Proceedings of the
46th IEEE Conference on Decision and Control
New Orleans, LA, USA, Dec. 12-14, 2007

Verifying stabilizing controllers via closed-loop noisy data: MIMO case

Arvin Dehghani, Brian D. O. Anderson, Alexander Lanzon, Andrea Lecchini-Visintini

***Abstract*— This article introduces novel tests which utilize a limited amount of experimental and possibly noisy data obtained from a stable closed-loop system, i.e. an interconnection of an existing known stabilizing controller and an unknown plant, to infer if the introduction of a prospective controller will stabilize the unknown plant. This extends our earlier results to include the MIMO systems.**

Index Terms*— Multiple Model Adaptive Control, Robust Control, Iterative Identification and Control.

I. INTRODUCTION

ADAPTIVE control algorithms would normally be expected to include a mechanism for ensuring that the introduction of a new controller will always yield a stable closed-loop. Yet, many state-of-the-art adaptive control design methodologies do not explicitly rule out the possibility of inserting a destabilizing controller in the closed-loop [1].

A renowned form of adaptive control is the iterative control and re-identification approach which emerged from the understanding of the interplay between identification and control design [2], [3], [4], [5], [6], [7]. The worst problem that this approach can engender is the so-called transient instability problem [8]. Here there is the possibility that the controller connected to the unknown plant at any particular time and frozen thereafter in combination with the plant provides an unstable closed loop. This happens partly because it is not always straightforward to accurately predict the new closed-loop transfer function that will result from changing a controller from one known controller to another known controller, when the first closed-loop transfer function is approximately known. All algorithms based on iterative identification and controller redesign need to include measures to avoid this sort of problem; if not, the new controller may *destabilize* the unknown plant [9], [10], [11].

Another different form of adaptive control is Multiple Model Adaptive Control (MMAC), see e.g. [12], [13], [14], [15], [9], [1], in which—for an unknown (frequently but not always assumed linear) plant belonging to a not usually finite set—a finite number of switchings among the controllers (nearly always assumed to be linear) of a given set takes place until a satisfactory one is obtained. It is assumed that each plant in the plant set will be satisfactorily controlled by at least one of the controllers. The potential problem with the MMAC approach, analogous to that with iterative identification and control, is that model-plant mismatch in the closed-loop may cause a new controller, chosen on the basis of its suitability for use with the current model, to actually *destabilize* the unknown true plant [1].

The remedy could be the Unfalsified Adaptive Control approach of [16], see also the references therein. This data-driven approach does not require explicit identification of a plant model and exploits information in the real-time measurement data to prospectively evaluate suitability, including closed-loop stability, and performance levels of all candidate controllers simultaneously *before* insertion of any one of them into the actual closed-loop to replace the existing controller. Nevertheless, it has been recently shown in [17] that the unfalsified adaptive control approach unfortunately gives no guarantee of protection against inserting a *destabilizing* controller in the closed-loop; moreover, such a destabilizing controller can remain in the loop for a long period of time resulting in very large closed-loop signals; for the example of [17] a maximum value of 1.228×10^6 was recorded for the plant input signal $u(t)$ when the the reference signal $r(t)$ was a sinusoid of magnitude 1. Indeed, one cannot even put a global upper bound on the time during which the destabilizing controller is attached.

There exist iterative control design methods which utilize the closed-loop data collected from an existing feedback interconnection in order to replace the current controller with a better performing controller, see e.g. [18], [19], [20] and the references therein. However, the existing data-based stability tests to ascertain internal stability with the new controller before its implementation in the closed-loop are based either on the identification of a parametric full order model of the current closed-loop transfer function or on the full estimation of frequency bounds on the magnitude of the current closed-loop transfer functions [21], [22], [23], [24].

This paper extends our earlier results [25] to MIMO systems and proposes novel tests using a *limited amount of experimental data, such as noisy frequency response data*, obtained with an existing known stabilizing controller to verify that the introduction of a new controller will stabilize the plant. These tests exploit phase information of the current closed-loop data to assess stability conditions, analogously to the Nyquist stability criterion, to ascertain closed-loop stability with the new controller. The proposed tests offer *significant robustness* to noise.

Arvin Dehghani is with the Department of Information Engineering, Research School of Information Sciences and Engineering, The Australian National University, Canberra, ACT 0200 Australia.

Brian D. O. Anderson is with the Department of Information Engineering, Research School of Information Sciences and Engineering, The Australian National University, Canberra, ACT 0200 Australia, and also with the National ICT Australia Ltd., Locked Bag 8001, ACT 2601, Australia.

Alexander Lanzon is with the Control Systems Centre, School of Electrical and Electronic Engineering, University of Manchester, Sackville Street, Manchester M60 1QD, UK.

Andrea Lecchini-Visintini is with the Department of Engineering, University of Leicester, LE17RH, UK.

Corresponding author: Arvin Dehghani.

Section II collects the required definitions and notations and is followed by Section III where a framework for implementing controllers in a specialized way is established, and preliminary results based on phase information are presented which define the experimental setting for stability tests. This leads to the development of the proposed novel closed-loop validation tests in Section IV. Section VI contains concluding remarks and future research direction.

II. Preliminaries

Let $\mathscr{H}_\infty$ denote the space of functions bounded and analytic in the open right-half complex plane and the same function spaces with prefix $\mathscr{R}$ be their real-rational proper subspaces. The transfer function of the plant P belongs to $\mathscr{R}^{m\times p}$. The transfer function of the controller is denoted by C and is assumed to belong to $\mathscr{R}^{p\times m}$. The eigenvalues of a matrix $A \in \mathbb{C}^{n\times m}$ are denoted by $\lambda_i(A)$, and its singular values by $\sigma_i(A)$, with the largest singular value $\overline{\sigma}(A)$ and the smallest singular value $\underline{\sigma}(A)$. The number $\operatorname{wno}(\cdot)$ denotes the winding number of the Nyquist diagram of a scalar transfer function, evaluated on a contour along the imaginary axis and indented to the right around any pure imaginary pole. Let also $G(j\omega)^* = G(-j\omega)^T$ denote the complex conjugate transpose of frequency-response function $G(j\omega)$ at each ω. What follows is a collection of internal stability results for the interconnection $[P, C]$ in Fig. 1, and definitions linked to coprime factor representation, see [26].

***Definition** 1:* The interconnection $[P, C]$ is "well-posed" if the transfer function matrix mapping $\left[\begin{smallmatrix} r \\ d \end{smallmatrix}\right]$ to $\left[\begin{smallmatrix} y \\ u \end{smallmatrix}\right]$ exists. Put another way, $[P, C]$ is well-posed if $(I - CP)^{-1} \in \mathscr{R}$.

Given such well-posedness, we can write

$$\begin{bmatrix} y \\ u \end{bmatrix} = \begin{bmatrix} P \\ I \end{bmatrix} (I - CP)^{-1} \begin{bmatrix} -C & I \end{bmatrix} \begin{bmatrix} r \\ d \end{bmatrix} = H(P, C) \begin{bmatrix} r \\ d \end{bmatrix}.$$

***Definition** 2:* The interconnection $[P, C]$ is said to be "internally stable" if it is well-posed and $H(P, C) \in \mathscr{RH}_\infty$.

***Definition** 3:* The ordered pair $\{N, M\}$, with $M, N \in \mathscr{RH}_\infty$, is a right-coprime factorization (*rcf*) of $P \in \mathscr{R}$ if M is invertible in $\mathscr{R}$, $P = NM^{-1}$, and N and M are right-coprime over $\mathscr{RH}_\infty$. Furthermore, the ordered pair $\{N, M\}$ is a normalized *rcf* of P if $\{N, M\}$ is a *rcf* of P and $M^*M + N^*N = I$.

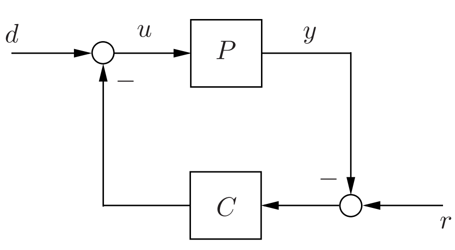

Fig. 1. Standard Feedback Configuration

***Definition** 4:* The ordered pair $\{\tilde{U}, \tilde{V}\}$, with $\tilde{U}, \tilde{V} \in \mathscr{RH}_\infty$, is a left-coprime factorization (*lcf*) of $C \in \mathscr{R}$ if $\tilde{V}$ is invertible in $\mathscr{R}$, $C = \tilde{V}^{-1}\tilde{U}$, and $\tilde{U}$ and $\tilde{V}$ are left-coprime over $\mathscr{RH}_\infty$. Furthermore, the ordered pair $\{\tilde{U}, \tilde{V}\}$ is a normalized *lcf* of C if $\{\tilde{U}, \tilde{V}\}$ is a *lcf* and $\tilde{V}\tilde{V}^* + \tilde{U}\tilde{U}^* = I$.

Let

$$G := \begin{bmatrix} N \\ M \end{bmatrix}, \qquad \tilde{K} := \begin{bmatrix} -\tilde{U} & \tilde{V} \end{bmatrix} \tag{1}$$

be the right graph symbol of P and the inverse left graph symbol of C respectively. Then the following results characterizes closed-loop stability in terms of coprime factorization.

***Theorem** 5:* [26, Proposition 1.9] Let G and $\tilde{K}$ be defined as in (1). Then the following are equivalent:

i. $[P, C]$ is internally stable;
ii. $(\tilde{K}G)^{-1} \in \mathscr{RH}_\infty$;
iii. $\det(\tilde{K}G)(j\omega) \neq 0 \ \forall\omega$ and $\operatorname{wno}\det(\tilde{K}G) = 0$.

III. Problem Formulation & Experimental Setting

The problem considered here is: given an unknown plant P, which is stabilized by a known controller C_0, and a limited amount of experimental data obtained with C_0, how can one verify—without actual insertion in the closed-loop—if the introduction of the new controller C_1 will stabilize the plant P? The following theorem defines the experimental framework for the stability tests proposed in this paper.

***Theorem** 6:* Let $[P, C_0]$ be internally stable. Let $C_0 = \tilde{V}_0^{-1}\tilde{U}_0$ and $C_1 = \tilde{V}_1^{-1}\tilde{U}_1$ be left coprime factorizations over $\mathscr{RH}_\infty$. Consider the configuration[1] in Fig. 2 and define the mapping $T : r \mapsto z$ to be

$$T = \begin{bmatrix} -\tilde{U}_1 & \tilde{V}_1 \end{bmatrix} \begin{bmatrix} P(I - C_0P)^{-1} \\ (I - C_0P)^{-1} \end{bmatrix} \tilde{V}_0^{-1} \tag{2}$$

and let $\arg$ denote the unwrapped phase[2]. Then the following are equivalent:

a) $[P, C_1]$ is internally stable;
b) $T^{-1} \in \mathscr{RH}_\infty$;
c) $\det T(j\omega) \neq 0 \ \forall\omega$ and $\operatorname{wno}\det T = 0$;
d) $\det T(j\omega) \neq 0 \ \forall\omega$ and $\arg\det T(j\infty) = \arg\det T(j0)$.

Proof: See [25]. ■

Note that the plant is unknown and hence one cannot explicitly construct T in closed form. However, the stable mapping resulting from $T : r \to z$ can be studied in a safe—in which no instability can occur—experimental framework, as shown in Fig. 2. The required properties of T can be experimentally inferred by utilizing the reference signal r

[1] The specialized implementation of the controller C_0 depicted in Fig. 2 is referred to as the "observer-form" implementation; see [25], [27], [26] for details.

[2] We refer to the *unwrapped* phase of a transfer function as the phase of the frequency response when it is in the form of a continuous function of the frequency [28].

and the constructed output signal z (computed as a filtered version of the measured signals $\begin{bmatrix} y \\ u \end{bmatrix}$ via $\tilde{K}_1$). The following section introduces data-based stability tests on the basis of experimental setting defined in this section and with the aim of verifying condition (d) in Theorem 6.

Before that, we shall present a form of the specialized implementation of the controller depicted in Fig. 2 by introducing an alternative controller implementation in the following subsection.

A. Alternative Implementation of Controllers

As discussed earlier, we adopt the observer-form implementation of the controller depicted in Fig. 2. One may be concerned in having to split up the physical controller in two coprime factors before injecting the reference signal r. The following implementation will circumvent the concerns.

Let $\begin{bmatrix} X \\ Y \end{bmatrix}$ be a right inverse of $\begin{bmatrix} -\tilde{U}_0 & \tilde{V}_0 \end{bmatrix}$. In other words, let $P_0 = XY^{-1}$ be some plant that stabilizes $C_0 = \tilde{V}_0^{-1}\tilde{U}_0$ and satisfies the corresponding Bezout identity. Note that P_0 does not have to be an estimate of P. Then, it is easy to see in Fig. 3 that

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix} r \quad \text{and} \quad \begin{bmatrix} y \\ u \end{bmatrix} = H(P, C_0) \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}.$$

Because $H(P, C_0) = G(\tilde{K}_0 G)^{-1}\tilde{K}_0$, it easily follows that

$$\begin{bmatrix} y \\ u \end{bmatrix} = G(\tilde{K}_0 G)^{-1} r$$

which is the mapping from r to $\begin{bmatrix} y \\ u \end{bmatrix}$ in Fig. 2. Note that the requirement $\begin{bmatrix} -\tilde{U}_0 & \tilde{V}_0 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = I$ can be relaxed to $\begin{bmatrix} -\tilde{U}_0 & \tilde{V}_0 \end{bmatrix}\begin{bmatrix} X \\ Y \end{bmatrix} = Z$ where Z is a unit in $\mathscr{RH}_\infty$ since the transfer function from r to $\begin{bmatrix} y \\ u \end{bmatrix}$ then becomes

$$\begin{bmatrix} y \\ u \end{bmatrix} = G(\tilde{K}_0 G)^{-1} Z r = G(\hat{\tilde{K}}_0 G)^{-1} r$$

with $\hat{\tilde{K}} = Z^{-1}\tilde{K}_0$, i.e. only changing the particular coprime factor representation of the controller.

An interesting observation is that there are several plants P_0 that stabilize C_0 and furthermore there are several coprime factorizations of $P_0 = Y^{-1}X$. This choice can be used in the synthesis of X and Y to determine the frequency and bandwidth characteristics of the physical reference signals r_1 and r_2.

Next, we shall propose our stability tests.

IV. Proposed Data-based Stability Tests

Using the experimental setting of Section III, we will now present data-based stability tests to verify condition (d) in Theorem 6. Let the following assumptions hold.

***Assumption** 7:* The factors $\tilde{V}_0$ and $\tilde{V}_1$ are such that $\tilde{V}_0(j\infty) = \tilde{V}_1(j\infty) = I$.

***Assumption** 8:* The transfer functions PC_0 and PC_1 are strictly proper.

Notice that the transfer function T can be written as

$$T = \tilde{V}_1(I - C_1 P)(I - C_0 P)^{-1}\tilde{V}_0^{-1} \tag{3}$$

for which under Assumptions[3] 7 and 8 we have

$$\det T(j\infty) = 1\,.$$

Thus, $\det T(j\infty)$ is strictly positive and known and will be used as a datum for the verification of condition (d) in Theorem 6. Condition (d) in Theorem 6 can be verified by using more sophisticated identification techniques. In principle, one could inject a white noise signal r or a full sine sweep, measure the corresponding output z and compute the *full* frequency response for T. However, this is not desirable on the grounds of complexity and hence one needs to determine an alternative sensible experiment. Two key points that have to be taken account of in designing the experiment are, first, that there is no need to estimate the full frequency response of T but what is instead needed is to measure its frequency response up to a certain finite frequency ω_0, as later explained. Second, the measurement can tolerate significant error, as its purpose is simply to facilitate computation of a certain integer (as both $T(j0)$ and $T(j\infty)$ are real) phase change. The following result, obtained by exploiting the structure of the transfer function T, paves the way for introduction of a mechanism to estimate ω_0.

[3]It is evident that Assumption 7 is without loss of generality and Assumption 8 captures a typical practical situation.

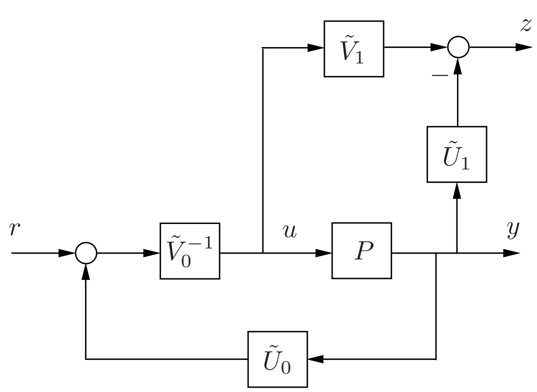

Fig. 2. Experimental setting

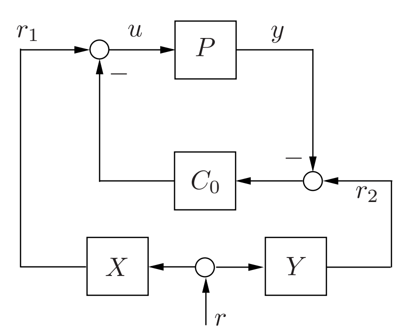

Fig. 3. Alternative experimental setting

***Lemma** 9:* Let the suppositions of Theorem 6 hold. Then the transfer function T has the following expression:

$$T = I + T' \tag{4}$$

$$T' = \begin{bmatrix} -(\tilde{U}_1 - \tilde{U}_0) & (\tilde{V}_1 - \tilde{V}_0) \end{bmatrix} \begin{bmatrix} P(I - C_0 P)^{-1} \\ (I - C_0 P)^{-1} \end{bmatrix} \tilde{V}_0^{-1} \tag{5}$$

Proof: The expression for T' is derived as follows

$$\begin{aligned} T' &= \tilde{K}_1 G (\tilde{K}_0 G)^{-1} - I \\ &= (\tilde{K}_1 - \tilde{K}_0) G (\tilde{K}_0 G)^{-1} \\ &= \begin{bmatrix} -(\tilde{U}_1 - \tilde{U}_0) & (\tilde{V}_1 - \tilde{V}_0) \end{bmatrix} \begin{bmatrix} P \\ I \end{bmatrix} (I - C_0 P)^{-1} \tilde{V}_0^{-1} \end{aligned}$$

The last expression coincides with (5). ■

The expression for the transfer function T presented in Lemma 9 shows that T is the sum of a known term (i.e. I) and a term which, under Assumptions 7 and 8, is strictly proper. Thus, it can be expected that measuring the frequency response of T up to a frequency, ω_0 say, where the response of T' has nearly vanished is enough to characterize the full frequency response of T in the direction of the following results.

***Theorem** 10:* Suppose the hypothesis of Theorem 6 and Assumption 7 and 8 hold. Define $T' \in \mathscr{RH}_\infty^{n\times n}$ by $T' = T - I$ as in Lemma 9. Let $\omega_0 \in [0, \infty)$ be a frequency such that

$$\rho(T'(j\omega)) < \sin(\frac{\pi}{2n}) \quad \forall \omega \geq \omega_0 \tag{6}$$

Then the condition

$$\begin{cases} \det T(j\omega) \neq 0 \ \forall \omega \in [0, \infty) & \text{and} \\ \pi \left[\frac{\arg\det T(j\omega_0)}{\pi} \right] = \arg\det T(j0) \,, \end{cases} \tag{7}$$

where $[\cdot]$ denotes the closest integer, is equivalent to condition (d) in Theorem 6.

Proof: Observe that $|\lambda_i(T')| \leq \rho(T')$ and that

$$\det T = \prod_{i=1}^{n} \lambda_i(I + T') = \prod_{i=1}^{n} [1 + \lambda_i(T')]$$

Since inequality (6) holds, $[1 + \lambda_i(T')]\,(j\omega)$ lies in the interior of a circle of center 1 and radius $\xi = \sin(\pi/2n)$ $\forall \omega \geq \omega_0$. Hence the angle θ depicted in Fig. 4 is precisely $\pi/2n$. Consequently, the angle of the complex number $1 + \lambda_i(T')$ for each i and each $\omega \in [\omega_0, \infty)$ lies in $\left(-\frac{\pi}{2n}, \frac{\pi}{2n}\right)$. Thus the angle of the complex number $\det T(j\omega)$ is in $(-\pi/2, \pi/2)$ for each $\omega \in [\omega_0, \infty)$ and $\det T$ can never complete a contour around the origin $\forall \omega \in [\omega_0, \infty)$. Since $\det T(j\omega)$ is a continuous function of frequency and is equal to unity at infinity frequency,

$$\pi \left[\frac{\arg\det T(j\omega_0)}{\pi} \right] = \arg\det T(j\infty) \,.$$

■

The following theorem is advanced for the strict MIMO case and provides a relaxation on the condition in (6) resulting in less experimental effort.

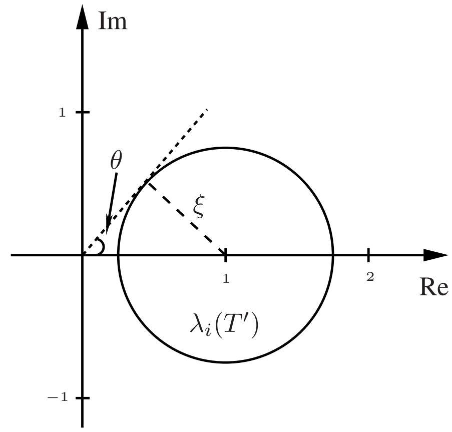

Fig. 4. Graphical representation of the condition on $\lambda_i(T')$. The angle θ is of the form $\pi/2n$.

***Theorem** 11:* Adopt the hypothesis of Theorem 6 and Assumption 7 and 8, and define $T' \in \mathscr{RH}_\infty^{n\times n}$, $n \geq 2$, by $T' = T - I$ as in Lemma 9. Let $\omega_0 \in [0, \infty)$ be a frequency such that

$$\rho(T'(j\omega)) < \sin(\frac{\pi}{n}) \quad \forall \omega \geq \omega_0 \tag{8}$$

Then the condition

$$\det T(j\omega) \neq 0 \ \forall \omega \ \text{and} \ 2\pi \left[\frac{\arg\det T(j\omega_0)}{2\pi} \right] = \arg\det T(j0) \tag{9}$$

with $[\cdot]$ denoting the closest integer is equivalent to condition (d) in Theorem 6.

Proof: The proof is similar to that of Theorem 10. ■

Evidently, checking the conditions in (6) or (8) requires finding the spectral radius of T which is not available. However, at each frequency one can use the inequality

$$\overline{\sigma}(T) \leq \|T\|_F = \sqrt{\sum_{i,j} |T_{ij}|^2}$$

along with

$$\rho(T(j\omega)) \leq \overline{\sigma}[T(j\omega)]$$

to find an upper bound on the eigenvalues of $T(j\omega)$. Alternatively the inequality

$$\overline{\sigma}[T(j\omega)] \leq \sqrt{n}\, \|T(j\omega)\|_1$$

can be utilized at each frequency. The reader is referred to [29, Chapter 2] for useful matrix norm inequalities.

The theorems presented in this section outline experimental tests to assess stability of $[P, C_1]$ before inserting controller C_1 into the closed-loop. Theorems 10 and 11 state necessary and sufficient conditions for the stability of $[P, C_1]$ and imply the estimation of the frequency response of the current closed-loop system up to a certain frequency ω_0.

It is also important to notice that since only the value $[\arg\det T(j\omega_0)/\pi]$ is used in condition (7), a rough estimate of $\arg\det T(j\omega_0)/\pi$ is enough and hence the test is considerably robust to estimation errors. Moreover, the structure of T' in (5) is such that $\overline{\sigma}(T'(j\omega))$ depends on the size of the controller change. A small controller change certainly implies a smaller frequency ω_0 and hence reduced experimental effort. The estimate of the frequency response of $T(j\omega)$ up to frequency ω_0 can be obtained by using either parametric [30] or non parametric [31] estimation methods.

V. Tools for Verifying if T is Minimum-phase

This section notes some *alternative* tools which may assist in verifying condition (b) in Theorem 6, i.e. checking if $T^{-1} \in \mathscr{RH}_\infty$, which evidently amounts to checking if $T \in \mathscr{RH}_\infty$ is a minimum-phase transfer function using the available closed-loop information. Thus, the question is how to determine if $T : r \mapsto z$ has right half-plane (RHP) zeros.

For a stable linear SISO system with real RHP zeros, it is known that its step response will exhibit undershoot [32, Sec. 5.7.1] [33]. We shall adopt a rather precise definition of undershoot [34].

***Definition** 12:* Let $T(s) \in \mathscr{RH}_\infty$ be a SISO system with $T(0) \neq 0$. Let $z(t)$ be the output response to a positive unit step input $r(t)$. Then it is said that $z(t)$ exhibits type u_z undershoot for $t > 0$ if there exist exactly u_z different values of t, say $0 < t_1 < \cdots < t_{u_z} < \infty$, such that for $i = 1, \cdots, u_z$, we have

$$z(t_i)T(0) < 0, \quad \left|\frac{dz(t)}{dt}\right|_{t=t_i} = 0, \quad T(0)\left|\frac{d^2z(t)}{dt^2}\right|_{t=t_i} > 0\,. \tag{10}$$

Under the conditions in (10), $z(t)$ is said to exhibit *initial* undershoot if $z(t)T(0) < 0 \ \ \forall t \in (0, t_1)$.

Note that initial undershoot refers to a step response which exhibits undershoot if its steady-state value has a sign opposite to that of its first non-zero derivative at time $t = 0$. This is also known as 'Type A' undershoot. In contrast, undershoot without initial undershoot is known as 'Type B'. The following summarizes the results in [34] on the relationship between the number of RHP zeros and undershoot.

***Fact** 13:* The output $z(t)$ has initial undershoot if and only if $T(s)$ has an odd number of real open right half-plane zeros.

***Fact** 14:* Suppose $T(s)$ has n_z real open right half-plane zeros, and $T(0) = 1$. Then the output $z(t)$ will display at least type $\mathrm{int}[(n_z + 1)/2]$ undershoot for $t > 0$, where $\mathrm{int}[\cdot]$ denotes integer part[4].

To the best of our knowledge, the aforementioned facts formalize the latest results on the relationship between real RHP zeros and undershoot in the SISO case. As for the MIMO case and in our settings, the question is how to calculate the step response of $\det T$. For that, a procedure is put forward in the sequel. For simplicity, the time or frequency dependence of the transfer functions or signals is dropped. For a 2×2 system of Fig. 5, one can

- Insert a step function at r_1 while $r_2 = 0$ and record $z_1 = T_{11}r_1$ and $z_2 = T_{21}r_1$;
- In a separate experiment, connect z_1 to the input r_2 of the system while $r_1 = 0$ and record the second output, say y_2, which is $y_2 = T_{11}T_{22}r_1$;
- In a separate experiment, connect z_2 to the input r_2 of the system while $r_1 = 0$ and record the first output, say y_1, which is $y_1 = T_{12}T_{21}r_1$;
- Calculate the step response by doing the subtraction $y_2 - y_1$.

Of course, the above procedure can be generalized to systems with higher input dimension. The results presented in this section can assist in verifying condition (b) in Theorem 6 by checking if $T \in \mathscr{RH}_\infty$ is a minimum-phase transfer function. This can be done by studying the step response of $T : r \mapsto z$ for undershoot, see Fact 13 and Fact 14, to determine if T has right half-plane zeros.

VI. Conclusions

We have proposed new validation tests for MIMO/SISO linear time-invariant systems which aim to project internal stability with the introduction of a new controller C_1 by utilizing a limited amount of experimental data obtained from the stable closed-loop interconnection $[P, C_0]$. Theorems 10 and 11 proposed a type of phase tests analogous to the Nyquist criterion and utilized the noisy frequency response information of the closed-loop mapping T up to a finite frequency ω_0 to check if C_1 will stabilize the unknown plant P. This was all achieved despite the restrictive assumption that no a priori information about the plant was available. This promises to address the so-called transient instability problem in the context of multiple model adaptive control (MMAC) and iterative identification and control ideas discussed in the introduction.

Our current research focuses on extending the proposed stability tests to also allow inference of some performance specifications of the closed-loop with C_1.

Acknowledgment

This work was partly supported by ARC Discovery-Projects Grants (DP0342683, DP0664427) and National ICT Australia. National ICT Australia is funded by the Australian Government's Department of Communications, Information

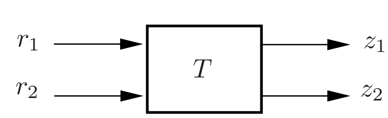

Fig. 5. A two-port system

[4] It is not hard to verify that complex RHP zeros do not have any influence on determining initial undershoot and hence Fact 13 will still be valid.

Technology, and the Arts and the Australian Research Council through Backing Australia's Ability and the ICT Research Centre of Excellence programs.

REFERENCES

[1] B. D. O. Anderson, "Failures of adaptive control theory and their resolution," *Communications in Information and Systems*, vol. 5, no. 1, pp. 1–20, 2005.

[2] R. J. P. Schrama, "Accurate identification for control: The necessity of an iterative scheme," *IEEE Transactions on Automatic Control*, vol. 37, pp. 991–994, 1992.

[3] R. J. P. Schrama and P. M. J. Van Den Hof, "An iterative scheme for identification and control design based on coprime factorizations," in *Proceedings of the American Control Conference*, Chicago, IL, USA, 1992, pp. 2842–2846.

[4] R. R. Bitmead, "Iterative control design approaches," in *Proceedings of the 12th IFAC World Congress*, vol. 9, Sydney, Australia, 1993, pp. 381–384.

[5] M. Gevers, "Towards a joint design of identification and control?" in *Essays on Control: Perspectives in the Theory and its Applications*, H. L. Trentelman and J. C. Willems, Eds. Birkhauser, New York, 1993, pp. 115–151.

[6] P. M. J. Van den Hof and R. J. P. Schrama, "Identification and control: Closed-loop issues," *Automatica*, vol. 31, no. 12, pp. 1751–1770, Dec. 1995.

[7] M. Gevers, "A decade of progress in iterative control design: from theory to practice," in *Symp. on Advanced Control of Chemical Processes*, vol. 2, Pisa, Italy, 2000, pp. 677–694.

[8] B. D. O. Anderson and M. Gevers, "Fundamental problems in adaptive control," in *Perspectives in Control Theory and Application*, D. Normand-Cyrot, Ed. Springer-Verlag, Berlin, 1998, pp. 9–21.

[9] B. D. O. Anderson, "Two decades of adaptive control pitfalls," in *Proceedings of the 8th International Conference on Control, Automation, Robotics and Vision*, Kunming, China, Dec. 2004.

[10] R. A. De Callafon and P. M. J. Van Den Hof, "Suboptimal feedback control by a scheme of iterative identification and control design," *Mathematical Modelling of Systems*, vol. 3, no. 1, pp. 77–101, 1997.

[11] M. Gevers, X. Bombois, B. Codrons, F. De Bruyne, and G. Scorletti, "The role of experimental conditions in model validation for control," in *Robustness in Identification and Control-Proceedings of Siena Workshop*, A. Garulli, A. Tesi, and A. Vicino, Eds. Springer-Verlag, July 1999, pp. 72–86.

[12] A. S. Morse, "Supervisory control of families of linear set-point controllers part i. exact matching," *IEEE Transactions on Automatic Control*, vol. 41, no. 10, pp. 1413–1431, Oct. 1996.

[13] K. S. Narendra and J. Balakrishnan, "Adaptive control using multiple models," *IEEE Transactions on Automatic Control*, vol. 42, no. 2, pp. 171–187, Feb. 1997.

[14] B. D. O. Anderson, T. Brinsmead, F. De Bruyne, J. Hespanha, D. Liberzon, and A. S. Morse, "Multiple model adaptive control. part 1: Finite controller coverings," *International Journal of Adaptive Control and Signal Processing*, vol. 10, no. 11, pp. 909–929, Aug. 2000.

[15] J. Hespanha, D. Liberzon, A. S. Morse, B. D. O. Anderson, T. Brinsmead, and F. De Bruyne, "Multiple model adaptive control. part 2: switching," *International Journal of Adaptive Control and Signal Processing*, vol. 11, no. 5, pp. 479–496, Apr. 2001.

[16] R. Wang, A. Paul, M. Stefanovic, and M. G. Safonov, "Cost-detectability and stability of adaptive control systems," in *Proceedings of the 44th IEEE Conference on Decision and Control and the 2005 European Control Conference*, Seville, Spain, Dec. 2005.

[17] A. Dehghani, B. D. O. Anderson, and A. Lanzon, "Unfalsified adaptive control: A new controller implementation and some remarks," in *Proceedings of the 2007 European Control Conference*, Kos, Greece, July 2007.

[18] H. Hjalmarsson, M. Gevers, S. Gunnarson, and O. Lequin, "Iterative feedback tuning: theory and applications," *The IEEE Control Systems Magazine*, vol. 18, no. 4, pp. 26–41, Aug. 1998.

[19] R. Hildebrand, A. Lecchini, G. Solari, and M. Gevers, "Asymptotic accuracy of iterative feedback tuning," *IEEE Transactions on Automatic Control*, vol. 50, no. 8, pp. 1182–1185, Aug. 2005.

[20] L. C. Kammer, R. R. Bitmead, and P. L. Bartlett, "Direct iterative tuning via spectral analysis," *Automatica*, vol. 36, no. 9, pp. 1301–1307, Sept. 2000.

[21] M. Gevers, X. Bombois, B. Codrons, G. Scorletti, and B. D. O. Anderson, "Model validation for control and controller validation in a prediction error identification framework–part i: Theory," *Automatica*, vol. 39, no. 3, pp. 403–415, Mar. 2003.

[22] S. G. Douma and P. M. J. Van den Hof, "Relations between uncertainty structure in identification for robust," *Automatica*, vol. 41, no. 3, pp. 439–457, Mar. 2005.

[23] B. Ninness and H. Hjalmarsson, "On the frequency domain accuracy of closed-loop estimates," *Automatica*, vol. 41, no. 7, pp. 1109–1122, July 2005.

[24] C. Kammer, "Stability assessment for cautious iterative controller tuning," *Automatica*, vol. 41, no. 10, pp. 1829–1834, Oct. 2005.

[25] A. Lanzon, A. Lecchini, A. Dehghani, and B. D. O. Anderson, "Checking if controllers are stabilizing using closed-loop data," in *Proceedings of the 45th IEEE Conference on Decision and Control*, San Diago, USA, Dec. 2006.

[26] G. Vinnicombe, *Uncertainty and Feedback: $\mathscr{H}_\infty$ loop-shaping and the ν-gap metric*. Imperial College Press, 2000.

[27] A. Dehghani, A. Lanzon, and B. D. O. Anderson, "A two degree of freedom $\mathscr{H}_\infty$ control design method for robust model matching," *International Journal of Robust and Nonlinear Control*, vol. 16, no. 10, pp. 467–483, July 2006.

[28] R. McGowan and R. Kuc, "A direct relation between a signal time series and its unwrapped phase," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 30, no. 5, pp. 719–726, Oct. 1982.

[29] C. A. Desoer and M. Vidyasagar, *Feedback Systems: Input-Output Properties*. Academic Press, 1975.

[30] L. Ljung, *System Identification–Theory For the Users*, 2nd ed. Upper Saddle River, N.J.: PTR, Prentice Hall, 1999.

[31] R. Pintelon and J. Schoukens, *System Identification: a frequency domain approach*. New York: Wiley-IEEE Press, 1979.

[32] S. Skogestad and I. Postlethwaite, *Multivariable Feedback Control: Analysis and Design*, 2nd ed. Sussex, England: John Wiley and Sons Ltd, 2005.

[33] K. Lau, R. H. Middleton, and J. H. Braslavsky, "Undershoot and settling time tradeoffs for nonminimum phase systems," *IEEE Transactions on Automatic Control*, vol. 48, no. 8, pp. 1389–1393, Aug. 2003.

[34] B. A. Leon de la Barra S., "On undershoot in siso systems," *IEEE Transactions on Automatic Control*, vol. 39, no. 3, pp. 578–581, Mar. 1994.