GLOBAL ADAPTIVE POLE PLACEMENT: DETAILED ANALYSIS OF A FIRST ORDER SYSTEM

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ABSTRACT

The paper studies an adaptive pole positioning problem for a linear first order system. The principal contribution is to specify an adaptive control law which secures global convergence. The key conditions required are that an externally available plant input be persistently exciting, and that an underbound (not necessarily tight) be known on the magnitude of the input coupling gain.

1. INTRODUCTION

One of the many remaining problems in adaptive control is the design of a globally stable controller for pole positioning of a linear, finite-dimensional system. Many papers have developed local convergence results; some have also alleged global convergence, or have stated global convergence results which depend on assumptions which are virtually uncheckable [1-13]. The difficulty in extending the local convergence results to global results is usually one of explaining how to cope with estimates of the plant parameters which, if they were the true parameters, would indicate that the plant was either uncontrollable, or unstabilizable, or overmodelled in dimension, or possessed some property of this nature.

In this paper, we show how the problem can be avoided by analysing a first order example in detail. The key is to define an appropriate strategy when the instantaneous estimates take on embarrassing values; the strategy will generalize to high order plants, but the intuitive understanding will be less.

In broad outline, the strategy is as follows. If the adaptive controller gain becomes infinite, or is viewed as becoming unacceptably large, we replace it by a fixed gain applied for $\bar{N}$ time instants, and then a second fixed gain for another $\bar{N}$ time instants, before attempting to revert to the usual gain. The integer $\bar{N}$ is related to an interval length in a persistency of excitation condition, and it turns out to be critical for our method that such a condition hold. It is also important to note that the condition is one of an externally applied input to the plant-with-feedback, as opposed to a condition (possibly uncheckable) on the plant input, viz the sum of the externally applied input and the feedback.

In addition to a persistency of excitation assumption there is one further assumption we must make that is not so standard as to be common to all treatments: we must know an underbound to the magnitude of the input coupling gain. We also assume that the desired closed-loop pole is stable.

2. FIRST ORDER ADAPTIVE POLE ASSIGNMENT EXAMPLE

The plant equation is assumed to be linear so that

$$y_k = ay_{k-1} + bu_{k-1} \tag{2.1}$$

where a and b are the unknown, time-invariant plant parameters.

There is given an external input u_k^* and the task is to implement a control law which asymptotically will position the closed loop pole at a prescribed δ with $|\delta| < 1$, i.e. asymptotically

$$u_k = \frac{\delta - a}{b} y_k + u_k^* \tag{2.2}$$

which ensures that

$$y_k = \delta y_{k-1} + bu_k^* \tag{2.3}$$

The overall controller contains an identifier and a feedback law, with the feedback law depending on the parameter estimates ($\hat{a}_k$ and $\hat{b}_k$) of a and b. The identifier is conventional. Thus suppose we use the stochastic-approximation type estimation scheme.* Then

$$\hat{\theta}_k = \hat{\theta}_{k-1} + \frac{\rho\psi_{k-1}(y_k - \psi_{k-1}^T\hat{\theta}_{k-1})}{\varepsilon + \|\psi_{k-1}\|^2} \tag{2.4}$$

or, if $\tilde{\theta}_k = \theta_o - \hat{\theta}_k$

$$\tilde{\theta}_k = (I - \frac{\rho\psi_{k-1}\psi_{k-1}^T}{\varepsilon + \|\psi_{k-1}\|^2})\tilde{\theta}_{k-1} \tag{2.5}$$

where $0 < \rho < 2$, $\varepsilon > 0$ and

$$\hat{\theta}_k = [\hat{a}_k \ \hat{b}_k]^T$$

$$\theta_o = [a \ b]^T$$

$$\psi_k = [y_k \ u_k]^T \tag{2.6}$$

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* In contrast with a conventional stochastic approximation algorithm, the gain here does not go to zero. We could also have used exponentially weighted recursive least squares.

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The reason that we use the estimation scheme characterised by (2.3) is that it has a number of desirable properties, which are easily established and now well known. In particular,

$$\text{Property 1:} \qquad \|\tilde{\theta}_k\| \leq \|\tilde{\theta}_{k-1}\| \qquad (2.7)$$

$$\text{Property 2:} \qquad \|\tilde{\theta}_k - \tilde{\theta}_{k-1}\| \to 0, \text{ as } k \to \infty. \qquad (2.8)$$

$$\text{Property 3:} \qquad \lim_{k\to\infty} \frac{(y_k - \psi_{k-1}^T \hat{\theta}_{k-1})}{\epsilon + \|\psi_{k-1}\|^2} = 0 \qquad (2.9)$$

These properties are true irrespective of whether $\|\psi_k\|$ is bounded or not.

Suppose now that we know a constant ρ *such that* $|b| \geq \rho$. This is a key, not altogether standard, but not unreasonable assumption. Let σ be any constant with $\rho > \sigma > 0$. The controller consists of the following rule. So long as $|\hat{b}_k| > \sigma$ then

$$u_k = \hat{k}_k y_k + u_k^* \qquad (2.10)$$

where $\hat{k}_k$ is obtained from the "adaptive pole assignment equation":

$$\hat{a}_k + \hat{b}_k \hat{k}_k = \delta \qquad (2.11)$$

When $|\hat{b}_k| \leq \sigma$, use $\hat{k}_j = \bar{k}_1$ for $j = k, k+1, \ldots, k+\bar{N}-1$ and $\hat{k}_j = \bar{k}_2$ for $\hat{k}_j = k+\bar{N}, \ldots k+2\bar{N}-1$; here $\bar{k}_1$ and $\bar{k}_2$ are two fixed but arbitrary gains, and $\bar{N}$ will be specified below. (It is desirable, in the interests of securing good transient performance, but not essential, that $|a + b\bar{k}_i| < 1$.) Next, at time $j = k + 2\bar{N}$, one reverts to (2.10) if $|\hat{b}_j| > \sigma$; otherwise for the next $2\bar{N}$ points, the $\bar{N}$ values of $\bar{k}_1$ followed by $\bar{N}$ values of $\bar{k}_2$ are again used.

Since $|\hat{a}_k|, |\hat{b}_k|$ are bounded, see (2.7), the above procedure assures that

$$|\hat{k}_k| < M \qquad (2.12)$$

for some M.

We remark that if $\hat{b}_0$ has opposite sign to b, it is conceivable (depending on step sizes and the choice of σ) that the condition $|\hat{b}_k| \leq \sigma$ will be encountered.

Our main result is

<u>Theorem</u> Suppose that u_k^* satisfies the persistency of excitation condition

$$\kappa_2 I \geq \sum_{k+j+2}^{j+\bar{N}} \begin{bmatrix} u_k^* \\ u_{k-1}^* \end{bmatrix} [u_k^* \; u_{k-1}^*] \geq \kappa_1 I > 0$$

for all k, some $\bar{N}$ and some positive κ_1 and κ_2. Then the above algorithm converges. More precisely

a) u_k, y_k are bounded

b) $\lim \|\tilde{\theta}_k\| \to 0; \quad \lim |\hat{k}_k - \frac{\delta - a}{b}| \to 0$

c) The limits are approached exponentially fast.

Before proving this result in the remainder of the paper, we comment on the role of the persistency of excitation condition. It is reasonably well known for the identification algorithm defined by (2.4) that if ψ_k is persistently exciting, then $\|\tilde{\theta}_k\| \to 0$ exponentially. Once $\|\tilde{\theta}_k\|$ is sufficiently small, any of the local convergence results for the adaptive pole positioning problem come into play. Hence our problem will basically be solved if we can show ψ_k is persistently exciting. But with the system having feedback, which is potentially unstabilizing and time-varying, it is not immediately clear how we *guarantee* that $\{\psi_k\}$ is persistently exciting. In the next section we show that one way is to insist that u_k^* be persistently exciting. An additional condition is that the adaptive gain be only slowly time-varying. This latter condition is guaranteed basically by property 2 of the estimation algorithm and the additional fact (which has not been proved yet) that the fixed controllers are only needed a finite number of times. This is, provided $|b| > \sigma$, $|\hat{b}_k| \leq \sigma$ only a finite number of times.

3. PRELIMINARY INVESTIGATION OF PERSISTENCY OF EXCITATION

In this section, we shall present a preliminary result on persistency of excitation. The main result, Proposition 3.1 below, depends on two lemmas.

<u>Lemma 3.1</u>. Suppose that (2.1) and (2.10) hold. Suppose also that there exists $\underline{\lambda} = [\lambda_0 \; \lambda_1]^T$

$$\left|\lambda^T \begin{bmatrix} y_k \\ y_{k-1} \end{bmatrix}\right| < c \qquad k\epsilon[j+1, j+\bar{N}] \qquad (3.1)$$

Then for $k\epsilon[j+2, j+\bar{N}]$

$$\left|\lambda^T \begin{bmatrix} u_k^* \\ u_{k-1}^* \end{bmatrix}\right| < [1 + |a + b\hat{k}_{j+1}|]c + \|\lambda\| \max_{k\epsilon[j,j+\bar{N}-1]} |y_k| \Delta \qquad (3.2)$$

where

$$\Delta = 2 \max_{k\epsilon[j,j+\bar{N}-1]} |[b\hat{k}_k - b\hat{k}_{j+1}]| \qquad (3.3)$$

<u>Proof</u> Rewrite (3.1) as

$$|\lambda(q^{-1})y_k| < c \quad k\epsilon[j+1, j+\bar{N}]$$

where $\lambda(q^{-1}) = \lambda_0 + \lambda_1 q^{-1}$ and q^{-1} is the backward shift operator. It follows that for $k\epsilon[j+2, j+\bar{N}]$,

$$|[-q^{-1}b\hat{k}_{j+1} + (1-aq^{-1})]\lambda(q^{-1})y_k| < |a+b\hat{k}_{j+1}|c + c = c_1$$

This implies that for $k\epsilon[j+2, j+\bar{N}]$,

$$|\lambda_0[-q^{-1}b\hat{k}_k + (1-aq^{-1})]y_k + \lambda_1[-q^{-1}b\hat{k}_{k-1} + (1-aq^{-1})]y_{k-1}|$$

$$< c_1 + 2\|\lambda\| \max_{k\epsilon[j,j+\bar{N}-1]} |y_k| \max_{k\epsilon[j,j+\bar{N}-1]} |[b\hat{k}_k - b\hat{k}_{j+1}]|$$

$= c_1 + c_2$

Using (2.1) and (2.10), we can rewrite this as

$$|\lambda_0 u^*_{k-1} + \lambda_1 u^*_{k-2}| < c_1 + c_2 \qquad k\varepsilon[j + 2, j + \bar{N}]$$

This yields (3.2)

Next, we link y_k to ψ_k.

Lemma 3.2. Suppose that (2.1) holds and there exists $\underline{\mu} = [\mu_0 \ \mu_1]^T$ with $\|\mu\| = 1$ and

$$\left|\mu^T \begin{bmatrix} y_k \\ u_k \end{bmatrix}\right| < d \quad k\varepsilon[j, j + \bar{N} - 1] \tag{3.4}$$

Then there exist λ_0, λ_1 not both zero such that

$$\left|[\lambda_0 \ \lambda_1]\begin{bmatrix} y_k \\ y_{k-1} \end{bmatrix}\right| < |b|d \quad k\varepsilon[j + 1, j + \bar{N}] \tag{3.5}$$

with

$$b(1 + a^2 + b^2)^{-\frac{1}{2}} \leq \|\lambda\| \leq (1 + a^2 + b^2)^{\frac{1}{2}} \tag{3.6}$$

Proof From (3.4) we obtain

$$|bq^{-1}\mu_0 y_k + bq^{-1}\mu_1 u_k| < |b|d \quad k\varepsilon[j + 1, j + \bar{N}]$$

and then from (2.1),

$$|[\mu_1 - (a\mu_1 - b\mu_0)q^{-1}]y_k| < |b|d \quad k\varepsilon[j + 1, j + \bar{N}]$$

Then (3.5) follows with $\lambda_0 = \mu$, $\lambda_1 = -a\mu_1 + b\mu_0$

Now since

$$\begin{bmatrix} \lambda_0 \\ \lambda_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ b & -a \end{bmatrix} \begin{bmatrix} \mu_0 \\ \mu_1 \end{bmatrix} = A \begin{bmatrix} \mu_0 \\ \mu_1 \end{bmatrix}$$

upper and lower bounds for $\|\lambda\|$ are given by $[\lambda_{max}(A^TA)]^{\frac{1}{2}}$ and $\lambda_{min}[A^TA]^{\frac{1}{2}}$. Weaker bounds are given by $[\text{trace } A^TA]^{\frac{1}{2}}$ and $[\det(A^TA)/\text{trace}(A^TA)]^{\frac{1}{2}}$ and these are readily computed to give (3.6).

Now we can state the restricted persistency of excitation result.

Proposition 3.1 Suppose that the plant equation (2.1) and the feedback law (2.10) hold. Suppose that for the values of k under consideration, $|y_k|$ and $|u_k|$ are bounded, and that $|\hat{k}_k| < M$ for all k, as in (2.12). If

$$\kappa_2 I > \sum_{k=j+2}^{j+\bar{N}} \begin{bmatrix} u^*_k \\ u^*_{k-1} \end{bmatrix} [u^*_k \ u^*_{k-1}] \geq \kappa_1 I > 0 \tag{3.7}$$

and with Δ as in (3.3) if there holds

$$\kappa_1 > \frac{N(1+a^2+b^2)}{b^2}\{[1+|a|+|b|M]|b|\sqrt{\Pi_1} + (a^2+b^2+1)^{\frac{1}{2}}\max_{k\varepsilon[j,j+\bar{N}-1]}|y_k|\Delta\} \tag{3.8}$$

for some positive Π_1, then

$$\Pi_2 I \geq \sum_{k=j}^{j+\bar{N}-1} \begin{bmatrix} y_k \\ u_k \end{bmatrix}[y_k \ u_k] \geq \Pi_1 I \tag{3.9}$$

Remarks If (3.7) holds and the time rate of variation of $\hat{k}_k$ is slow enough, i.e. Δ is sufficiently small, then (3.8) is guaranteed to hold for some Π_1 and then (3.9) holds. Thus the idea of the Proposition is that persistency of excitation of u^* carries through to persistency of excitation of ψ provided that (1) y_k, u_k are bounded and (2) the time-variation of the gain is suitably slow. Results of this type have been used elsewhere [14]; the restriction to plants with a single pole is not essential.

Proof The upper bound of (3.9) follows from the boundedness of $|u_k|$ and $|y_k|$. The lower bound will be established by contradiction. Thus suppose the lower bound fails. Then there exists $\mu = [\mu_0 \ \mu_1]^T$ with $\|\mu\| = 1$ such that

$$\left|\mu^T \begin{bmatrix} y_k \\ u_k \end{bmatrix}\right| < \sqrt{\Pi_1} \quad k\varepsilon[j, j+\bar{N} - 1]$$

By lemma 3.2, there exists $\lambda = [\lambda_0 \ \lambda_1]^T$ such that

$$\left|[\lambda_0 \ \lambda_1]\begin{bmatrix} y_k \\ y_{k-1} \end{bmatrix}\right| < |b|\sqrt{\Pi_1} \quad k\varepsilon[j + 1, j + \bar{N}]$$

By lemma 3.1, this implies for $k\varepsilon[j + 2, j + \bar{N}]$,

$$\left|[\lambda_0 \ \lambda_1]\begin{bmatrix} u^*_k \\ u^*_{k-1} \end{bmatrix}\right| < [1 + |a + b\hat{k}_{j+1}|]|b|\sqrt{\Pi_1} + \|\lambda\|\max_{k\varepsilon[j,j+\bar{N}-1]}|y_k|\Delta$$

and so using (3.7) and the bounds on $\|\lambda\|$ of (3.6),

$$b^2(1+a^2+b^2)^{-1}\kappa_1 < N\{[1+|a|+|b|M]|b|\sqrt{\Pi_1} + (a^2+b^2+1)^{\frac{1}{2}}\max_{k\varepsilon[j,j+\bar{N}-1]}|y_k|\Delta\}$$

This violates (3.8).

4. PROOF OF MAIN RESULT

An error equation Define the error e_k between the plant output and an identifier output as

$$e_k = y_k - \psi^T_{k-1}\hat{\theta}_{k-1} \tag{4.1}$$

Then we have

$$e_k = ay_{k-1} + b\hat{k}_{k-1}y_{k-1} + bu^*_{k-1} - \hat{a}_{k-1}y_{k-1} - \\ - \hat{b}_{k-1}\hat{k}_{k-1}y_{k-1} - \hat{b}_{k-1}u^*_{k-1}$$

$$= [a-\hat{a}_{k-1} + (b-\hat{b}_{k-1})\hat{k}_{k-1}]y_{k-1} + (b-\hat{b}_{k-1})u^*_{k-1} \quad (4.2)$$

Define

$$\alpha_{k-1} = a - \hat{a}_{k-1} + (b - \hat{b}_{k-1})\hat{k}_{k-1} \quad (4.3)$$

<u>Division into two cases</u> We now divide consideration into two possibilities, $\limsup_{k\to\infty} |\alpha_k| \neq 0$ and $|\alpha_k| \to 0$

<u>Case 1</u> Assume that $\limsup_{k\to\infty} |\alpha_k| \neq 0$. (A rather lengthy argument by contradiction will show this is impossible.

Suppose $J = \{t_1, t_2 \ldots\}$ is an infinite subset of the integers such that $|\alpha_{t_i}| \to \mu > 0$. It is then easy to see per (4.2) and (4.3) and the fact that $\hat{a}_k$, $\hat{b}_k$ and $\hat{k}_k$ are bounded, that

$$|y_{t_{i-1}}| < C_1 + C_2|e_{t_i}|$$

$$< C_1 + C_2 \max |e_\tau|$$

$$0 \leq \tau \leq t_i$$

for some constants C_1, C_2. From the boundedness of the $\hat{k}_k$ and u^*_k,

$$|\psi_{t_{i-1}}| < C_3 + C_4 \max_{0\leq\tau\leq t_i} |e_\tau|$$

This and (2.9) imply (see [15] for a detailed argument) that $e_{t_i} \to 0$ and $\| \psi_{t_{i-1}} \|$ is a bounded sequence.

Now, choose a subset of J such that each element of the subset is separated by at least $2\bar{N}-1$ instants. Call this subset $\bar{T}$. Next form $S = \bar{T} \cup \bar{T}_1 \cup \ldots \cup \bar{T}_{2\bar{N}-1}$ where $\bar{T}_i = \bar{T}$ translated i steps to the right, i.e. if $\bar{T} = \{t_1, t_2, \ldots\}$ then $\bar{T}_i = \{\bar{t}_1 + i, \bar{t}_2 + i \ldots\}$.

Note that on S, $|y_k|$ and $|u_k|$ are bounded (since they are bounded on $\bar{T}$, since u^*_k is bounded, and since $\hat{k}_k$ is bounded).

Now suppose S consists of a union of intervals $[\bar{t}_i, \bar{t}_i + 2\bar{N}-1]$. Because of the way $\hat{k}_k$ is chosen, there exists a subinterval of $[\bar{t}_i, \bar{t}_i + 2\bar{N}-1]$ of length $\bar{N}$ such that either $\hat{k}_k = \bar{k}_1^i$ on this subinterval, or $\hat{k}_k = \bar{k}_2$ on this subinterval, or $\hat{k}_k$ is chosen to satisfy $\hat{a}_k + \hat{b}_k \bar{k}_k = \delta$ on the subinterval†.
If the latter is the case, we know

† Note that the two fixed gains, $\bar{k}_1$ and $\bar{k}_2$, need not be always the same. So that $\bar{k}_1$ and $\bar{k}_2$ used on a second $2\bar{N}$ interval may be different to those used on the first $2\bar{N}$ interval.

that for i sufficiently large, $\hat{k}_k$ will be slowly varying, since Property 2, see (2.8), makes this true for $\hat{a}_k$ and $\hat{b}_k$. So on the subinterval, either $\hat{k}_k$ is constant or slowly varying.

Consequently by Proposition 3.1 (see also the Remark following the Proposition), since u^*_k is persistently exciting, there will be a subinterval of length $\bar{N}$ of $[\bar{t}_i, \bar{t}_i + 2\bar{N} - 1]$ for all sufficiently large i on which $\psi_k{}^i = [y_k\ u_k]^T$ will be persistently exciting. Call the subinterval $[\hat{t}_i, \hat{t}_i + \bar{N}-1]$. Then the basic result of [16] shows that for some constant $\lambda < 1$

$$\| \tilde{\theta}_{\hat{t}_i + \bar{N}-1} \| < \lambda \| \tilde{\theta}_{\hat{t}_i} \|^2$$

and therefore, by (2.7),

$$\| \tilde{\theta}_{t_i + 2\bar{N}-1} \| \leq \| \tilde{\theta}_{\hat{t}_i + \bar{N}-1} \| < \lambda \| \tilde{\theta}_{\hat{t}_i} \|^2 \leq \lambda \| \tilde{\theta}_{\bar{t}_i} \|^2$$

for $i \geq$ some i_0. It follows that $\| \tilde{\theta}_{t_i} \| \to 0$ exponentially fast. But this contradicts the assumption that $|\alpha_{t_i}| \to \mu$.

Case 2: <u>Assume that</u> $\alpha_k \to 0$. <u>Suppose that</u> $\hat{k}_k = \bar{k}_1$ <u>on the set</u> I_1 and $\hat{k}_k = \bar{k}_2$ <u>on the set</u> I_2. <u>We shall first show</u> (by contradiction) that I_1 <u>cannot be an infinite set</u> - If it is, then on I_1

$$|a - \hat{a}_k + (b - \hat{b}_k)\ \bar{k}_1| \to 0$$

while on I_2

$$|a - \hat{a}_k + (b - \hat{b}_k)\ \bar{k}_2| \to 0$$

Now I_1 is a union of a number of intervals of length $\bar{N}$ and I_2 similarly. Further, the last point of any I_1 interval is adjacent to the first point of an I_2 interval. Let $\bar{I}_1$ denote the set of last points of I_1 intervals. Then

$$|a - \hat{a}_k + (b - \hat{b}_k)\ \bar{k}_1| \to 0 \quad (4.4)$$

$$|a - \hat{a}_{k+1} + (b - \hat{b}_{k+1})\ \bar{k}_2| \to 0 \quad (4.5)$$

on $\bar{I}_1$. Also, by (2.8),

$$|\hat{a}_k - \hat{a}_{k+1}| + |\hat{b}_k - \hat{b}_{k+1}| \to 0 \quad (4.6)$$

on $\bar{I}_1$. Since $\bar{k}_1$, $\bar{k}_2$ are different, (4.4) and (4.6) together imply $|a - \hat{a}_k| \to 0$ and $|b - \hat{b}_k| \to 0$ on $\bar{I}_1$. Then by Property 1, see (2.7), and the fact that $\bar{I}_1$ is finite, $|a - \hat{a}_k| \to 0$ and $|b - \hat{b}_k| \to 0$ for all k. But then it is impossible to have $|\hat{b}_k| < \sigma$ an infinite number of times. So it is impossible that I_1 be an infinite set.

Parameter and control law convergence Since I_1 is finite, it follows that for all k from some point on, $|\hat{b}_k| > \sigma$. Then we could probably argue using many approaches. However, an efficient argument yielding the exponential convergence is as follows. Since $\alpha_k \to 0$ and from some point on $\hat{a}_k + \hat{b}_k \hat{k}_k = \delta$, we have

$$a + b\hat{k}_k \to \delta \tag{4.7}$$

Then since

$$y_k = (a + b\hat{k}_k)y_{k-1} + bu_k^* \tag{4.8}$$

we see using (4.7) and $|\delta| < 1$ that y_k is bounded. Also, u_k is bounded. The persistency of excitation condition on u_k^* then translates via Proposition 3.1 for suitable large k into one on ψ_k, whence $\tilde{\theta}_k$ converges exponentially fast to zero [16], then α_k converges exponentially fast to zero, as does $(a + b\hat{k}_k) - \delta$.

5. CONCLUSION

The most important part of the paper is the description of the control law, and in particular that used when the conventional law cannot be used. The choice of a temporarily constant law over a suitably long interval helps secure a persistency of excitation condition which promotes parameter convergence; the choice of one temporarily constant law followed by another is what allows us to rule out any but a finite number of uses of the non-conventional law.

It is interesting to speculate whether a global convergence result could be found that did not rely on having persistency of excitation in some way. The question is perhaps artificial, in the sense that there is only dubious value in any adaptive algorithms where persistency of excitation is lacking [16,17].

Another form of first order problem which might be considered is one where a persistently exciting reference trajectory y_j^* is given, and one wishes to implement a control law of the form $u_j = k(y_j - y_j^*)$ with $a + bk = \delta$. If $a = \delta$, so that the correct law has $k = 0$, problems must be expected, since there can be no persistency of excitation of ψ_j for $k = 0$ and as $\hat{k}_j \to 0$, persistency of excitation will be lost, even if y_j^* is persistently exciting. However, if $a \neq \delta$, an analogous theory to that given here can be carried through, save that one must take precautions to prevent $\hat{k}_j = 0$. These precautions are the same as those used to prevent $\hat{k}_j$ becoming too large.

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