NEW RESULTS ON STATIONARY STOCHASTIC FEEDBACK PROCESSES

M. R. Gevers

Laboratoire d'Automatique et d'Analyse des Systèmes, Louvain University, Bâtiment Maxwell, B-1348-Louvain-la-Neuve, Belgium (on leave in 1980 at University of Newcastle, Australia)

B.D.O. Anderson

Department of Electrical and Computer Engineering, The University of Newcastle, NSW 2308, Australia.

Abstract. We consider stationary stochastic discrete-time vector processes made up of two components processes y and u, such that the joint (y,u)-process has a rational spectral density $\phi_{yu}(z)$. Such processes can be represented by a white noise driven matrix transfer function model, and (in most cases) by a closed-loop model. We present a number of new results on the connections between these two representations, and on their properties : stability, invertibility, identifiability, uniqueness of the spectral factorization, detection of feedback, continuity of spectral factors.

Keywords. Multivariate stochastic processes ; spectral factorization ; feedback processes ; identifiability ; feedback detection.

1. INTRODUCTION

We consider stationary stochastic processes made up of two vector component processes y and u. Such processes arise, for example, in feedback processes or in any situation where a vector stochastic process can be thought of as made up of two subprocesses that play different roles. We assume that the joint (y,u)-process has a rational spectral density matrix function $\phi_{yu}(z)$. Assume that $\{W(z), Q\}$ is a spectral factorization of $\phi_{yu}(z)$, i.e.

$$\phi_{yu}(z) = W(z)QW^*(z) \qquad (1.1)$$

where $W(z)$ is real rational and stable, Q is real nonnegative definite symmetric and $W^*(z)$ denotes $W^T(z^{-1})$. Then the following representation will be called a joint model for (y,u)[1]

$$\begin{bmatrix} y_i \\ u_i \end{bmatrix} = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} \begin{bmatrix} w_i \\ v_i \end{bmatrix} \qquad (1.2)$$

where

$$E\left\{ \begin{bmatrix} w_i \\ v_i \end{bmatrix} \begin{bmatrix} w_j^T & v_j^T \end{bmatrix} \right\} = Q\delta_{ij} \quad . \qquad (1.3)$$

Here $\dim w_i \geqslant \dim y_i = p$, $\dim v_i \geqslant \dim u_i = q$.

We shall consider that the (y,u)-process may also originate from a linear closed-loop transfer function model (see Fig. 1) :

$$y_i = F(z)u_i + G(z)w_i \qquad (1.4a)$$
$$u_i = H(z)y_i + K(z)v_i \qquad (1.4b)$$

[1] For reasons of brevity, the argument z will often be omitted in the sequel.

where F,G,H,K are causal, real rational transfer function matrices, and (w,v) is a white noise process with covariance given by (1.3).

The following assumptions will be made throughout the paper and will not be repeated in the sequel :

A1 : (y,u) is a stationary full rank bounded stochastic process

A2 : there is a delay in the closed-loop, i.e. ,

$$F(\infty)H(\infty) = 0, \text{ where } F(\infty) = \lim_{z\to\infty} F(z) \; .$$

As an alternative to model (1.4), we shall also sometimes use a difference equation representation for the feedback process (y,u). Let A(z), B(z), C(z), D(z), M(z), N(z) be polynomial matrices such that $A^{-1}(B\vdots C)$ is a left coprime polynomial matrix fraction description (MFD) of $(F\vdots G)$, and, similarly, $D^{-1}(M\vdots N) = (H\vdots K)$, a left coprime polynomial MFD (Rosenbrock, 1970 ; Wolowich, 1974 ; Kailath, 1980). Then the model (1.4) can be replaced by the following closed-loop MFD model :

$$A(z)y_i = B(z)u_i + C(z)w_i \qquad (1.5a)$$
$$D(z)u_i = M(z)y_i + N(z)v_i \qquad (1.5b)$$

Notice that A,B,C are determined from F,G up to left multiplication by unimodular matrices, and similarly for D,M,N.

Given a real rational spectrum $\phi_{yu}(z)$, we know that the spectral factorization (1.1) is not unique. Actually we have the following result, which is a discrete-time extension of Youla (1961).

Spectral Factorization Theorem : Let $\phi(z)$ be a nxn real rational full rank spectral density matrix :

a) Then there exists a unique factorization of the form $\phi(z) = \overline{W}(z)\overline{Q}\,\overline{W}^*(z)$, in which $\overline{W}(z)$ is nxn real rational, stable, minimum phase and such that $\overline{W}(\infty) = I$, with $\overline{Q}$ positive definite symmetric.

b) Any other factorization of the form $\phi(z)= W(z)QW^*(z)$ in which $W(z)$ is real rational, and Q is nonnegative definite symmetric, is such that $W(z) = \overline{W}(z)V(z)$, where $V(z)$ is a real rational scaled paraunitary matrix, i.e. $V(z)QV^*(z) = \overline{Q}$. Moreover $V(z)$ is stable if and only if $W(z)$ is stable.

The unique (canonical) spectral factor $\{\overline{W}(z), \overline{Q}\}$ defined in part a) of the theorem, will be called the normalized minimum phase spectral factor (NMSF) of $\phi(z)$. Two spectral factorizations $\{W_1(z),Q_1\}$ and $\{W_2(z),Q_2\}$ of $\phi(z)$ will be called equivalent if

$$\phi(z) = W_1(z)Q_1W_1^*(z) = W_2(z)Q_2W_2^*(z) \quad .(1.6)$$

There is a bijective relation (one-to-one and onto) between the closed-loop model (1.4) and the joint model (1.2) :

$$\begin{cases} W_{11} = (I-FH)^{-1}G & W_{12} = (I-FH)^{-1}FK \\ W_{21} = (I-HF)^{-1}HG & W_{22} = (I-HF)^{-1}K \end{cases} \quad (1.7)$$

$$\begin{cases} F = W_{12}W_{22}^{-1} & G=W_{11}-W_{12}W_{22}^{-1}W_{21} \\ H = W_{21}W_{11}^{-1} & K=W_{22}-W_{21}W_{11}^{-1}W_{12} \end{cases} \quad (1.8)$$

Because of this bijective relation we can define the normalized minimum phase realization (NMR) of the (y,u)-process as the unique closed-loop model $\{\overline{F},\overline{G},\overline{H},\overline{K}\}$ obtained from the NMSF $\overline{W}(z)$ by (1.8) and with covariance $\overline{Q}$. For reasons that will become clear later we shall define the absence of feedback from y to u in the following way.

Definition 1 : Consider a process (y,u) with rational spectrum $\phi_{yu}(z)$. There is no feedback from y to u (or the process is $(y\not\to u)$ (feedback free) if and only if the NMSF $\{\overline{W}(z), \overline{Q}\}$ of $\phi_{yu}(z)$ has $\overline{W}_{21}(z) = 0$ and $\overline{Q}$ block-diagonal (or equivalently the NMR $\{\overline{F},\overline{G},\overline{H},\overline{K},\overline{Q}\}$ has $\overline{H}(z) = 0$ and $\overline{Q}$ block-diagonal).

Starting from the relations (1.7)-(1.8) and the spectral factorization theorem, we present a number of new results that answer the following questions :

- Given a closed-loop model, what are the conditions on F,G,H,K (or on A,B,C,D,M,N) that guarantee the stability of W(z) (or, equivalently, the stationarity of (y,u) ?
- What are the conditions on F,G,H,K that will make W(z) minimum phase ?
- Given a rational spectrum $\phi_{yu}(z)$, under what conditions does a stable closed-loop model exist for (y,u) ?
- Under what conditions can F,G,H,K be uniquely identified from a given spectrum $\phi_{yu}(z)$?
- Assuming that a closed-loop system is given and that w_i and v_j are uncorrelated for all i,j (i.e. Q is block-diagonal), under what conditions can this be observed from the spectrum $\phi_{yu}(z)$?
- Assuming a closed-loop model { F,G,H,K} with covariance Q has H = 0 and Q block-diagonal, is the process $(y\not\to u)$ feedback-free ?
- Conversely, assuming a (y,u)-process has no feedback from y to u, under what conditions do all closed-loop models of that process have H = 0 and Q block-diagonal ?

It is clear from this list that our new results on stochastic feedback processes cover a large amount of material. Here we present without proof a survey of our main results in a unified treatment. The proofs are presented in a number of papers that deal specifically with the various issues raised : identifiability of feedback systems, stability of closed-loop systems, detection of feedback between stochastic processes, equivalence between different representations for joint processes (Anderson and Gevers, 1981a, 1981b ; Gevers and Anderson, 1981a, 1981b).

2. PROPERTIES OF CLOSED-LOOP MODELS

In this section we examine under what conditions a closed-loop model produces a stationary (y,u)-process, and also under what conditions (on F,G,H,K) the corresponding joint model W(z) is minimum phase. Conversely we present conditions on $\phi_{yu}(z)$ which ensure that a stable feedback model F,G,H,K exists for the (y,u)-process.

We consider a process (y,u) described by the closed-loop model (1.4). In addition we assume that (1.4) may have been obtained from a model of the form (see Fig. 2) :

$$\begin{cases} y_i = F_2(F_1u_i + G_1w_i) & (2.1a) \\ u_i = H_2(H_1y_i + K_1v_i) & (2.1b) \end{cases}$$

This arises when the noises w_i and v_i enter the plant and the regulator at some internal part, and are then referred to the output. The model (1.4) is then equivalent with (2.1) provided

$$F = F_2F_1 \ , \ G = F_2G_1 \ , \ H = H_2H_1 \ , \ K = H_2K_1 \quad (2.2)$$

We first introduce some notations and definitions.

Definition 2 : Let H(z) be a proper rational transfer function matrix. The unstable part of H(z), written $H_+(z)$, is the sum of those terms in a partial fraction expansion of

H(z) that have poles in $|z| \geq 1$. The stable part of H(z) is then $H_-(z) \triangleq H(z) - H_+(z)$.

Denoting by $\delta[H]$ the McMillan degree of H(z) (Mc Millan, 1952 ; Kalman, 1965) we have $\delta[H] = \delta[H_+] + \delta[H_-]$.

Definition 3 : Let F and H be two proper transfer function matrices. There is no unstable pole-zero cancellation in the product FH if $\delta[(FH)_+] = \delta[F_+] + \delta[H_+]$.

We now have our main result on stability of closed-loop models.

Theorem 2.1 : Consider the joint process (y,u) generated by the closed-loop model (1.4) or (1.5). Then (y,u) is stationary (i.e. W(z) has all its poles in $|z| < 1$) if and only if either one of the following two conditions hold :

1) $\det \begin{bmatrix} A & -B \\ -M & D \end{bmatrix} \neq 0$ for $|z| \geq 1$ (2.3)

2) $\delta[F_+ \vdots G_+] = \delta[F_+]$, $\delta[H_+ \vdots K_+] = \delta[H_+]$ (2.4)

and the closed loop is stable.

The closed loop is stable if and only if

a) $(I-FH)^{-1}, (I-FH)^{-1}F, (I-HF)^{-1}$ and $(I-HF)^{-1}H$ have all their poles in $|z| < 1$, or (2.5)

b) $(I-FH)^{-1}$ has all its poles in $|z| < 1$ and there is no unstable pole-zero cancellation in FH. (2.6)

Comments : Condition (2.3) insures that the closed-loop is stable and that the entering noises are stationary. Condition (2.4) means that all instabilities of the process noise model come from referring the noise to the output, and similarly for the regulator noise. This corresponds to a model like Fig. 2 where G_1 and K_1 are stable, but F_1 , F_2 , H_1 , H_2 can be unstable provided the closed-loop is stable. Conditions for closed-loop stability have been studied by Desoer and Chan (1976) where (2.5) was derived ; (2.3),(2.4) and (2.6) are new results.

The next result gives conditions on the closed-loop model under which the corresponding W(z) is minimum phase[2].

Theorem 2.2 : Consider the closed-loop model (1.4) with G(z) and K(z) square and the corresponding W(z) derived from (1.7). Then W(z) is minimum phase if and only if

1) $\delta[F_+ \vdots G_+] = \delta[G_+]$, $\delta[H_+ \vdots K_+] = \delta[K_+]$ (2.7)

2) G and K are minimum phase (2.8)

Conditions (2.7) implies that F does not contain any instabilities that are not in G, and similarly for H and K. This corresponds to a situation like that of Fig. 2 where F_1 and H_1 are stable.

Finally we show that almost every stationary (y,u)-process with a rational spectrum $\phi_{yu}(z)$ admits a stable closed-loop representation.

Theorem 2.3 : Consider a joint process (y,u) with real rational spectrum $\phi_{yu}(z)$ and assume that $\phi_{yu}(z)$ is positive definite on $|z| = 1$. Let W(z) be any stable minimum phase spectral factor of $\phi_{yu}(z)$. Then the closed-loop model (1.4), obtained from W(z) by (1.8), is stable.

The only significant restriction of Theorem 2.3 is that the spectrum $\phi_{yu}(z)$ not have any zero on the unit circle. Therefore any strictly minimum phase W(z) produces a stable closed-loop model (F,G,H,K). However this is not a necessary condition.

Example : The closed-loop model obtained from $F = \frac{1}{z-1}$, $G = 1$, $H = -1.5$, $K = 1$ is stable, even though the corresponding W(z) is not strictly minimum phase.

Theorem 2.3 justifies the interest of studying closed-loop models of the form (1.4) for the representation of jointly stationary processes.

3. IDENTIFIABILITY OF CLOSED-LOOP MODELS

We now examine under what conditions closed-loop models can be identified from the spectrum $\phi_{yu}(z)$ of the joint process. We consider here that a physical (but unknown) closed-loop system of the form (1.4) exists but that only the spectrum $\phi_{yu}(z)$ is known. The identifiability of feedback systems of a more general form (with "one-sided" correlation between process noise and regulator noise) is considered in Anderson and Gevers (1981b).

First we define precisely what we mean by identifiability. Loosely speaking the question is : can we identify the transfer matrices F,G,H,K and the noise covariance Q of a closed-loop system (1.4) from knowledge of $\phi_{yu}(z)$. Since the relation between W(z) and {F,G,H,K} is one-to-one, the critical step is the non-unique spectral factorization. Now it turns out that, if a priori knowledge is available on the delay structure (e.g. a delay in F, or in H, or in both) of the system and on the noise covariance, then this restricts, via the relation (1.7)-(1.8), the class of spectral factorizations that can be considered.

Definition 4 : Given the spectrum $\phi_{yu}(z)$ of a joint process (y,u) generated by (1.4), we shall say that the spectral factorization $\phi_{yu}(z) = W(z)\, QW^{*}(z)$ is admissible if W(z) and Q are consistent with the a priori knowledge on the delay structure and the noise correlation of the closed-loop system.

For example, if there is a delay in F and H, then $W(\infty)$ must be block-diagonal (see (1.7)).

[2] We say that W(z) is minimum phase if it is square and if $W^{-1}(z)$ is causal and analytic in $|z| > 1$.

If the noises are uncorrelated, then Q must be block-diagonal. To any set of a priori conditions there corresponds a class of admissible spectral factorizations $\{W(z),Q\}$ and it is always possible to define a unique (canonical) element in this class which we shall denote $\{\tilde{W}(z),\tilde{Q}\}$. For example, the NMSF is a canonical admissible spectral factor if it is known that there is a delay in F and H.

Definition 5 : Consider the spectrum $\phi_{yu}(z)$ generated by the closed-loop system (1.4). Let $\{\tilde{W}(z),\tilde{Q}\}$ be a canonical admissible spectral factor of $\phi_{yu}(z)$, and let $\{\tilde{F},\tilde{G},\tilde{H},\tilde{K}\}$ be obtained from $\tilde{W}(z)$ by (1.8). Then the system (1.4) is identifiable if

$$F = \tilde{F},\ G = \tilde{G}V_1\ ,\ H = \tilde{H},\ K = \tilde{K}V_2 \qquad (3.1)$$

where $V_1(z)$ and $V_2(z)$ are scaled paraunitary matrices such that

$$V_1(z)Q_{11}V_1^*(z)=\tilde{Q}_{11},V_2(z)Q_{22}V_2^*(z) = \tilde{Q}_{22} \qquad (3.2)$$

Here Q_{ii} and $\tilde{Q}_{ii}$ are the (i,i) blocks of Q and $\tilde{Q}$. Notice that we allow the matrices $\tilde{G}$ and $\tilde{K}$ to differ from the true G and K by right multiplication by scalar paraunitary matrices. This does not influence the input-output characteristics of the model. The main result on identifiability of closed-loop models of the form (1.4) is as follows.

Theorem 3.1 : The closed-loop system (1.4) is identifiable if it is known a priori that either one of the following conditions hold :

1) w_i and v_j are uncorrelated for all i,j. (3.3)

2) $\delta[F_+ \vdots G_+] = \delta[G_+]$, $\delta[H_+ \vdots K_+] = \delta[H_+]$; there is a delay in both F and H ; G and K are minimum phase while $G(\infty)$ and $K(\infty)$ are nonsingular.

Comment : Part 1) of Theorem 4.1 is an improvement over the results of Ng, Goodwin and Anderson (1977) where, in addition to condition (3.3), a delay was required in both F and H. Here a delay is required in only one of F and H by the standing assumption A.2. Part 2) is a new result ; note that by Theorem 2.2 the conditions of part 2) imply that W(z) must be minimum phase, which considerably restricts the class of admissible spectral factors.

Other results on identifiability of feedback systems can be found in Anderson and Gevers (1981b).

4. GENERIC CLOSED-LOOP SYSTEMS WITH UNCORRELATED NOISES

Most useful results on feedback systems are obtained by studying the class of equivalent spectral factors of a given spectrum. This is the basis for our identifiability results of the previous section. Deeper results can be obtained by noting that almost all feedback systems obey a certain generic property which we now define.

Definition 6 : Consider the closed-loop system (1.4) with the equivalent model (1.5) obtained from left coprime MFD's, and let W(z) be the corresponding joint model and $\phi_{yu}(z)$ the spectrum. Let r = highest power of z^{-1} in det $C(z)C^*(z)$ and s = highest power of z^{-1} in det $N(z)N^*(z)$. Then the system $\{F,G,H,K\}$ will be called generic if

1) W(z) has minimal degree, i.e. $\delta[W(z)] = 1/2\delta[\phi_{yu}(z)]$ (4.1)

2) z^r det $C(z)C^*(z)$ and z^s det $N(z)N^*(z)$ have no common zeros. (4.2)

When G and K are square, the zeros of det CC^* are the zeros of det C and their inverses, and similarly for the zeros of det NN^*. Notice that the zeros of det C are zeros of G and poles of F, while the zeros of det N are zeros of K and poles of H. Therefore (4.2) will almost always hold. The minimal degree condition (4.1) will be satisfied if there is no pole-zero cancellation between a pole of W(z) and a zero of $W^*(z)$. Therefore (4.1) and (4.2) are very natural conditions which will almost always be satisfied. The case of nonsquare G and K is treated in Gevers and Anderson (1981a).

We now present our main result for generic systems. It has important consequences in terms of the identifiability of feedback systems, the detection of feedback, and the detection of correlation between process noise and regulator noise from the spectrum $\phi_{yu}(z)$.

Theorem 4.1 : Consider the closed-loop model (1.4) with the following additional assumptions :

1) the model is generic

2) $F(\infty) = H(\infty) = 0$; $G(\infty)$ and $K(\infty)$ have full rank

3) Q is block-diagonal.

Let W(z) be the corresponding joint process model, and $\phi_{yu}(z) = W(z)QW^*(z)$. If $\{\hat{W}(z),\hat{Q}\}$ is any other minimal degree spectral factor of $\phi_{yu}(z)$ with $\hat{W}(\infty)$ block-diagonal and nonsingular, then

a) $\hat{Q}$ is block-diagonal

b) the scaled paraunitary matrix $V(z) = \hat{W}(z)^{-1} W(z)$ is block-diagonal : $V(z)=\mathrm{diag}(V_1(z), V_2(z))$

c) the model $\hat{F},\hat{G},\hat{H},\hat{K}$ corresponding to $\hat{W}(z)$ is generic and $F = \hat{F}$, $G = \hat{G}V_1$, $H = \hat{H}$, $K = \hat{K}V_2$.

An important consequence of Theorem 4.1 is that, in a generic closed-loop system with a delay in F and H and with G and K square, the absence of correlation between the process noise and the regulator noise can be tested from the spectrum $\phi_{yu}(z)$ of the joint process or, more precisely, from the NMSF of $\phi_{yu}(z)$. If the NMSF is generic and has a block-diagonal $\bar{Q}$, then the physical closed-loop process has orthogonal noise sources and it can be identified. A similar but weaker result was first obtained by Sin and Goodwin (1980).

5. FEEDBACK-FREE PROCESSES

We have seen in Theorem 2.3 that almost all stationary joint processes with a real rational spectrum can be represented by a feedback model (1.4). One interesting question about jointly stationary processes is wether there actually is feedback from one of the subprocesses, say y, to the other subprocess u. There are of course several ways of defining the absence of feedback from y to u (see e.g. Caines, 1976). We have adopted Definition 1 (see section 1) because it can be shown to be equivalent with a number of other intuitively appealing definitions.

Denoting by X^j the space spanned by the components of $(x_{-\infty},\dots,x_{j-1}, x_j)$ and by $r|X^j$ the least mean square projection of r on X^j, we have the following result.

Theorem 5.1 : Consider a process (y,u) with rational spectrum $\phi_{yu}(z)$. Then Definition 1 for the absence of feedback from y to u is equivalent with any one of the following definitions :

1) $u_i \perp (y_j - y_j|U^{j-1}) \qquad i \geq j \qquad (5.1)$
2) $u_i \perp (y_j - y_j|Y^{j-1},U^{j-1}) \qquad i \geq j \qquad (5.2)$
3) $u_i|U^{i-1},Y^i = u_i|U^{i-1} \qquad (5.3)$

Definition (5.4) says that there is no feedback from y to u if u is orthogonal to the process obtained from the past y's after removing the effect of pastu's, and (5.2) is equivalent to (5.1). Finally (5.3) states that the prediction of u based on past u's is not affected by knowledge of past and present y's in a feedback-free case.

A natural question now is whether any closed-loop model F,G,H,K,Q that represents a (y→u) feedback-free process inherits the properties that H = 0 and Q is block-diagonal. Conversely, suppose a closed-loop model has H = 0 and Q block-diagonal ; is the process (y→u) feedback-free ? The results of the previous sections enable us to answer these questions.

Theorem 5.2 : Consider a closed-loop system (1.4) with a delay in F(z), and assume that H(z) = 0 and Q is block-diagonal. Let W(z) be the corresponding joint model and $\phi_{yu}(z)$ the corresponding spectrum. Then the NMSF $\{\bar{W}(z),\bar{Q}\}$ of $\phi_{yu}(z)$ has $\bar{W}_{21} = 0$ and $\bar{Q}$ block-diagonal, and the process is (y→u) feedback-free.

The converse is true for all generic systems.

Theorem 5.3 : Consider a (y→u) feedback-free joint process with spectrum $\phi_{yu}(z)$ and assume that the NMR of $\phi_{yu}(z)$ is generic. Then all equivalent minimal degree spectral factors $\{W(z),Q\}$ of $\phi_{yu}(z)$ with $W(\infty)$ block-diagonal and nonsingular have $W_{21}(z) = 0$ and Q block-diagonal and the corresponding closed-loop models have H(z) = 0.

Theorem 5.3 shows that, in a generic situation, if a process is (y→u) feedback-free, then any minimal degree spectral factorization with $W(\infty)$ block-diagonal and nonsingular (and not just the NMSF) will show this, i.e. will have $W_{21} = 0$ and Q block-diagonal.

Caines (1976) called a joint process (y,u) obeying Definition 1 "strong feedback-free". The new result here is that the strong feedback-free property can normally (i.e. except in non-generic cases) be detected from every square feedback representation of (y,u) with the proper normalization at $z = \infty$, and not just from the NMSF.

6. CONTINUITY OF SPECTRAL FACTORS

Many of the results presented above allow us to derive properties of closed-loop models or to identify such models from the joint spectrum $\phi_{yu}(z)$. However in most practical situations $\phi_{yu}(z)$ is not known but only an estimate $\hat{\phi}_{yu}(z)$ is available. Our results of this last section show that the NMSF $\{\bar{W}(z),\bar{Q}\}$ depends continuously on $\phi_{yu}(z)$, and therefore consistent estimates of $\phi_{yu}(z)$ yield consistent estimates of $\{\bar{W}(z),\bar{Q}\}$.

Theorem 6.1 : Suppose there is given a hermitian nxn joint spectrum $\phi_{yu}(\omega), \omega\in[-\pi,\pi]$, with $\phi_{yu}(-\pi) = \phi_{yu}(\pi)$, satisfying $0 < c_1 I \leq \phi_{yu}(\omega) \leq c_2 I < \infty$ for some positive constants c_1 and c_2. Define a norm by

$$||\phi_{yu}||_\infty = \sup_{\omega\in[-\pi,\pi]} ||\phi_{yu}(\omega)|| \qquad (6.1)$$

Then the NMSF $\bar{W}(z)$ for $|z| \geq 1$ and $\bar{Q}$ depend continuously on $\phi_{yu}(z)$.

It follows easily that in all cases where a closed-loop model (1.4) can be identified from the spectrum $\phi_{yu}(z)$, approximate estimates of F,$\tilde{G}$,H,$\tilde{K}$ and Q can be obtained from an approximate estimate $\hat{\phi}_{yu}(z)$ of $\phi_{yu}(z)$. Here $\tilde{G}$ and $\tilde{K}$ are the minimum phase equivalents of the true G and K ; recall Definition 5 and the comment thereafter.

It also follows that the absence of feedback from y to u can be detected from an estimate $\hat{\phi}_{yu}(z)$ of $\phi_{yu}(z)$. If a process is (y→u) feedback-free the NMSF $\{\hat{\bar{W}}(z),\hat{\bar{Q}}\}$ will have $\hat{\bar{W}}(z)$ approximately upper block triangular and $\hat{\bar{Q}}$ approximately block-diagonal.

Finally, our result on the detection of orthogonal noises in closed-loop models can also be extended to the case where only estimates of $\phi_{yu}(z)$ are available.

Theorem 6.1 : Consider a closed-loop system (1.4) with F,G,H,K generic, $F(\infty) = H(\infty)$ and $G(\infty)$ and $K(\infty)$ nonsingular. Let $\hat{\phi}_{yu}(z)$ be an approximation of the true spectrum $\phi_{yu}(z)$. Suppose the NMSF $\{\hat{\bar{W}}(z),\hat{\bar{Q}}\}$ of $\hat{\phi}_{yu}(z)$ has $\hat{\bar{Q}}$ approximately block-diagonal, that $\hat{\bar{W}}$ has a coprime MFD that has approximately the form

$$\begin{bmatrix} \hat{\bar{A}} & -\hat{\bar{B}} \\ -\hat{\bar{M}} & \hat{\bar{D}} \end{bmatrix}^{-1} \begin{bmatrix} \hat{\bar{C}} & 0 \\ 0 & \hat{\bar{N}} \end{bmatrix} \qquad (6.2)$$

and that this form is generic. Then the true noise covariance matrix is approximately block-diagonal and F,G,H,K are approximately identifiable from $\hat{\phi}_{yu}(z)$.

7. CONCLUDING REMARKS

We have presented an overview of new results on stationary feedback processes. We believe our study of feedback processes has interesting implications in that it provides answers to a number of questions that deal with the existence and stability of the closed-loop model, its invertibility, the connections between different representations producing the same joint spectrum, the identifiability of such models, the detection of feedback, the detection of orthogonal noises in the forward and feedback paths and the consistency of the model estimates obtained from estimated spectra. The proofs, together with other results and examples, will be presented in a series of papers that deal with each of the specific issues.

REFERENCES

Anderson, B.D.O., and M.R. Gevers (1981a). On multivariable pole-zero cancellations and the stability of feedback systems, IEEE Trans. Circuits & Systems, to appear.

Anderson, B.D.O., and M.R. Gevers, (1981b). Identifiability of linear stochastic systems operating under linear feedback, submitted for publication.

Caines, P.E.(1976). Weak and strong feedback-free processes, IEEE Trans. Autom. Control, AC-21, 737-739.

Desoer, C.A., and W.S. Chan (1976). The feedback inter-connection of multivariable systems, simplifying theorems for stability, Proc. IEEE, 64, 1, 139-144.

Gevers, M.R., and B.D.O. Anderson (1981a). Representations of jointly stationary stochastic feedback processes, Int. J. Control, to appear.

Gevers, M.R., and B.D.O. Anderson (1981b). On jointly stationary feedback-free stochastic processes. IEEE Trans. Autom. Control, to appear.

Kailath, T. (1980). Linear Systems, Prentice-Hall.

Kalman, R.E. (1965). Irreductible realizations and the degree of a matrix of rational functions, SIAM J. Appl. Math., 13, 2, 520-544.

McMillan, B. (1952). Introduction to formal realizability theory, Bell Syst. Tech. J., 31, 541-560.

Ng, T.S., G.C. Goodwin and B.D.O. Anderson (1977). Identifiability of MIMO linear dynamic systems operating in closed-loop, Automatica, 13, 477-485.

Rosenbrock, H.H. (1970). State Space and Multivariable Theory, New-York, Wiley.

Sin, K.S.,and G.C. Goodwin (1980). Checkable conditions for identifiability of linear systems operating in closed-loop, IEEE Trans. Autom. Control, AC-25, 722-729.

Wolovich, W.A. (1974). Linear Multivariable Systems, Springer-Verlag, New York.

Youla, D.C. (1961). On the factorization of rational matrices, IRE Trans. Inform. Theory, IT-7, 172-189.

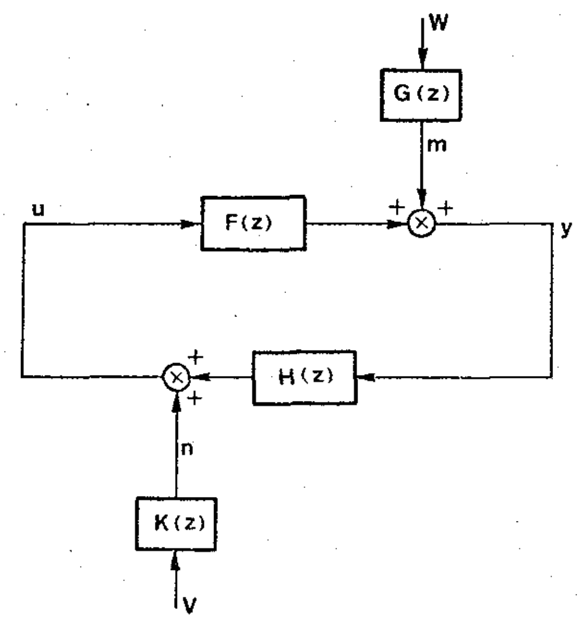

Figure 1

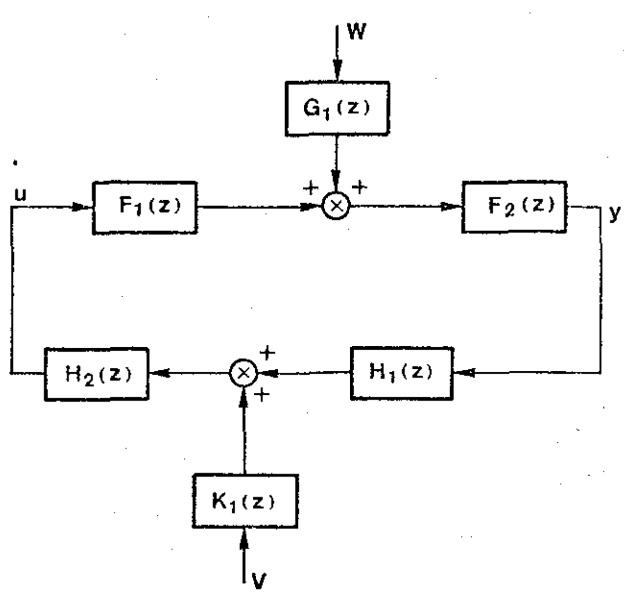

Figure 2