \theta_j, u, d(t))$, and $g(x, \hat{x}_{\theta_j}, \theta_j, u)$ is a design function and is introduced to make the state estimation error satisfy conditions in assumption A.3.

Remark 4: The estimators above are provided with a very general form with the aim of making it suitable to nonlinear systems as general as possible. The specific forms for $f_m(x, \theta_j, u)$ and $g(x, \hat{x}_{\theta_j}, \theta_j, u)$ are problem dependent and can not be given at this stage because we are dealing with nonlinear systems with a general form of (1). However, as can be seen later in Lemma 2 and in Section IV, they can be specified under certain conditions for some systems.

Define estimation error signals as $e_\theta = \hat{x}_\theta - x, \theta = \theta_1, \cdots, \theta_M$.

The designed estimators are required to satisfy the following condition.

- A.3 The vector function $g(x, \hat{x}_{\theta_j}, \theta_j, u)$ is designed such that $\dot{V}(e_{\theta_j}) \leq -K_3(e_{\theta_j}) + K_{4,\theta_j}(x, e_{\theta_j})$ when both θ_j and θ^* are in the same set, where V and K_3 are known positive definite continuous functions of e_{θ_j} with $V(0) = 0$ and $K_3(0) = 0$, which satisfy $lim_{\|e_{\theta_j}\|\to\infty} V(e_{\theta_j}) = \infty$ and $lim_{\|e_{\theta_j}\|\to\infty} K_3(e_{\theta_j}) = \infty$, and K_{4,θ_j} is a known nonnegative function of x and e_{θ_j} and is bounded for any x and e_{θ_j} and for any given θ_j.

If $K_{4,\theta_j}(x, e_{\theta_j}) = 0$, the condition $\dot{V}(e_{\theta_j}) \leq -K_3(e_{\theta_j}) + K_{4,\theta_j}(x, e_{\theta_j})$ will imply that e_{θ_j} tends to zero asymptotically. However, in the presence of parameter uncertainty and nonparameterized uncertainties in $d(t)$, it is in general very hard if not impossible to design state estimators such that $K_{4,\theta_j}(x, e_{\theta_j}) = 0$. Here, we allow $K_{4,\theta_j}(x, e_{\theta_j})$ to be nonzero to ease the difficulty in the estimator design.

The following lemma shows that assumption A.3 can indeed be satisfied by the proper design of $g(x, \hat{x}_{\theta_j}, \theta_j, u)$ for some systems under certain conditions.

Lemma 2: For system (1), suppose that $\|f(x, \theta^*, u, d) - f_m(x, \theta^*, u)\| \leq \rho_m(x, u)$, and suppose also that, for any $\theta \in \Theta$, there exists a positive function $\rho(x, \theta, u)$ continuous with respect to its arguments and a function $\gamma(r)$ of class $\mathcal{K}_\infty$ such that $\|f_m(x, \theta, u) - f_m(x, \theta^*, u)\| \leq \rho(x, \theta, u)\gamma(\|\theta - \theta^*\|)$. Suppose also that M^* is such that for any $\theta^* \in S_j$ and j, $\|\theta_j - \theta^*\| \leq M^*$ and P is a positive definite symmetric matrix such that $H^T P + PH = -I$ with I being an identity matrix. If we choose $g(x, \hat{x}_{\theta_j}, \theta_j, u) = -K_g[\rho^2(x, \theta_j, u)\gamma^2(M^*) + \rho_m^2(x, u)]Pe_{\theta_j}$ with K_g a positive design constant, then the multiple estimators defined by (2) can ensure that assumption A.3 is satisfied.

Proof: Suppose without loss of generality that $\theta^* \in S_{j_0}$; we only need to show that $\dot{V}(e_{\theta_j}) \leq -K_3(e_{\theta_j}) + K_{4,\theta_j}(x, e_{\theta_j})$ for $j = j_0$, which can be proved using standard arguments. The detailed proof is omitted due to lack of space. ¶

Remark 5: The reason for having $\|f_m(x, \theta, u) - f_m(x, \theta^*, u)\| \leq \rho(x, \theta, u)\gamma(\|\theta - \theta^*\|)$ is that we would like to have as weak a condition as possible. It includes a Lipschitz condition as a special case with $\gamma(\|\theta - \theta^*\|) = \|\theta - \theta^*\|$ and $\rho(x, \theta, u)$ a constant. If θ enters linearly into $f_m(x, \theta, u)$, the condition can be satisfied naturally. Moreover, it allows nonlinear functions that do not satisfy Lipschitz condition with respect to their unknown parameters. For example, consider $f_m(x, \theta) = xe^{\theta x}$. After some calculations, it can be shown $|f_m(x, \theta) - f_m(x, \theta^*)| = x^2 e^{[\theta + \zeta_\theta(\theta^* - \theta)]x}|\theta - \theta^*| \leq x^2 e^{\theta x} e^{x^2/2} e^{(\theta - \theta^*)^2/2}|\theta - \theta^*|$, where ζ_θ depends on θ and θ^* and satisfies $0 \leq \zeta_\theta \leq 1$.

To employ the idea of dwell-time-switching proposed in [12], we also need to design monitoring signals.

It is obvious that $\dot{V}(e_{\theta_j}) \leq -K_3(e_{\theta_j}) + K_{4,\theta_j}(x, e_{\theta_j})$ on

$[0, t]$ implies that

$$V(e_{\theta_j}(t)) \leq V(e_{\theta_j}(0)) - \int_0^t (K_3(e_{\theta_j}) - K_{4,\theta_j}(x, e_{\theta_j}))d\tau \quad (3)$$

Based on (3), the first family of monitoring signals are defined as

$$\dot{W}_\theta = -\lambda W_\theta + 1 - sgn[V(e_\theta(0)) - \int_0^t (K_3(e_\theta) - K_{4,\theta}(x, e_\theta))d\tau - V(e_\theta)] \quad (4)$$

where $\theta = \theta_1, \cdots, \theta_M$, λ is a positive constant, and $W_\theta(0) = 0$ for all $\theta = \theta_1, \cdots, \theta_M$. The "sgn" function is defined as follows: $sgn(x) = 1$ if $x \geq 0$; $sgn(x) = -1$ if $x < 0$. For that value of θ_j - and there is necessarily one- such that (3) holds, the value of the "sgn" function in (4) is one, and W_{θ_j} is zero over $[0, t]$.

Note also that $\dot{V}_{\theta_j}(x(t)) \leq -K_{1,\theta_j}(x(t)) + K_{2,\theta_j}(x(t))$ implies that

$$V_{\theta_j}(x(t)) \leq V_{\theta_j}(x(t')) - \int_{t'}^t (K_{1,\theta_j}(x) - K_{2,\theta_j}(x))d\tau \quad (5)$$

Based on (5), the second family of monitoring signals are defined as

$$\bar{W}_\theta(t', t) = \int_{t'}^t e^{-\lambda(t-\tau)}(1 - sgn[V_\theta(x(t')) - \int_{t'}^\tau (K_{1,\theta}(x) - K_{2,\theta}(x))ds - V_\theta(x(\tau))])d\tau \quad (6)$$

where $\theta = \theta_1, \cdots, \theta_M$, λ is a positive constant, and $t > t'$. For that value of θ_j - and there is necessarily one- such that (5) holds on $[t', t]$, the value of the "sgn" function in (6) is one, and $\bar{W}_{\theta_j}$ is zero over $[t', t]$.

In dwell-time-switching, we have a piecewise constant switching signal $\sigma(t) \in \{1, 2, \cdots, M\}$, which is used to determine a controller amongst M candidate controllers ($u_\theta = \phi(x, \theta), \theta = \theta_1, \cdots, \theta_M$) to be switched on. We let $u = u_{\theta_{\sigma(t)}} = \phi(x, \theta_{\sigma(t)})$. For example, if $\sigma(t) = 1$, then $u = u_{\theta_1} = \phi(x, \theta_1)$ is switched on.

With the multiple estimators and monitoring signals at hand, we are ready to provide our dwell-time-switching MMAC scheme, which is presented in steps.

Switching control mechanism (SCM):

- Step 1. Choose a dwell-time constant τ_D.
- Step 2. Let $t_0 = 0$ and $k = 0$, and pick a candidate controller in the candidate controller family.
- Step 3. For $t_k \leq t < t_k + \tau_D$, let $\sigma(t) = \sigma(t_k)$ and $u(t) = u_{\theta_{\sigma(t)}}(t)$.
- Step 4. For $t \geq t_k + \tau_D$, monitor all $W-$ signals and only one $\bar{W}-$ signal $\bar{W}_{\theta_{\sigma(t_k)}}(t_k, t)$, let $S_{min,t} = \{i | W_{\theta_i}(t) = 0\}$. If $\sigma(t_k) \in S_{min,t}$ and $\bar{W}_{\theta_{\sigma(t_k)}}(t_k, t) = 0$, no new controller is switched on and let $\sigma(t) = \sigma(t_k)$ and $u(t) = u_{\theta_{\sigma(t)}}(t)$. If there exists a smallest time instant, say $\bar{t}$ such that $\sigma(t_k) \in S_{min,\bar{t}}$ but $\bar{W}_{\theta_{\sigma(t_k)}}(t_k, \bar{t}) > 0$ or $\sigma(t_k)$ does not belong to $S_{min,\bar{t}}$, then increment k by 1, let $t_k = \bar{t}$, pick $\sigma(\bar{t})$ as any element in $S_{min,\bar{t}}$ that is different from $\theta_{\sigma(t_j)}$, $0 \leq j \leq k-1$, and let a new controller $u(t) = u_{\theta_{\sigma(\bar{t})}}(t)$ be switched on at $\bar{t}$. Go back to Step 3.

Let $S_W = \{\theta_i | W_{\theta_i}(\tau_D) = 0\}$ and $\bar{S}_W = \{\theta_i | W_{\theta_i}(\tau_D) \neq 0\}$, and denote the number of elements in S_W as l_W.

Now, we are ready to give a result regarding to the maximum number of switching times.

Lemma 3: For system (1) under assumptions $A.1$ to $A.3$, suppose that the multiple estimators are defined by (2), and that $W-$ and $\bar{W}-$signals are defined by (4) and (6), respectively. If **SCM** is employed, then the maximum number of switchings is equal to l_W if $l_W < M$ and $M-1$ if $l_W = M$.

Proof: Assume $\theta^* \in S_{j_0}$ with $1 \leq j_0 \leq M$. Now, the key point in this proof is to notice the fact that $W_{\theta_{j_0}}(t) \equiv 0$ for all $t \geq 0$. Note also that once for some t_0, $W_{\theta_j}(t_0) \neq 0$, we will always have $W_{\theta_j}(t) \neq 0$ for all $t \geq t_0$. This together with the way **SCM** is defined proves that any candidate controller can only be switched on once. This fact leads to the conclusions of the lemma. ¶

Remark 6: From Lemma 3, it is easy to see that switching can happen at most once if $l_W = 1$, that is, if there is one and only one element in S_W. Whether $l_W < M$ or not, switching can happen at most $M-1$ times.

Remark 7: Note that the results presented in the above lemmas do not depend on the choice of τ_D. This means that τ_D can be chosen as an arbitrarily small constant in order to allow fast switching to happen.

The main result is presented in the following theorem.

Theorem 1: For system (1) that satisfies assumptions A.1 to A.3, suppose that the multiple estimators defined by (2) are designed, and suppose also that the $W-$signals and the $\bar{W}-$signals defined by (4) and (6) are used in **SCM** to generate a switching controller for system (1). Assume further that no candidate controller $u_j = \phi(x, \theta_j)$ with $j = 1, \cdots, M$ can make the system $\dot{x} = f(x, \theta^*, \phi(x, \theta_j), d)$ escape in finite time. Then, for any x_0, all the closed-loop signals are bounded and $x(t)$ enters asymptotically to a neighborhood of the origin.

Proof (Outline): Because there is no-finite-time escape and only a finite number of switchings can happen, a fixed candidate controller will stay in the loop after a finite time. The fixed candidate controller $u_j = \phi(x, \theta_j)$ must either be the "correct" one, say $u_{j_0} = \phi(x, \theta_{j_0})$ or satisfy $\dot{V}_{\theta_j}(x(t)) \leq -K_{1,\theta_j}(x(t)) + K_{2,\theta_j}(x(t))$ for all $t > \bar{t}$ when $j \neq j_0$. In either case, the theorem can be proved based on Assumption A.2.

IV. Application of the proposed MMAC scheme to a class of nonlinearly parameterized uncertain systems

As an application of the proposed MMAC scheme, we consider the adaptive control problem of the follow class of systems

$$\begin{aligned} \dot{x}_1 &= x_2 + \xi_1(x_1, \theta^*) + \psi_1(x, d(t)) \\ &\vdots \\ \dot{x}_i &= x_{i+1} + \xi_i(x_1, \cdots, x_i, \theta^*) + \psi_i(x, d(t)) \\ &\vdots \\ \dot{x}_n &= \xi_n(x_1, \cdots, x_n, \theta^*) + u + \psi_n(x, d(t)) \end{aligned} \quad (7)$$

where θ^* is an unknown parameter vector that belongs to a closed and bounded set $\Theta \in R^q$, and $\xi_i(x_1, \cdots, x_i, \theta^*), i = 1, \cdots, n$ are scalar smooth nonlinear functions with $\xi_i(0, \cdots, 0, \theta^*) = 0, i = 1, \cdots, n$. $\psi_i(x, d(t)), i = 1, \cdots, n$ are uncertainties from modeling errors and disturbances $d(t)$.

We need the following assumptions to solve adaptive control problem of systems given by (7).

- B.1 For each $1 \leq i \leq n$, there exist a known positive smooth function $\rho_i(x_1, \cdots, x_i, \theta)$ and a known function $\gamma_i(r)$ of class $\mathcal{K}_\infty$ such that $|\xi_i(x_1, \cdots, x_i, \theta) - \xi_i(x_1, \cdots, x_i, \bar{\theta})| \leq \rho_i(x_1, \cdots, x_i, \theta)\gamma_i(\|\theta - \bar{\theta}\|)$, where $\theta, \bar{\theta} \in \Theta$.
- B.2 For each $1 \leq i \leq n$, there exists a known positive smooth function $\varphi_i(x_1, \cdots, x_i)$ such that $|\psi_i(x, d(t))| \leq \varphi_i(x_1, \cdots, x_i)$.

When θ^* enters nonlinearly into all nonlinear functions, the adaptive control problem of the above systems under assumptions B.1 and B.2 is still not completely solved. In this section, we shall combine our dwell-time-switching based MMAC with backstepping design ([22]) to provide a solution, which is novel based on our knowledge and is at least a new alternative to the existing approaches for (7).

Since Θ is closed and bounded, it is possible to find finite number of sets, that is, $S_j, j = 1, \cdots, M$ such that $\Theta = \cup_{j=1}^M S_j$ and $\|\theta - \theta_j\| \leq M^*$ if $\theta, \theta_j \in S_j$, where M^* can be made sufficiently small by proper partitioning.

For simplicity, we consider only the case that $n = 3$ because the candidate controllers for $n > 3$ can be designed in exactly the same way. Rewrite (7) for $n = 3$ as

$$\begin{aligned} \dot{x}_1 &= x_2 + \xi_1(x_1, \theta_j) \\ &+ \xi_1(x_1, \theta^*) - \xi_1(x_1, \theta_j) + \psi_1(x, d(t)) \\ \dot{x}_2 &= x_3 + \xi_2(x_1, x_2, \theta_j) + \xi_2(x_1, x_2, \theta^*) \\ &\quad -\xi_2(x_1, x_2, \theta_j) + \psi_2(x, d(t)) \\ \dot{x}_3 &= \xi_3(x_1, x_2, x_3, \theta_j) + \xi_3(x_1, x_2, x_3, \theta^*) \\ &\quad -\xi_3(x_1, x_2, x_3, \theta_j) + u + \psi_3(x, d(t)) \end{aligned} \quad (8)$$

Since backstepping design is now quite standard [22], we provide our candidate controller directly below without offering the detailed design procedure.

$$\begin{aligned} u_{\theta_j}(t) &= \phi(x, \theta_j) \\ &= -c_3 z_{j,3} - z_{j,2} - \xi_3 + \frac{\partial \alpha_{j,2}}{\partial x_2}(x_3 + \xi_2) \\ &+ \frac{\partial \alpha_{j,2}}{\partial x_1}(x_2 + \xi_1) \\ &- k_3\gamma_3^2(M^*)\rho_3^2 z_{j,3} - l_3\varphi_3^2 z_{j,3} \\ &- (\frac{\partial \alpha_{j,2}}{\partial x_2})^2(k_3\gamma_2^2(M^*)\rho_2^2 z_{j,3} + l_3\varphi_2^2 z_{j,3}) \\ &- (\frac{\partial \alpha_{j,2}}{\partial x_1})^2(k_3\gamma_1^2(M^*)\rho_1^2 z_{j,3} + l_3\varphi_1^2 z_{j,3}) \end{aligned} \quad (9)$$

where $z_1 = x_1$, $z_{j,2} = x_2 - \alpha_{j,1}(x_1, \theta_j)$, and $z_{j,3} = x_3 - \alpha_{j,2}(x_1, x_2, \theta_j)$, and the $\alpha-$ functions are defined as

$$\begin{aligned} \alpha_{j,1} &= -c_1 z_1 - \xi_1 - l_1\varphi_1^2 z_1 - k_1\gamma_1^2(M^*)\rho_1^2 z_1. \\ \alpha_{j,2} &= -c_2 z_{j,2} - z_1 - \xi_2 + \frac{\partial \alpha_{j,1}}{\partial x_1}(x_2 + \xi_1) \\ &- k_2\gamma_2^2(M^*)\rho_2^2 z_{j,2} - l_2\varphi_2^2 z_{j,2} \\ &- k_2(\frac{\partial \alpha_{j,1}}{\partial x_1})^2\gamma_1^2(M^*)\rho_1^2 z_{j,2} - l_2(\frac{\partial \alpha_{j,1}}{\partial x_1})^2\varphi_1^2 z_{j,2} \end{aligned} \quad (10)$$

where $c_i, k_i, l_i, 1 \leq i \leq 3$ are positive design constants.

Choose $V_{\theta_j} = \frac{1}{2}z_1^2 + \frac{1}{2}z_{j,2}^2 + \frac{1}{2}z_{j,3}^2$, it can be shown that

$$\dot{V}_{\theta_j} \leq -c_1 z_1^2 - c_2 z_{j,2}^2 - c_3 z_{j,3}^2 + \beta_3, \quad (11)$$

where β_3 satisfies

$$\begin{aligned} \beta_3 &\leq \frac{3\gamma_1^2(\|\theta_j - \theta^*\|)}{4k_1\gamma_1^2(M^*)} + \frac{3}{4l_1} \\ &+ \frac{2\gamma_2^2(\|\theta_j - \theta^*\|)}{4k_2\gamma_2^2(M^*)} + \frac{2}{4l_2} \\ &+ \frac{\gamma_3^2(\|\theta_j - \theta^*\|)}{4k_3\gamma_3^2(M^*)} + \frac{1}{4l_3}. \end{aligned} \quad (12)$$

Define $\bar{\beta}_3 = \sum_{i=1}^3(\frac{4-i}{4k_i} + \frac{4-i}{4l_i})$ and let $K_{1,\theta_j}(x(t)) = c_1 z_1^2 + c_2 z_{j,2}^2 + c_3 z_{j,3}^2$ and $K_{2,\theta_j}(x(t)) = \bar{\beta}_3$.

Now, we turn to designing multi-estimators satisfying assumption A.3. Under Assumption B1 and Assumption B2, we can design multi-estimators for systems given by (8) as

$$\begin{aligned} \dot{\hat{x}}_{\theta_j} &= H(\hat{x}_{\theta_j} - x) + f_m(x, \theta_j, u) \\ &\quad + g(x, \hat{x}_{\theta_j}, \theta_j), j = 1, \cdots, M \end{aligned} \quad (13)$$

where $H = -I$, and $\hat{x}_{\theta_1}(0) = \cdots = \hat{x}_{\theta_M}(0)$.

The function $f_m(x, \theta_j, u)$ is defined as

$$f_m(x, \theta_j, u) = (\, x_2 + \xi_1 \quad x_3 + \xi_2 \quad \xi_3 + u\,)^T \quad (14)$$

and $g(x, \hat{x}_{\theta_j}, \theta_j)$ is defined as

$$\begin{aligned} g &= -K_g P e_{\theta_j}(\rho_1^2 + \rho_2^2 + \rho_3^2)\gamma^2(M^*) \\ &\quad -K_g P e_{\theta_j}(\varphi_2^2 + \varphi_2^2 + \varphi_3^2) \end{aligned} \quad (15)$$

where $e_{\theta_j} = \hat{x}_{\theta_j} - x$, $P = \frac{1}{2}I$ since $H = -I$, $\gamma(M^*) = max\{\gamma_1(M^*), \gamma_2(M^*), \gamma_3(M^*)\}$, and K_g is a positive design constant.

We have the following result based on Theorem 1.

Corollary 1: For system (8) that satisfies Assumptions B1 and B2, suppose that candidate controllers are given by (9) and multi-estimators are designed through (13) to (15). Define the $W-$ and $\bar{W}-$ signals as in the previous subsection, then the switching controller generated by **SCM** ensures that, for any x_0, all the closed-loop signals are bounded and $x(t)$ enters asymptotically into a neighborhood of the origin, whose size can be made as small as possible by choosing the design constants $c_i, k_i, l_i, 1 \leq i \leq 3$ sufficiently large.

V. CONCLUSIONS

In this paper, we have advanced the idea of dwell-time-switching from linear systems to a class of general nonlinear systems with nonlinear parameterization through proposing a dwell-time-switching based multiple model adaptive control (MMAC) scheme called **SCM**, which is our major contribution. In **SCM**, we have proposed *combining* the idea of monitoring the adequacy of candidate models (in particular their estimation performances) in most MMAC schemes with the idea of monitoring the performance of candidate controllers in unfalsified control, which represents another significant contribution.

Sufficient conditions for closed-loop stability are established for the proposed MMAC scheme. These sufficient conditions provide general guidelines for the design of candidate controllers, multiple estimators and monitoring signals. Given that the candidate controllers, multiple estimators and monitoring signals have been designed properly such that these sufficient conditions are satisfied, a finite time switching result has been derived and a characterization on the maximum number of switchings has been provided, which are also contributions of this paper.

By employing the idea of combining our dwell-time-switching based MMAC with backstepping design, a constructive design has been provided for the adaptive control problem of a special class of nonlinearly parameterized systems, which can satisfy all the sufficient conditions to ensure closed-loop stability. The resulting control scheme is novel for the considered class of nonlinear systems according to our knowledge. At the very least, it is a new alternative to the existing approaches.

Our results do not address yet the question of where to select the nominal parameters θ_j, or how many there should be. Further, our results were obtained under the condition that all the state variables are available for controller design. One of our future research topics is to extend the results to systems where only the outputs are available.

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