SANJIT K. MITRA

Circuit Theory and Design

Proceedings of the 1981 European Conference on Circuit Theory and Design

The Hague, The Netherlands
25-28 August, 1981

edited by R. Boite and P. Dewilde

1981
Delft University Press / Delft
North-Holland Publishing Company / Amsterdam • New York • Oxford

BLOCK KALMAN FILTERING

Brian D. O. Anderson
Department of Electrical and Computer Engineering
University of Newcastle
Australia

and

Sanjit K. Mitra and N. S. Ramesh
Department of Electrical and Computer Engineering
University of California, Santa Barbara
Santa Barbara, CA 93106

Abstract

The concept of block signal processing is extended to the Kalman filtering problem. Block processing algorithms are developed for state/signal estimation in noisy linear systems and it is shown that smoothed estimates are naturally obtained. Further, block processing is shown to reduce sensitivity to computational errors, and promises a reduction in the amount of computation, although at a cost of delay in the production of estimates, as compared with conventional Kalman filtering.

1. INTRODUCTION

The concept of block processing of signals, wherein a block of input data samples, rather than a single data sample, is processed at a given time instant has received considerable attention lately for the implementation of time-invariant and periodic digital filters [1-4].

In this paper we extend the concept to the problem of state/signal estimation in noisy linear systems where all processes are stationary*. The aim is to develop block processing algorithms and to show that smoothed estimates can be naturally obtained by block processing; such estimates are more accurate than the conventional filtered estimates which are the usual output of a Kalman filter. We further show that this approach reduces the amount of computation required to obtain the estimates and that generally the block estimation is superior from the standpoint of sensitivity to roundoff errors in the computations. There are however two possible costs. One stems from the fact that estimates are produced with a larger delay using block processing than with the conventional Kalman filter. The second stems from the fact that, at least with the simplest block processing scheme, estimates are provided at every instant of time only of the signal process; estimates of the state process are available, but only at instants of time separated by one block length.

The structure of the paper is as follows. In Section 2, we describe the concept of block signal processing, in Section 3 we note two ways in which Kalman filtering might be tied to block processing and in Section 4 and 5 we explore one of these ideas in some detail. In Section 6, we comment on stability and sensitivity, and some computational issues are examined in Section 7. Section 8 describes various embellishments to the basic block filtering idea and Section 9 contains concluding remarks.

*While nonstationary processes can be considered, most of the usual advantages of block processing (which flow from time-invariance of the various systems involved) will be lost.

2. BLOCK-STATE STRUCTURES

The concept of block processing was first suggested by Gold and Jordan [1]. Burrus [2] was one of the first to systematically treat the block implementation of digital filters, and since then several new block structures have been proposed by various authors e.g. [3,4]. We make use of the block-state structures [3,4] in this paper as this is the most convenient form for formulating the Kalman filter problem.

Consider an s-input, ℓ-output time-invariant system represented in state-space form as

$$x(n+1) = ax(n) + bu(n) \tag{1a}$$

$$y(n) = c'x(n) + du(n) \tag{1b}$$

with state dimension N. We shall refer to this model as the unblocked model, to distinguish it from the block model to be introduced next.

A straightforward extension of state-equations-derived block model [3,4] from 1-input, 1-output to the s-input, ℓ-output case results in the following block system model (BSM),

$$X_{k+1} = A\,X_k + B\,U_k \tag{2a}$$

$$Y_k = C'\,X_k + D\,U_k \tag{2b}$$

where X_k, U_k, Y_k are the block state, input and output vectors respectively. With a block length of L, the block system is related to the unblocked system by

$$X_k = x(kL) \tag{3a}$$

$$U_k = \begin{bmatrix} u(kL) \\ u(kL+1) \\ \cdot \\ \cdot \\ \cdot \\ u(kL+L-1) \end{bmatrix} \tag{3b}$$

$$Y_k = \begin{bmatrix} y(kL) \\ y(kL+1) \\ \cdot \\ \cdot \\ \cdot \\ y(kL+L-1) \end{bmatrix} \quad (3c)$$

Note that u(n) and y(n) are themselves vector quantities in general.

$$A = a^L \quad (4a)$$

$$B = [a^{L-1}b,\ a^{L-2}b,\ \ldots\ ,\ ab,\ b] \quad (4b)$$

$$C' = \begin{bmatrix} c' \\ c'a \\ \cdot \\ \cdot \\ \cdot \\ c'a^{L-1} \end{bmatrix} \quad (4c)$$

$$D = [D_{ij}]^{+},\ D_{ij} = \begin{cases} 0, & i < j \\ d, & i = j \\ c'a^{i-j-1}b, & i > j \end{cases} \quad (4d)$$

3. KALMAN FILTERING

The Kalman filtering [5] problem may be stated briefly as follows: Let

$$z(n) = y(n) + v(n),\ n = 0,1,2,\ldots \quad (1c)$$

by noisy observations of the output (signal) y(n) of a system. We wish to obtain the minimum variance estimate of the state x(n) as well as the signal y(n) of the system in real time using the observations (sometimes only signal estimation is of interest). It is further assumed that both the system input u(n) as well as the noise process v(n) are white, Gaussian, and uncorrelated with each other and with the initial system state x(0); u(n) is unmeasurable. We show in this paper that when all processes are stationary (as above) the block approach is particularly appealing in signal estimation problems where estimates of only y(n) are of interest.

There are two possible ways in which one may consider developing a block Kalman filter for a time invariant system {a, b, c, d}

(a) {a,b,c,d} —BLOCK→ {A,B,C,D} ——→ Derive Block Kalman filter

(b) {a,b,c,d} ——→ Derive Kalman filter ——→ Block the Kalman filter

In the next section we discuss the relative merits of these two approaches.

4. COMPARISON OF BLOCK AND SCALAR ESTIMATES

Approach (a)

Let {A, B, C, D} be the BSM of the system (1) with

$$Z_k = Y_k + V_k,\ k = 0,1,2,\ldots \quad (2c)$$

being the noisy observations of the block signal Y_k. Note that

$$Z_k = (z'(kL),\ z'(kL+1),\ \ldots,\ z'(kL+L-1))'.$$

Let $\hat{X}_{k/\ell}$, $\hat{Y}_{k/\ell}$ denote the best estimates of X_k, Y_k respectively, based on observations till block instant ℓ. Likewise, let $\hat{x}(n/m)$ and $\hat{y}(n/m)$ represent the best estimates of x(n) and y(n), respectively based on observations till time instant m. The Kalman filter is usually constructed so as to yield one or more of $\hat{X}_{k/k-1}$, $\hat{Y}_{k/k-1}$, $\hat{X}_{k/k}$ and $\hat{Y}_{k/k}$. Using (3), we have, considering first the one-step prediction estimates,

$$\hat{X}_{k/k-1} = \hat{x}(kL/Z_0,Z_1,\ldots,Z_{k-1})$$
$$= \hat{x}(kL/z(0),z(1),\ldots,z(kL-1)) \triangleq \hat{x}(kL/kL-1) \quad (5)$$

and

$$\hat{Y}_{k/k-1} = \begin{bmatrix} \hat{y}(kL/kL-1) \\ \hat{y}(kL+1/kL-1) \\ \cdot \\ \cdot \\ \cdot \\ \hat{y}(kL+L-1/kL-1) \end{bmatrix} \quad (6)$$

We note that the block state estimation produces samples of the unblocked state estimates L time instants apart, and the block signal estimation produces a vector of unblocked signal estimates. It is important to note that the block signal estimate $\hat{Y}_{k/k-1}$ is <u>not</u> a blocked version of $\hat{y}(n/n-1)$; rather, the first component represents a one step prediction, the second a two step prediction and so on. It is intuitively evident, and can be verified by calculation, that a multi-step prediction produces larger error variance than a single-step prediction. Hence the block signal estimate $\hat{Y}_{k/k-1}$ is inferior to the collection of estimates $\hat{y}(n/n-1)$ with n = kL, kL+1,..., kL+L-1.

Now consider the true filtered estimates $\hat{X}_{k/k}$ and $\hat{Y}_{k/k}$:

$$\hat{X}_{k/k} = \hat{x}(kL/Z_0,Z_1,\ldots,Z_k)$$
$$= \hat{x}(kL/z(0),z(1),\ldots z(kL+L-1))$$
$$= \hat{x}(kL/kL+L-1) \quad (7)$$

and

$$\hat{Y}_{k/k} = \begin{bmatrix} \hat{y}(kL/kL+L-1) \\ \hat{y}(kL+1/kL+L-1) \\ \cdot \\ \cdot \\ \cdot \\ \hat{y}(kL+L-1/kL+L-1) \end{bmatrix} \quad (8)$$

The estimate $\hat{X}_{k/k}$ thus represents a sampled version of the fixed lag smoothed state estimate $\hat{x}(j/j+L-1)$ of the unblocked system, and the block signal estimate $\hat{Y}_{k/k}$ is a vector of smoothed estimates of y(j) with different lags depending on the particular

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+Each D_{ij} is an ℓ x s matrix.

value of j. Since smoothing in general produces more reliable estimates we see that the above estimates are superior to the set of scalar estimates $\hat{x}(n/n)$ and $\hat{y}(n/n)$. Of course, there is a price paid: the estimate of y(kL), viz the first entry of $\hat{Y}_{k/k}$, does not become available until time kL+L-1.

Approach (b)

In this approach we block the Kalman filter corresponding to the unblocked system {a, b, c, d}. The Kalman filter can be organized as a linear system with $\hat{x}(n/n)$ serving as the state vector and $\hat{y}(n/n)$ as output; so it may be blocked to obtain

$$\hat{\underline{X}}_{k/k} = \hat{x}(kL/kL) \tag{9}$$

$$\hat{\underline{Y}}_{k/k} = \begin{bmatrix} \hat{y}(kL/kL) \\ \hat{y}(kL+1/kL+1) \\ \vdots \\ \hat{y}(kL+L-1/kL+L-1) \end{bmatrix} \tag{10}$$

[The underbar on the block variables distinguishes them from those in Appendix A]. No smoothing is involved in the above block estimates in contrast to (7) and (8). The Kalman filter for the unblocked signal model may also be organized as a linear system with state $\hat{x}(k/k-1)$ and output $\hat{y}(k/k-1)$, and blocking leads to $\hat{\underline{X}}_{k/k-1} = \hat{x}(kL/kL-1)$ and $\hat{\underline{Y}}_{k/k-1} = [\hat{y}'(kL/kL-1), \ldots \hat{y}'(kL+L-1/kL+L-2)]'$.

In either case the blocking process may be more complex since it may involve blocking a time varying system. Recall that the Kalman filter can be time-varying (though asymptotically time-invariant) even if the system model is time-invariant with stationary input and measurement noise [5]. For these reasons we restrict ourselves largely to approach (a) in this paper.

5. KALMAN FILTER FOR BLOCK SIGNAL MODEL

Consider the block signal model,

$$X_{k+1} = AX_k + BU_k \tag{2a}$$

$$Y_k = C'X_k + DU_k \tag{2b}$$

$$Z_k = Y_k + V_k \tag{2c}$$

Appendix A contains a derivation of the Kalman filter/predictor for the above model. The derivation is not standard because the signal model contains the direct feedthrough term with gain D, which is nonzero even if the unblocked system has no feedthrough (4d). Fig. 1 is a block diagram representation of the Kalman filter/predictor.

5.1 RELATION BETWEEN UNBLOCKED KALMAN FILTER AND KALMAN FILTER OF A BLOCKED SIGNAL MODEL

As shown in Section 4, the block and unblocked one-step predictors are related by (5),

$$\hat{X}_{k/k-1} = \hat{x}(kL/kL-1)$$

and hence the one step error covariances are related directly by,

$$\Sigma_{k/k-1} = \Sigma(kL/kL-1)$$

(see Appendix B for details on the notation). Unfortunately the Kalman gains k(n) for the unblocked model and K_n for the BSM are not simply related, but the steady state gains $\bar{k}$ and $\bar{\mathcal{K}}$ respectively of the unblocked Kalman filter for the unblocked signal model and Kalman filter for the BSM are related as follows: Let

$$\hat{x}(k+1/k) = \alpha\, \hat{x}(k/k-1) + \bar{k}\, z(k) \tag{11}$$

represent the steady state unblocked Kalman filter for the unblocked model, where

$$\alpha = a - \bar{k}\, c' \tag{12}$$

and $\bar{k}$ is the steady state Kalman gain. Now blocking (11) [3,4] we obtain,

$$\begin{aligned} \hat{X}_{k+1/k} &= \hat{x}(\overline{k+1}\, L/\overline{k+1}\, L-1) \\ &= \alpha^L\, \hat{x}(kL/kL-1) + \bar{\mathcal{K}} \begin{bmatrix} z(kL) \\ z(kL+1) \\ \vdots \\ z(kL+L-1) \end{bmatrix} \\ &= \alpha^L\, \hat{X}_{k/k-1} + \bar{\mathcal{K}}\, Z_k \end{aligned} \tag{13a}$$

where

$$\bar{\mathcal{K}} = [\alpha^{L-1}\bar{k},\ \alpha^{L-2}\bar{k}, \ldots, \alpha\bar{k}, \bar{k}] \tag{13b}$$

$$= [(a-\bar{k}\, c')^{L-1}\, \bar{k},\ (a-\bar{k}\, c')^{L-2}\, \bar{k}, \ldots, \bar{k}] \tag{13c}$$

Now (13a) has been arrived by taking the Kalman filter for the unblocked filter model, and then blocking it. On the other hand, as earlier stated, we are interested in a Kalman filter for the blocked signal model. But as the line of Table I labelled "STATE" illustrates the quantities obtained with the one-step predictor are the same for both arrangements (even though the other quantities obtained are all different). It follows that (13a) is just as applicable as an equation describing the Kalman filter for the blocked state model. The above remark applies to the Kalman filter set up as a one-step predictor. In Appendix A, the relation of the filter generating $\hat{X}_{k/k-1}$ to that generating $\hat{X}_{k/k}$ is described.

6. STABILITY AND SENSITIVITY

It is readily observed from (13) that the Kalman filter for the blocked state model is stable if the unblocked Kalman filter is stable (α has all eigenvalues inside the unit circle) and vice versa; for if $\{\lambda_i\}$ are the eigenvalues of α then $\{\lambda_i^L\}$ are the eigenvalues of α^L. Further, the blocking process moves the eigenvalues towards the origin, promoting stability and also reducing the effects due to truncation/roundoff in the computations [3, 4].

7. COMPUTATIONAL BURDEN

We consider the computations involved in updating the state estimate only, taking the view that covariance and gain updates (or calculation of the limiting forms of these matrices) are done offline. Suppose the unblocked measurement vector has dimension ℓ and the signal model state vector has dimension N. Then each update of the unblocked one-step predictor for the unblocked model involves the following operations:

(NxN matrix)(N vector) + (Nxℓ matrix)(ℓ vector)

If the block length is L, the corresponding operations are

(NxN matrix)(N vector) + (NxℓL matrix)(L vector)

for the blocked predictor.

Since there is one block update per L unblocked system updates, we need to compare the block complexity with the complexity of $\{$(NxN matrix)(N-vector) + (Nxℓ matrix)(ℓ vector)$\}_{L\ times}$. The reduction in complexity is clear, and the reduction carries through to signal, as opposed to state, estimation.

8. LINEAR FUNCTIONAL, INTERPOLATED STATE ESTIMATORS AND OTHER MATTERS

Earlier, we derived block state and block signal estimators for the BSM. We wish to re-emphasize that while the block signal estimator $\hat{Y}_{k/k}$ represents estimates of the unblocked signal y(n) for all n, the block state estimator $\hat{X}_{k/k-1}$ is actually a sampled version of the scalar state estimates $\hat{X}(n/n-1)$. At first glance this may seem unacceptable for control applications where it is generally assumed that the state estimate must be available for each instant n. But other recent unpublished work by the authors has shown that this need not always be the case. Further, in many control applications what one really needs is not the state estimate but a linear functional of the state estimate (usually of lower dimension than the state dimension N) for all time instants n. We derive below a block version of this linear functional, and also show a technique by which it is possible to obtain interpolated state estimates between sample instants kL.

8.1 LINEAR FUNCTIONAL ESTIMATOR

Let f(n) = g'x(n) be a collection of r linear functionals to be estimated. Now the pair of equations,

$$x(n+1) = ax(n) + bu(n) \tag{14a}$$

$$f(n) = g'x(n) \tag{14b}$$

may be blocked to give

$$X_{k+1} = AX_k + BU_k \tag{15}$$

$$F_k = G'X_k + JU_k \ , \tag{16}$$

where A, B, X_k, U_k are as before in the BSM and,

$$G' = \begin{bmatrix} g' \\ g'a \\ \vdots \\ g'a^{L-1} \end{bmatrix} \tag{17a}$$

$$J = [J_{ij}], \qquad J_{ij} = \begin{cases} 0, & i \leq j \\ g'a^{i-j-1}b, & i > j \end{cases} \tag{17b}$$

It follows from (16) that

$$\begin{aligned} \hat{F}_{k/k} &= G'\hat{X}_{k/k} + JE\{U_k | \tilde{Z}_k\} \\ &= G'\hat{X}_{k/k} + JQD'(C'\textstyle\sum_{k/k-1} C + \tilde{R})^{-1} \tilde{Z}_k \end{aligned} \tag{18}$$

using (A.17) and (A.20), where $\hat{F}_{k/k}$ represents the blocked linear functional estimator,

$$\hat{F}_{k/k} = \begin{bmatrix} f'(kL/\overline{k+1}\ L-1) \\ f'(kL+1/\overline{k+1}\ L-1) \\ \vdots \\ f'(kL+L-1/\overline{k+1}\ L-1) \end{bmatrix} \tag{19}$$

Also, $\tilde{Z}_k$ is the innovation sequence defined in the appendix.

8.2 INTERPOLATED STATE ESTIMATOR

It may be desirable to obtain an estimate of x(kL+α) for some fixed α in the range 0 < α < L in addition to x(kL) for k=0,1,2,.... We refer to this estimate as the interpolated state estimate and denote it by $\hat{x}(kL+\alpha|k)$.

$$\hat{x}(kL+\alpha|k) \triangleq \hat{x}(kL+\alpha|Z_0,Z_1,\ldots Z_k) \tag{20}$$

Now from the state equation

$$x(k+1) = ax(k)+bu(k)$$

it is easy to observe that

$$x(kL+\alpha) = a^{\alpha}x(kL) + HU_k \tag{21}$$

with

$$H = [a^{\alpha-1}b,\ a^{\alpha-2}b, \ldots, ab,\ b,\ 0,\ \ldots, 0] \tag{22}$$

where U_k is the block input as before, see (3b). From (20), (21), it follows that

$$\begin{aligned} \hat{x}(kL+\alpha|k) &= a^{\alpha}\hat{x}(kL/Z_0,Z_1,\ldots Z_k) \\ &\quad + HE\{U_k|Z_0,Z_1,\ldots,Z_k\} \\ &= a^{\alpha}\hat{X}_{k/k} + HQD'(C'\textstyle\sum_{k/k-1}C+R)^{-1}\tilde{Z}_k \end{aligned} \tag{23}$$

using (A.17) and (A.20).

Fig. 2 is a block diagram showing the various estimates we have obtained so far.

8.3 FIXED LAG SMOOTHING

We have already seen that the signal estimator $\hat{Y}_{k/k}$ estimates Y_k more accurately than the blocked version of $\hat{y}(k/k)$ because of the *smoothing* involved. It is possible to further improve the estimate of Y_k by using further smoothing; the price paid being increased computation and delay in obtaining the estimate. We sketch below one approach to fixed-lag smoothing of the signal. For details on fixed-lag smoothing, see [5].

We wish to obtain the block smoothed estimates $\hat{Y}_{k-M/k}$ for all k and a fixed M.

In general,

$$\hat{Y}_{k-i/k} = \hat{Y}_{k-i/k-1} + K_k^{(i)}\tilde{Z}_k, \quad (24)$$

with

$$K_k^{(i)} = E\{Y_{k-i}/\tilde{Z}_k\}, \; i = 1,2,..,M, \quad (25)$$

and $\tilde{Z}_k$ the innovation at time k. A block diagram is shown in Fig. 3. The calculation of the gain in (25) is achievable by methods of [5], virtually identically with the conventional case.

Now from the definitions it follows that

$$\hat{Y}_{k/k+M} = \begin{bmatrix} \hat{y}(kL/kL + (M+1)L-1) \\ \hat{y}(kL+1/kL + (M+1)L-1) \\ \vdots \\ \hat{y}(kL+L-1/kL + (M+1)L-1) \end{bmatrix} \quad (26)$$

It is seen that the minimum smoothing interval is ML time instants, and hence if the block length L is comparable to the dominant time constant of the system we can obtain almost all the improvement which smoothing can offer with M = 3 or 4 [5], and a small increase in the estimator complexity.

9. CONCLUSIONS

We have shown the possibility of applying the block processing technique to the problem of state/signal estimation in noisy linear systems. It is seen that smoothed estimates of the signal can be naturally obtained using this approach (some of the signal estimates are of course delayed due to the blocking process), and further that the blocking process promotes stability and reduces the sensitivity to computational errors. There also exists the possibility of reduced computational efforts associated with any use of block processing, though the derivation of this advantage presupposes a time-invariant filter, which in turn presupposes that the underlying random processes are stationary.

We have dealt exclusively with linear systems and the regular Kalman filter in this paper; it is likely that the concept of blocking could be applied to non-linear systems and their corresponding estimators such as the extended Kalman filter (EKF) [6].

It is conjectured that the smoothing effect in the signal estimates would result in more accurate linearization (at least when all the system nonlinearity is associated with the measurement equation) in the case of the EKF, and reduce the possibility of filter divergence. This is an area with considerable research promise.

APPENDIX A

In this appendix we derive the Kalman filter for the BSM (2), with observations

$$Z_k = Y_k + V_k = C'X_k + DU_k + V_k$$

Here, U_k, V_k are assumed to be zero mean white gaussian processes with

$$\text{Cov}\begin{bmatrix} U_k \\ V_k \end{bmatrix} = \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \quad (A.1)$$

The initial state X_0 has zero mean, $\text{Cov}\{X_0\} = P_0$, and is uncorrelated with U_k, V_k for all k. Further we assume that R is positive definite.

The direct feedthrough term with gain D in (2b) prevents use of the standard Kalman filter such as in [5]. Though it is possible to derive a Kalman filter directly for (2a), (2b), (2c), in the interests of brevity we perform a transformation to the standard form.

Define

$$\bar{V}_k \triangleq DU_k + V_k \quad (A.2)$$

$$\bar{U}_k \triangleq U_k - QD'(DQD' + R)^{-1}\bar{V}_k \quad (A.3)$$

It is readily observed from (A.1), (A.2) and (A.3) that,

$$\text{Cov}\begin{bmatrix} \bar{U}_k \\ \bar{V}_k \end{bmatrix} = \begin{bmatrix} \bar{Q} & 0 \\ 0 & \bar{R} \end{bmatrix} \quad (A.4)$$

where

$$\bar{Q} = Q-QD'(DQD' + R)^{-1}DQ \quad (A.5)$$

$$\bar{R} = DQD' + R \quad . \quad (A.6)$$

Substituting (A.2), (A.3) in (2) we obtain,

$$\begin{aligned} X_{k+1} &= AX_k + B\bar{U}_k + BQD'\bar{R}^{-1}(DU_k + V_k) \\ &= (A-BQD'\bar{R}^{-1}C')X_k + B\,\bar{U}_k + BQD'\bar{R}^{-1}Z_k \\ &= \bar{A}X_k + B\,\bar{U}_k + BQD'\bar{R}^{-1}Z_k\,, \end{aligned} \quad (A.7)$$

with

$$\bar{A} = (A - BQD'\bar{R}^{-1}C')$$

and

$$\begin{aligned} Z_k &= C'X_k + DU_k + V_k \\ &= C'X_k + \bar{V}_k \quad . \end{aligned} \quad (A.8)$$

We note that (A.7), (A.8) represents a system with uncorrelated input and measurement noise processes, and an input $BQD'\bar{R}^{-1}Z_k$ which may be considered deterministic at time instant k since Z_k is available at time k. We use the standard Kalman filter derived in [5]. See Appendix B for details on the notations used.

$$\hat{X}_{k+1/k} = \bar{A}\,\hat{X}_{k/k} + BQD'\bar{R}^{-1}Z_k, \; \hat{X}_{0/-1} = 0$$

TIME UPDATE (A.9)

$$\hat{X}_{k+1/k+1} = \hat{X}_{k+1/k} + L_{k+1}\tilde{Z}_{k+1}$$

MEASUREMENT UPDATE (A.10)

where

$$L_k = \sum_{k/k-1} C(C'\sum_{k/k-1} C + \bar{R})^{-1} . \qquad (A.11)$$

The corresponding error covariances are,

$$\sum_{k+1/k} = \bar{A} \sum_{k/k} \bar{A}' + B\bar{Q}B', \qquad (A.12)$$

$$\sum_{k/k} = \sum_{k/k-1} - \sum_{k/k-1} C(C'\sum_{k/k-1} C+\bar{R})^{-1} C'\sum_{k/k-1}.$$

$$\sum_{0/-1} = P_0 \qquad (A.13)$$

We can combine (A.9) and (A.10) to obtain,

$$\hat{X}_{k+1/k} = \bar{A}(I-L_kC')\hat{X}_{k/k-1} + K_kZ_k \qquad (A.14a)$$

and

$$\hat{X}_{k+1/k+1} = (I-L_{k+1}C')\bar{A}\hat{X}_{k/k} + (I-L_{k+1}C')BQD'\bar{R}^{-1} Z_k$$

$$+ L_{k+1}Z_{k+1} \qquad (A.14b)$$

with

$$K_k = \bar{A}L_k + BQD'\bar{R}^{-1}$$

$$= (A\sum_{k/k-1}C + BQD')(C'\sum_{k/k-1}C + \bar{R})^{-1} \qquad (A.15)$$

after some algebraic manipulation. Similarly (A.12) and (A.13) may be combined to yield,

$$\sum_{k+1/k} = \bar{A}[\sum_{k/k-1} - \sum_{k/k-1}C(C'\sum_{k/k-1}C+\bar{R})^{-1} C'\sum_{k/k-1}]\bar{A}'$$

$$+ B\bar{Q}B' \qquad \text{(RICCATI EQN)} \qquad (A.16)$$

Further, the innovation sequence $\{\tilde{Z}_k\}$ is white and

$$\text{Cov } \{\tilde{Z}_k\} = C'\sum_{k/k-1}C + \bar{R} . \qquad (A.17)$$

The signal estimates are

$$\hat{Y}_{k/k-1} = C'\hat{X}_{k/k-1} + DE\{U_k|\tilde{Z}_{k-1}\} = C'\hat{X}_{k/k-1} \qquad (A.18)$$

and

$$\hat{Y}_{k/k} = C'\hat{X}_{k/k} + DE\{U_k|\tilde{Z}_k\} . \qquad (A.19)$$

Now

$$E\{U_k \tilde{Z}_k'\} = E\{U_k(Z_k - C'\hat{X}_{k/k-1})'\}$$

$$= E\{U_kZ_k'\}$$

$$= E\{U_k(C'X_k + DU_k + V_k)'\}$$

$$= QD' . \qquad (A.20)$$

Further,

$$E\{U_k|\tilde{Z}_k\} = E[U_k\tilde{Z}_k'][E\{\tilde{Z}_k\tilde{Z}_k'\}]^{-1}\tilde{Z}_k$$

So combining (A.19) and (A.20) we obtain, using (A.17),

$$\hat{Y}_{k/k} = C'\hat{X}_{k/k} + M_k\tilde{Z}_k \qquad (A.21)$$

where

$$M_k = DQD'(C'\sum_{k/k-1} C + \bar{R})^{-1} . \qquad (A.22)$$

APPENDIX B

NOTATION

$\hat{X}_{k/\ell}$ = Minimum variance estimate of X_k based on the observation set $\{Z_0,Z_1,\ldots,Z_\ell\}$

$\tilde{X}_{k/\ell} = X_k - \hat{X}_{k/\ell}$ = Estimation error

$\sum_{k/\ell}$ = Cov $\{\tilde{X}_{k/\ell}\}$ = Error covariance

$\tilde{Z}_k = Z_k - C'\hat{X}_{k/k-1}$ = Innovation at block time k. $\{\tilde{Z}_k\}$ is called the innovation sequence.

$\hat{Y}_{k/\ell}$ = Minimum variance estimate of the signal Y_k based on the observations $\{Z_0,Z_1,\ldots,Z_\ell\}$

$\hat{x}(k/\ell)$, $\tilde{x}(k/\ell)$, $\sum(k/\ell)$, $\tilde{z}(k)$ and $\hat{y}(k/\ell)$ represent the above quantities for the unblocked model with the observation set $\{z(0),z(1),\ldots,z(\ell)\}$.

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	Kalman Filter for BSM Approach (a)		Blocked Kalman Filter Approach (b)	
	ONE STEP PREDICTOR	TRUE FILTER	ONE STEP PREDICTOR	TRUE FILTER
STATE	$\hat{x}(kL/kL-1)$	$\hat{x}(kL/kL+L-1)$	$\hat{x}(kL/kL-1)$	$\hat{x}(kL/kL)$
SIGNAL	$\hat{y}(kL/kL-1)$ $\hat{y}(kL+1/kL-1)$ . . . $\hat{y}(kL+L-1/kL-1)$	$\hat{y}(kL/kL+L-1)$ $\hat{y}(kL+1/kL+L-1)$ . . . $\hat{y}(kL+L-1/kL+L-1)$	$\hat{y}(kL+j/kL+j-1)$ $j=0,1,..,L-1$	$\hat{y}(kL+j/kL+j)$ $j=0,1,..,L-1$

Table I Comparison of the two approaches to block Kalman filtering.

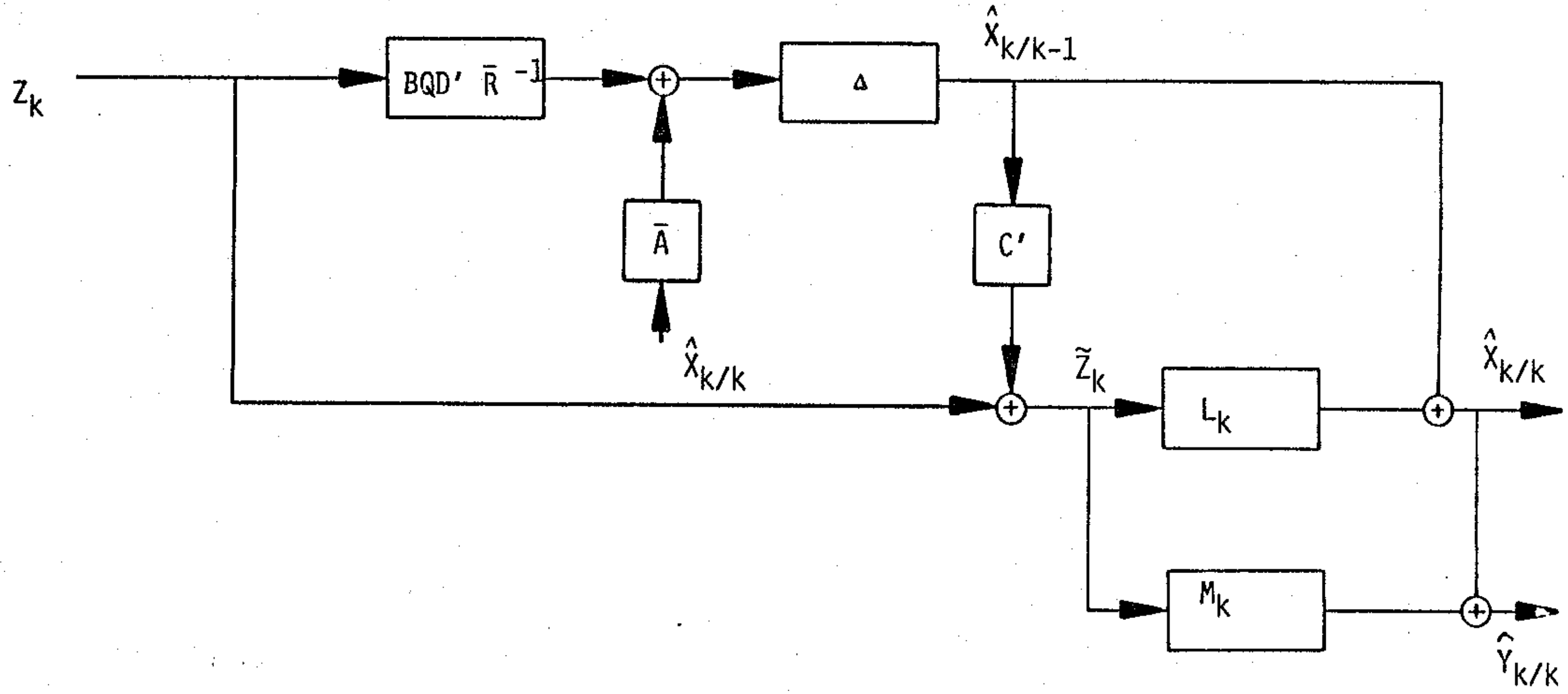

Figure 1. Block State and Signal Filter (Predictor)

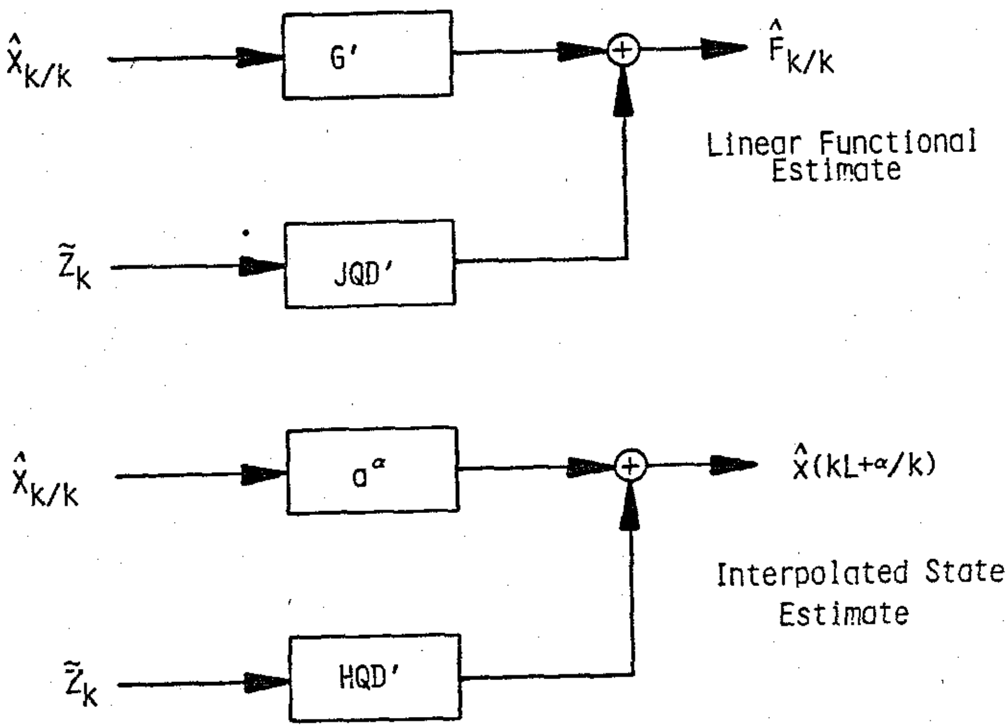

Figure 2. Block Linear Functional, Interpolated State Estimators