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On the Properties of Giant Component in Wireless Multi-hop Networks

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***Abstract*—In this paper, we study the giant component, the largest component containing a non-vanishing fraction of nodes, in wireless multi-hop networks in $\Re^d$ ($d = 1,\ 2$). We assume that n nodes are randomly, independently and uniformly distributed in $[0,1]^d$, and each node has a uniform transmission range of $r = r(n)$ and any two nodes can communicate directly with each other iff their Euclidean distance is at most r. For $d = 1$, we derive a closed-form analytical formula for calculating the probability of having a giant component of order above pn with any fixed $0.5 < p \leq 1$. The asymptotic behavior of one dimensional network having a giant component is investigated based on the derived result, which is distinctly different from its two dimensional counterpart. For $d = 2$, we derive an asymptotic analytical upper bound on the minimum transmission range at which the probability of having a giant component of order above qn for any fixed $0 < q < 1$ tends to one as $n \to \infty$. Based on the result, we show that significant energy savings can be achieved if we only require a large percentage of nodes (e.g. 95%) to be connected rather than requiring all nodes to be connected. The results of this paper are of practical significance in the design and analysis of wireless ad hoc networks and sensor networks.**

I. Introduction

A network is connected iff (if and only if) for any pair of two nodes, there is at least one path between them. In other words, all nodes in the network are parts of a single component. In the past several years, the connectivity problem in wireless multi-hop networks has been widely investigated and significant outcome has been achieved [1], [2], [3]. However, from a practical point of view, requiring all nodes to be connected may be a too stringent condition to satisfy. It has been shown by simulations that the transmission range required for a large percentage of nodes to be connected is much less than the transmission range for all nodes to be connected [4], [5], [6]. Fig. 1 shows the average value of the ratio of the transmission range required for 95% of nodes to be connected to the transmission range required for a connected network. As shown in the figure, when the number of nodes is 1000, the transmission range required for 95% nodes to

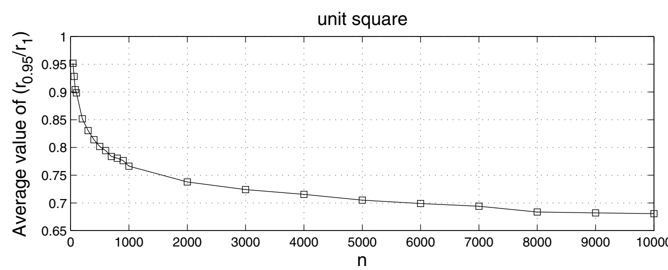

Fig. 1. Simulation results: Average value of ratio $r_{0.95}/r_1$. r_1: transmission range required for a connected network; $r_{0.95}$: transmission range required for 95% of nodes to be connected. The ratio shown is the average value, and each average value is obtained over 2000 random topologies, in which a total of n nodes are uniformly and randomly distributed on a unit square.

be connected is 24% less than that required for a connected network. Based on a conservative estimate that the required transmission power increases with the square of the required transmission range, this means an energy saving of at least 42%. In addition, the ratio decreases as the total number of nodes n increases. As we will show in section VI, the ratio will go to zero when $n \to \infty$. This means that the energy saving is more significant in a network with a large number of nodes. In practice, many network applications do not necessarily require all nodes to be connected and having a few disconnected nodes is not critical. Examples of applications of this type include habitat monitoring for wild animals, monitoring ocean temperature, and so on. Hence, it is important to investigate the largest connected component containing a non-vanishing fraction of nodes, termed the *giant component* [5], [6].

In this paper, we investigate the probability of having a giant component of order above a given fraction of nodes in both one and two dimensional spaces. The *order* of the giant component is defined as the number of nodes in the giant component. Based on this result one can derive the minimum transmission range at which a given fraction of nodes are connected with a high probability. The advantage of using this minimum transmission range, rather than a higher transmission range required for a connected network, is that both power consumption and interference can be reduced while

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meaningful services can still be provided. We also investigate the asymptotic behavior of one dimensional network having a giant component, which is distinctly different from two dimensional results in the literature.

The rest of this paper is organized as follows. Section II reviews related work. Section III describes the models and some notations. In section IV we derive a closed-form analytical formula for computing the probability of having a giant component of order above pn $(0 < p \leq 1)$ for $d = 1$. In section V, we investigate the asymptotic behavior of the giant component for $d = 1$. In section VI we derive an asymptotic analytical upper bound on the minimum transmission range at which the probability of having a giant component of order above qn $(0 < q < 1)$ tends to one as $n \to \infty$ for $d = 2$. Section VII concludes this paper.

II. Related work

The concept of the giant component has been extensively investigated in the literature for *Bernoulli random graphs*, and an analytical formula relating the giant component size and the average node degree[2] has been found [7]. However, the Bernoulli random graph is not suitable for modeling wireless multi-hop networks, hence, it is inappropriate to apply the results on the giant component size obtained from Bernoulli random graphs directly into wireless multi-hop networks.

In [5], Hekmat *et al.* investigated the giant component size in a log-normal shadowing environment, where a total of n nodes are randomly and uniformly distributed on a square and a link exists between two nodes if the power received at one node from the other node, as determined by the log-normal shadowing model [5], is greater than a given threshold. Based on the analytical results obtained in Bernoulli random graphs, the authors proposed an empirical formula relating the giant component size and the average node degree in random geometric graphs. In [8], Németh *et al.* investigated the giant component size by using a fractal propagation model where the probability of having a link between two nodes is determined by their Euclidean distance and two non-negative constants. They found that the giant component size can be characterized by a single parameter, viz., the average node degree. However, both papers investigated the giant component size empirically rather than analytically.

In [6], Raghavan *et al.* investigated the phase transition behaviors for the emergence of a giant component in wireless sensor networks with the same network model as in this paper. The authors proposed an empirical formula for the critical transmission range at which the network has a giant component with a high probability, and they showed that the critical range is approximately inversely proportional to $\sqrt{n}$.

In [9], Bradonjić *et al.* studied the giant component using a network model based on a *geographical threshold graph* which is almost the same as the random geometric graph except that the link existence between any two nodes is determined not only by the Euclidean distance between them but also by the node weights assigned for them. The authors derived the conditions for the absence and existence of a giant component.

[2]Giant component size is the ratio of the number of nodes in the giant component to the total number of nodes.

III. Preliminaries

A. Network model

Generally, a wireless multi-hop network can be represented by an undirected graph $G(V, E)$ with a set of vertices V and a set of edges E. Each vertex of the set V uniquely represents a node and each edge of the set E uniquely represents a wireless link, and vice versa. In this paper, we model wireless multi-hop networks by widely used *random geometric graphs* [1], [10]. Typically, a random geometric graph is defined as follows:

Definition 1 ([10]). Let X_1, X_2, ..., X_n be n points which are independently, randomly and uniformly distributed in $[0, 1]^d$ $(d = 1,\ 2)$; let $\mathcal{X}_n = \{X_1, X_2, ..., X_n\}$. Let $r = r(n)$ be a real number in $(0, 1)$. A random geometric graph $G_d(\mathcal{X}_n, r)$ is an undirected graph having $\mathcal{X}_n$ as its vertex set, and with an edge connecting each pair of vertices X_i and X_j in $\mathcal{X}_n$ if $||X_i - X_j|| \leq r$, where norm $||\cdot||$ means the Euclidean norm.

B. Notation

The *degree* of a node, is the number of its neighbors directly connected to it. A node of degree zero is called an *isolated node*. In what follows, the order of the giant component in a graph G is denoted by $L(G)$.

Throughout this paper, let c be any fixed real number. It is clear that e^{-c} can be any fixed positive real number.

IV. Giant component in one-dimensional networks

In this section, we derive a closed-form analytical formula for the probability of having a giant component of order above pn $(0.5 < p \leq 1)$ in one-dimensional space $(d = 1)$, denoted by $P(n, r, p)$. It is given in the following theorem.

Theorem 1. *Consider a random geometric graph $G_1(\mathcal{X}_n, r)$ in $\Re^d$ $(d = 1)$. Let p be a fixed real number in $(0.5, 1]$. Let $P(n, r, p)$ be the probability that $G_1(\mathcal{X}_n, r)$ has a giant component of order above pn. Then,*

$$\begin{aligned} & P(n,r,p) \\ = & \sum_{i=\lceil np\rceil}^{n-1}\Bigg[2\sum_{j=0}^{\min\{i,\lfloor\frac{1}{r}\rfloor-1\}}\binom{i}{j}(-1)^j(1-(j+1)r)^n \\ & +(n-i+1) \\ & \cdot\sum_{j=0}^{\min\{i-1,\lfloor\frac{1}{r}\rfloor-2\}}\binom{i-1}{j}(-1)^j(1-(j+2)r)^n\Bigg] \\ & +\sum_{j=0}^{\min\{n-1,\lfloor\frac{1}{r}\rfloor\}}\binom{n-1}{j}(-1)^j(1-jr)^n. \qquad (1) \end{aligned}$$

In order to prove Theorem 1, the following three lemmas, viz., Lemma 1, Lemma 2 and Lemma 3, are needed.

Lemma 1 (Lemma 1 in [11]). *Let $[x, x+y]$ be a subinterval of length y on a unit interval $[0, 1]$. Let two of k given vertices be*

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placed on the borders of this subinterval. Let $P(k,y,r)$ be the probability that the remaining $k-2$ vertices placed randomly and uniformly on $[0,1]$ are inside $[x,x+y]$ and the k vertices form a connected subgraph of length y. Then

$$P(k,y,r) = \sum_{j=0}^{\min\{k-1,\lfloor y/r\rfloor\}} \binom{k-1}{j}(-1)^j(y-jr)^{k-2}. \quad (2)$$

Lemma 2. *Let $\mathcal{F}_k^1$ denote the event that there exists a connected subgraph with exactly k ($k<n$) vertices in $G_1(\mathcal{X}_n,r)$ and both endpoints of this subgraph are not within distance r from the borders of the unit interval, and none of the remaining $n-k$ vertices is connected to this subgraph. Then*

$$Pr\{\mathcal{F}_k^1\} = (n-k+1) \cdot \sum_{j=0}^{\min\{k-1,\lfloor\frac{1}{r}\rfloor-2\}} \binom{k-1}{j}(-1)^j(1-(j+2)r)^n. \quad (3)$$

Proof: There are $\binom{n}{k}$ distinct combinations for selecting k vertices from a total of n vertices. Consider a subinterval $[x,x+y]$, where x and $x+y$ are the positions of the left border and the right border respectively. For any given k vertices, there are $\binom{k}{2}$ different combinations for selecting 2 vertices as endpoints, and two permutations of each selection in placing them on the borders of $[x,x+y]$; the probability that the remaining $k-2$ vertices placed randomly and uniformly on $[0,1]$ are inside $[x,x+y]$ *and* the k vertices form a connected subgraph is given by Eq. 2. Then

$$\begin{aligned} Pr\{\mathcal{F}_k^1\} &= \binom{n}{k}2\binom{k}{2}\int_0^{1-2r}\left[\int_r^{1-r-y}dx\right] P(k,y,r)(1-y-2r)^{n-k}dy \\ &= \binom{n}{k}2\binom{k}{2}\int_0^{1-2r} P(k,y,r)(1-y-2r)^{n-k+1}dy. \quad (4)\end{aligned}$$

Dividing the integration interval $[0,1-2r]$ into subintervals, i.e., $[0,r)$, $[r,2r)$,..., and using Lemma 1, Eq. 4 becomes

$$\begin{aligned} Pr\{\mathcal{F}_k^1\} &= \binom{n}{k}2\binom{k}{2}\sum_{i=0}^{\lfloor\frac{1}{r}\rfloor-3}\int_{ir}^{(i+1)r}(1-y-2r)^{n-k+1} \left[\sum_{j=0}^{L(i)}\binom{k-1}{j}(-1)^j(y-jr)^{k-2}\right]dy \\ &+\binom{n}{k}2\binom{k}{2}\int_{\lfloor\frac{1}{r}\rfloor r-2r}^{1-2r}(1-y-2r)^{n-k+1} \left[\sum_{j=0}^{L(i)}\binom{k-1}{j}(-1)^j(y-jr)^{k-2}\right]dy, \quad (5)\end{aligned}$$

where $L(i)=\min\{k-1,i\}$. Then taking the inner sums outside the integrals, and letting $L'=\min\{k-1,\lfloor 1/r\rfloor-2\}$, Eq. 5 becomes

$$\begin{aligned} Pr\{\mathcal{F}_k^1\} &= \binom{n}{k}2\binom{k}{2}\sum_{j=0}^{L'}\binom{k-1}{j}(-1)^j \cdot\left(\int_{jr}^{1-2r}(y-jr)^{k-2}(1-y-2r)^{n-k+1}dy\right) \\ &= \binom{n}{k}2\binom{k}{2}\sum_{j=0}^{L'}\binom{k-1}{j}(-1)^j \cdot(1-jr-2r)^n\left(\int_0^1 t^{k-2}(1-t)^{n-k+1}dt\right). \quad (6)\end{aligned}$$

Note that the integral on the right hand side of Eq. 6 is the *Beta Function*. Therefore, it follows

$$\int_0^1 t^{k-2}(1-t)^{n-k+1}dt = \frac{(k-2)!(n-k+1)!}{n!}. \quad (7)$$

Inserting Eq. 7 into Eq. 6, Eq. 3 can readily be obtained. ■

Lemma 3. *Let $\mathcal{F}_k^2$ denote the event that there exists a connected subgraph with exactly k ($k<n$) vertices in $G_1(\mathcal{X}_n,r)$ and the leftmost vertex of the subgraph is located within distance r from the left border of the unit interval and the remaining $n-k$ vertices are all located on the right side of this subgraph and none of them is connected to this subgraph. Then*

$$Pr\{\mathcal{F}_k^2\} = \sum_{j=0}^{\min\{k,\lfloor\frac{1}{r}\rfloor-1\}}\binom{k}{j}(-1)^j(1-(j+1)r)^n. \quad (8)$$

Proof: Refer to an extended version of this paper, [14], for the proof, which is omitted here due to space limitation. ■

Proof of Theorem 1: It is clear that

$$P(n,r,p) = \sum_{i=\lceil np\rceil}^{n} Pr\{L(G_1(\mathcal{X}_n,r))=i\}.$$

$P(n,r,p)$ can then be obtained combining two calculations: (1) $Pr\{L(G_1(\mathcal{X}_n,r))=n\}$. This probability is actually the probability that the network $G_1(\mathcal{X}_n,r)$ is connected, denoted as P_{con}. It is given by Corollary 1 in [11] as

$$P_{con} = \sum_{j=0}^{\min\{n-1,\lfloor 1/r\rfloor\}}\binom{n-1}{j}(-1)^j(1-jr)^n. \quad (9)$$

(2) $Pr\{L(G_1(\mathcal{X}_n,r))=i\}$ for $\lceil np\rceil \le i < n$. This probability is equal to the probability that there exists a connected subgraph with exactly i ($\lceil np\rceil \le i < n$) vertices and none of the remaining $n-i$ vertices is connected to this subgraph. There are three different sub-cases in which this event may happen, i.e., (a) Both endpoints of this subgraph are not within a distance r from the borders of the unit interval. Lemma 2 provides the probability for this case. (b) The left (right) endpoint of this subgraph is within a distance r from the left (right) border of the unit interval. Lemma 3 provides the probability for this case. (c) Both endpoints of this subgraph

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are within distance r from the borders of the unit interval. This can only happen when $i = n$, but here we require $i < n$. Hence, the probability of this case is zero.

Note that in Theorem 1 it is required that $p > 0.5$. In Lemmas 2 and 3, the connected subgraph with exactly i vertices is not necessarily the largest connected subgraph which we are interested in. To ensure that the connected subgraph with exactly i vertices is the largest connected subgraph, it suffices that we restrict $p > 0.5$.

Finally, the probability $P(n, r, p)$ can be readily derived as

$$P(n, r, p) = \sum_{i=\lceil np \rceil}^{n-1} \left[2Pr\{\mathcal{F}_i^2\} + Pr\{\mathcal{F}_i^1\}\right] + P_{con}. \quad (10)$$

Substituting Eq. 3, Eq. 8 and Eq. 9 into Eq. 10, we can readily obtain Eq. 1. ∎

V. Asymptotic behavior of the giant component in one-dimensional networks

In this section, we investigate the asymptotic behavior of the giant component by studying the probability $P(n, r, p)$ when $n \to \infty$. We assume that $r = r(n)$ is a function of n.

Based on Theorem 1, the following three theorems can be obtained.

Theorem 2. *Adopt the same assumptions as in Theorem 1. If $n(1-r)^n \to e^{-c}$ as $n \to \infty$, or equivalently if $r = \frac{\log n + c + o(1)}{n}$,*

$$\lim_{n\to\infty} P(n, r, p) = e^{-pe^{-c}} + (1-p)e^{-c}e^{-pe^{-c}}.$$

In addition, almost surely there are only components whose orders are $\Theta(n)$.

Proof: Refer to [14]. ∎

Theorem 3. *Adopt the same assumptions as in Theorem 1. If $n(1-2r)^n \to 0$ and $n(1-r)^n \to \infty$ as $n \to \infty$, then $\lim_{n\to\infty} P(n, r, p) = 0$. In addition, there is almost surely no isolated vertex or finite component as $n \to \infty$.*

Proof: Refer to [14]. ∎

Theorem 4. *Adopt the same assumptions as in Theorem 1. If $n(1-2r)^n \to e^{-c}$ as $n \to \infty$, or equivalently if $r = \frac{\log n + c + o(1)}{2n}$, then $\lim_{n\to\infty} P(n, r, p) = 0$. In addition, there is almost surely a non-vanishing probability of having isolated vertices and finite components.*

Proof: Refer to [14]. ∎

Remark. The above results reveal an interesting finding that is different from higher-dimensional networks (e.g. $d = 2$). As we shall see, for $d \geq 2$, if $nr^d \to \infty$ as $n \to \infty$, almost surely the network only consists of isolated nodes and a unique giant component as $n \to \infty$ [12]. In addition, when the last isolated node vanishes, the network becomes connected almost surely [1], [10]. However, for $d = 1$, there may be multiple giant components (Theorem 2); and when the last isolated vertex vanishes, the network may still be disconnected (Theorem 3).

VI. Giant component in two-dimensional networks

For a two-dimensional wireless multi-hop network, it is difficult to obtain an analytical formula comparable to the one-dimensional case. In this section, we derive an asymptotic analytical *upper bound* on the minimum transmission range at which the probability of having a giant component of order above qn tends to one as $n \to \infty$, where q is any fixed real number in $(0, 1)$. In what follows, let r_q denote this minimum transmission range.

Our main results for the upper bound on the minimum transmission range r_q is given in the following theorem.

Theorem 5. *Consider $G_2(\mathcal{X}_n, r)$ in $\Re^2$. Let q be any fixed real number within $(0, 1)$. Let c be any fixed real number. Let $f(n)$ be a function of n satisfying*

$$f(n) > 0, \quad \lim_{n\to\infty} f(n) = \infty \quad \textit{and} \quad \lim_{n\to\infty} \frac{f(n)}{\log n} = 0. \quad (11)$$

If $\pi r^2 = \frac{f(n)+c}{n}$, then, $\lim_{n\to\infty} P\{L(G_2(\mathcal{X}_n, r)) \geq qn\} = 1$.

Remark. At the first glance, the above result appears abnormal as it suggests the probability of having a giant component of order qn as $n \to \infty$ is independent of q. Here we offer the following intuitive explanation for the result. It is well known that the width of the phase transition region from an almost disconnected network to an almost connected network approaches zero as $n \to \infty$ [13]. This means at large n, the probability of having a connected network as a function of r is almost like a step function such that at a certain value of r (termed the critical transmission range), a tiny variation in r causes a large change in the probability. The above result indicates that the same phenomenon may also be observed for the probability of having a giant component. Possibly, a refined set of conditions of $f(n)$ in Eq. 11 could allow distinguishing the different values of q.

In order to prove Theorem 5, we shall use Poissonization and De-Poissonization techniques [10]. Let $\{X_1, X_2, X_3, ...\}$ be a series of points which are randomly, independently and uniformly distributed in $[0, 1]^2$ in $\Re^2$. Given $\lambda > 0$, let N_λ be a Poisson random variable with mean λ, independent of $\{X_1, X_2, X_3, ...\}$, and let $\mathcal{P}_\lambda := \{X_1, X_2, ..., X_{N_\lambda}\}$. Define $G_2(\mathcal{P}_\lambda, r)$ as an undirected graph having $\mathcal{P}_\lambda$ as its vertex set, and with an edge connecting each pair of vertices X_i and X_j in $\mathcal{P}_\lambda$ if $||X_i - X_j|| \leq r$. With $\mathcal{P}_\lambda$ and $\mathcal{X}_n$ being related, we shall start by proving result about $G_2(\mathcal{P}_\lambda, r)$ (i.e. the following Lemma 4), and then deduce result about $G_2(\mathcal{X}_n, r)$ from this.

Lemma 4. *Consider $G_2(\mathcal{P}_{m(n)}, r)$ in $\Re^2$, where $m(n) = \lfloor n - n^{\frac{3}{4}} \rfloor$. Let q be any fixed real number within $(0, 1)$. Let c be any fixed real number. Let $f(n)$ be a function of n satisfying Eq. 11. If $\pi r^2 = \frac{f(n)+c}{n}$, then $\lim_{n\to\infty} P\{L(G_2(\mathcal{P}_{m(n)}, r)) \geq qn\} = 1$.*

Proof: Refer to [14]. ∎

Proof of Theorem 5: Let $m(n) = \lfloor n - n^{\frac{3}{4}} \rfloor$. Define

$(\mathcal{P}_{m(n)}, r)$ and $Y(\mathcal{X}_n, r)$ as

$$\begin{aligned} Y(\mathcal{P}_{m(n)}, r) &:= P\{L(G_2(\mathcal{P}_{m(n)}, r)) < qn\} \\ Y(\mathcal{X}_n, r) &:= P\{L(G_2(\mathcal{X}_n, r)) < qn\}. \end{aligned}$$

Define J_n as $J_n := \{j : j \in \mathbb{N},\ \lfloor n - 2n^{\frac{3}{4}} \rfloor \le j \le n\}$.

By Chebyshev's inequality, we have

$$\begin{aligned} Y(\mathcal{P}_{m(n)}, r) &= \sum_{j=0}^{\infty} \frac{(m(n))^j}{j!} e^{-m(n)} Y(\mathcal{X}_j, r) \\ &= \sum_{j \in J_n} \frac{(m(n))^j}{j!} e^{-m(n)} Y(\mathcal{X}_j, r) + o(1),\ as\ n \to \infty. \quad (12) \end{aligned}$$

Let $\mathcal{E}(\mathcal{X}_n, \mathcal{X}_j)$ denote the event that all nodes in $(\mathcal{X}_n \setminus \mathcal{X}_j)$ are isolated in $G_2(\mathcal{X}_n, r)$. Then, for fixed r, any fixed $q \in (0, 1)$, and any $j \in J_n$, it can be obtained that

$$\begin{aligned} Y(\mathcal{X}_n, r) &\le P\{\mathcal{E}(\mathcal{X}_n, \mathcal{X}_j)\} + Y(\mathcal{X}_j, r) \\ &\sim \left((1 - \pi r^2)^{n-1}\right)^{n-j} + Y(\mathcal{X}_j, r) \\ &\le \left(\frac{e^{-c}}{e^{f(n)}}\right)^{n-j} + Y(\mathcal{X}_j, r) \\ &= o(1) + Y(\mathcal{X}_j, r), \quad as \quad n \to \infty. \quad (13) \end{aligned}$$

Substituting Eq. 13 into Eq. 12, it can be obtained that

$$\begin{aligned} & Y(\mathcal{P}_{m(n)}, r) \\ \ge\ & \sum_{j \in J_n} \frac{(m(n))^j}{j!} e^{-m(n)} (Y(\mathcal{X}_n, r) - o(1)) + o(1) \\ =\ & Y(\mathcal{X}_n, r) \sum_{j \in J_n} \frac{(m(n))^j}{j!} e^{-m(n)} + o(1) \\ =\ & Y(\mathcal{X}_n, r) + o(1), \quad as \quad n \to \infty. \quad (14) \end{aligned}$$

Because $Y(\mathcal{P}_{m(n)}, r) = o(1)$ as $n \to \infty$ by Lemma 4, from Eq. 14, we have

$$o(1) \ge Y(\mathcal{X}_n, r) + o(1), \quad as \quad n \to \infty,$$

which yields

$$P\{L(G_2(\mathcal{X}_n, r)) < qn\} = Y(\mathcal{X}_n, r) = o(1), \quad as \quad n \to \infty.$$

The results follows immediately. ■

Remark. Let r_1 denote the minimum transmission range above which a network is connected with probability one as $n \to \infty$. By Theorem 2.1 of [1], $\sqrt{\frac{\log n + c'}{n\pi}}$ is a lower bound of r_1 where c' is any fixed real number. By Theorem 5, $\sqrt{\frac{f(n)+c}{n\pi}}$ is an upper bound of r_q. Hence, we have

$$\lim_{n\to\infty} \frac{r_q}{r_1} \le \lim_{n\to\infty} \frac{\sqrt{\frac{f(n)+c}{n\pi}}}{\sqrt{\frac{\log n + c'}{n\pi}}} = \lim_{n\to\infty} \sqrt{\frac{f(n) + c}{\log n + c'}} = 0.$$

The implication of the above result is that when $n \to \infty$, the transmission range required for having a giant component is vanishingly small compared to the transmission range for a connected network. Therefore, in a large scale network, a significant energy saving can be achieved by requiring most nodes, instead of all nodes, to be connected. Furthermore, in a network where almost (but not) all nodes are connected, a large leap in transmission range may be required to connect the remaining few nodes and the transmission range required for a large scale network to be connected is dominated by these few nodes, i.e., rare events.

VII. Conclusion

In this paper, we investigated the order of the giant component in wireless multi-hop networks. In one dimensional networks, we derived a closed-form formula for calculating the probability $P(n, r, p)$ that a network has a giant component of order above pn with any fixed $0.5 < p \le 1$. We also studied the asymptotic behavior of the derived analytical result as $n \to \infty$. Interesting results are found on the asymptotic behavior of one dimensional network having a giant component which is distinctly different from two dimensional counterpart. In two dimensional networks, we derived an asymptotic analytical upper bound on the minimum transmission range r_q. Based on the result, we further showed that $\frac{r_q}{r_1} \to 0$ as $n \to \infty$. This indicates that a significant energy saving may be achieved if we only require a giant component rather than a connected network, especially for a network with a large number of nodes.

References

[1] P. Gupta and P. Kumar, "Critical power for asymptotic connectivity in wireless networks," *Stochastic Analysis, Control, Optimization and Applications*, pp. 547–566, 1998.

[2] C. Bettstetter, "On the Minimum Node Degree and Connectivity of a Wireless Multihop Network," in *3rd ACM International Symposium on Mobile Ad Hoc Networking and Computing*, Lausanne, 2002, pp. 80–91.

[3] F. Xue and P. Kumar, "The number of neighbors needed for connectivity of wireless networks," *Wireless Networks*, vol. 10, no. 2, pp. 169–181, 2004.

[4] P. Santi and D. M. Blough, "The Critical Transimitting Range for Connectivity in Sparse Wireless Ad Hoc Networks," *IEEE Transactions on Mobile Computing*, vol. 2, no. 1, pp. 25–39, 2003.

[5] R. Hekmat and P. V. Mieghem, "Connectivity in Wireless Ad-hoc Networks with a Log-normal Radio Model," *Mobile Networks and Applications*, vol. 11, no. 3, pp. 351–360, June, 2006.

[6] U. N. Raghavan, H. P. Thadakamalla, and S. Kumara, "Phase Transition and Connectivity in Distributed Wireless Sensor Networks," in *13th International Conference on Advanced Computing and Communications*, Coimbatore, India, December, 2005.

[7] B. Bollobas, *Random Graphs*. Academic Press, 1985.

[8] G. Németh and G. Vattay, "Giant Clusters in Random Ad Hoc Networks," *Physical Review E*, vol. 67, no. 3, March, 2003.

[9] M. Bradonjić, A. Hagberg, and A. G. Percus, "Giant Component and Connectivity in Geographical Threshold Graphs," in *Algorithms and Models for the Web-Graph (WAW 2007)*, San Diego, CA USA, December, 2007, pp. 209–216.

[10] M. Penrose, *Random Geometric Graphs*, 1st ed. Oxford: Oxford University Press, 2003.

[11] E. Godehardt and J. Jaworski, "On the Connectivity of a Random Interval Graph," *Random Structures and Algorithms*, vol. 9, no. 1 and 2, pp. 137–161, 1996.

[12] R. Meester and R. Roy, *Continuum Percolation*. Cambridge, UK: Cambridge University Press, 1996.

[13] G. Han and A. M. Makowski, "Poisson convergence can yield very sharp transitions in geometric random graphs," in *the Inaugural Workshop, Information Theory and Applications*, San Diego (CA), February, 2006.

[14] Online, "http://www.ee.usyd.edu.au/∼guoqiang/papers/Ta09The.pdf".