Proc. 1981 Joint Auto. Control Conference.

# ON REDUCED ORDER ADAPTIVE OUTPUT ERROR IDENTIFICATION AND ADAPTIVE IIR FILTERING

by

B. D. O. Anderson
Department of Electrical & Computer Engineering
University of Newcastle,
New South Wales, 2308, Australia.

C. Richard Johnson, Jr.
Department of Electrical Engineering
Virginia Polytechnic Institute
and State University
Blacksburg, VA 24061, U.S.A.

ABSTRACT

The reduced-order application of Landau's adaptive output error identifier (and therefore also the adaptive IIR filter HARF) results in a perturbed error system where the perturbation signal is a moving-average of the unmodeled portion of the unknown plant output (or desired signal in adaptive filter parlance). Proven in this paper is that if this perturbation signal is sufficiently small and a reduced-order dimension model is sufficiently excited then the output and parameter estimates of this adaptive identifier/filter remain bounded. The influence of various operating conditions on this quantitatively defined bound are noted. This robustness property is crucial in all real applications, which due to nonlinearities and distributed effects are subject to reduced-order modeling.

## 1. INTRODUCTION

The finite-dimensional, linear, differential or difference equation is a mathematical device that has found tremendous usefulness in the study of dynamic systems due to the accuracy with which such descriptions match the real behavior of numerous physical systems. These finite dimensional models are ultimately all approximations of infinite dimensional phenomena. Yet the theory of adaptive identification and control requires at least sufficient model order prespecification to prove exact recovery from a priori parameter uncertainty [1]-[4]. The more practical question concerns the effect of insufficiently dimensioned models on adaptive identification (and control).

Since the question of reduced-order model selection - even from (presumed) exact higher-order descriptions - for specific purposes such as prediction or control remains incompletely resolved, it may appear pretentious to resort to the "magic" of an adaptive solution to extract a satisfactory reduced-order model for a incompletely understood system given only input-output operating data. Though specialized search algorithms, e.g. [5], have been shown to have desirable local convergence properties, this paper avoids this philosophical debate by asking a quite different question for a specific algorithm: How do adaptive schemes, such as identifiers, which are presently proven to behave "satisfactorily" only when the model order is not underestimated, survive in engineering practice where this condition is undeniably violated? An initial response to this question is obtained by appropriate definition of "satisfactory" behavior.

For proof of convergence adaptive identification schemes have been converted to stability problems [6]-[11], where convergence to a point is required for mathematically exact solution. In practice, convergence to a region near this point is satisfactory and occasionally indiscernible from the "exact" solution. This seemingly trivial shift from asymptotic stability allows a subtle but significant shift in the methods of evaluation of adaptive identifiers. This shift is supported by the practical need for such answers, which can bolster the confident usage of theoretically convergent identifiers, and the success of practical usage, which provides confidence that meaningful results exist. This paper employs this concept of a region of attraction for adaptive parameter estimators by focusing on a specific class of algorithms developed initially as output error identifiers [6]-[7], subsequently recognized as adaptive infinite impulse response filters [8] [12], and recently expanded to include model reference adaptive controllers [13]-[14]. Only their reduced-order identification and/or filtering usage will be addressed in this paper.

This paper builds on a qualitative Lyapunov analysis in [15] where the adaptive identifier/filter (AIF) of [8] was shown to maintain a bounded output and parameter error despite reduced-order usage, if the AIF dimension and not the full "real" system dimension was sufficiently excited. This concept of reduced-order sufficient excitation is again utilized for the original establishment of quantitative relationships between the output and parameter error bound and the unmodelable portion of the real output to be tracked. The next section follows the suggestion in [10] by transforming the AIF of [8] into a perturbed, nonlinear, time-varying state difference equation with the state being a concatenation of several past output and present parameter estimate errors and with the perturbation due to the non-ideal, i.e. reduced-order modeling, situation. (A similar reparameterization of the identifier in [6]-[7] with $\lambda = \infty$ is also possible.) The third section supports the inclusion of the sufficient excitation requirement and outlines the approach of the proofs. Section four provides the major original theoretical results: (i) some amount of mismodelling is allowed by retention of identifier stability no matter how large the initial output and parameter estimate errors and (ii) a quantitative relationship exists between the amount of mismodelling and the output and parameter estimate bounds. (Due to the space limitations the reader is referred to [16] for detailed proofs of the stated lemmas and theorems.) The implications and limitations of these new results are also discussed. The conclusion notes that these general results may be best extended by incorporating any additional structural knowledge regarding the general nature of the neglected system dynamics.

## 2. REDUCED-ORDER ADAPTIVE IDENTIFIER/FILTER ERROR MODEL

Before obtaining the equations for a reduced order identifier/filter, we review the standard equations for an output error identifier/filter. As shown in [11] the adaptive IIR filter of [8]

$$\hat{y}(k) = \sum_{i=1}^{n} \hat{a}_i(k)z(k-i) + \sum_{j=1}^{m} \hat{b}_j(k)u(k-j) \quad \text{(a priori output)} \tag{2.1}$$

$$\hat{a}_i(k+1) = \hat{a}_i(k) + \mu_i z(k-i)v(k), \mu_i > 0 \quad \text{(AR parameter update)} \tag{2.2}$$

$$\hat{b}_j(k+1) = \hat{b}_j(k) + \rho_j u(k-j)v(k), \rho_j > 0 \quad \text{(MA parameter update)} \tag{2.3}$$

$$v(k) = y(k) - z(k) + \sum_{i=1}^{n} c_i[y(k-i) - z(k-i)] \quad \text{(smoothed output error)}$$

$$= \frac{y(k) - \hat{y}(k) + \sum_{i=1}^{n} c_i[y(k-i) - z(k-i)]}{1 + \sum_{i=1}^{n} \mu_i z^2(k-i) + \sum_{j=1}^{m} \rho_j u^2(k-j)}, \tag{2.4}$$

$$z(k) = \sum_{i=1}^{n} \hat{a}_i(k+1)z(k-i) + \sum_{j=1}^{m} \hat{b}_j(k+1)u(k-1) \quad \text{(a posteriori output)} \tag{2.5}$$

with the desired output of

$$y(k) = \sum_{i=1}^{n} a_i y(k-i) + \sum_{j=1}^{m} b_j u(k-j) \tag{2.6}$$

can be written in the form of the error model of [9]

$$\underline{e}(k+1) = \underline{A}\underline{e}(k) + \underline{b}w(k) \tag{2.7}$$

$$v(k) = \underline{h}^T\underline{e}(k) + dw(k) \tag{2.8}$$

$$w(k) = \underline{\phi}^T(k)\underline{x}(k) - \alpha\underline{x}^T(k)\underline{\Gamma}\underline{x}(k)v(k), \quad \alpha > \tfrac{1}{2}, \quad \underline{\Gamma} = \underline{\Gamma}^T > 0 \tag{2.9}$$

$$\underline{\phi}(k+1) = \underline{\phi}(k) - \underline{\Gamma}v(k)\underline{x}(k) \tag{2.10}$$

given (2.4) and the following definitions:

$$\underline{e}^T(k) \triangleq [y(k-1)-z(k-1) \;\; y(k-2)-z(k-2) \;\; \ldots y(k-n)-z(k-n)] \tag{2.11}$$

$$\underline{A} \triangleq \begin{bmatrix} a_1 & a_2 & \cdots & a_n \\ 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 \;\; 1 & 0 \end{bmatrix} \tag{2.12}$$

$$\underline{b}^T \triangleq [1 \quad 0 \;\ldots\; 0] \tag{2.13}$$

$$\underline{h}^T \triangleq [c_1 + a_1 \quad c_2 + a_2 \;\ldots\; c_n + a_n] \tag{2.14}$$

$$d \triangleq 1$$

$$w(k) \triangleq \sum_{i=1}^{n} [a_i - \hat{a}_i(k+1)]z(k-i) + \sum_{j=1}^{m} [b_j - \hat{b}_j(k+1)]u(k-j) \tag{2.16}$$

$$\underline{\phi}^T(k) \triangleq [a_1 - \hat{a}_1(k) \;\ldots\; a_n - \hat{a}_n(k) \;\; b_1 - \hat{b}_1(k) \;\ldots\; b_m - \hat{b}_m(k)] \tag{2.17}$$

$$\underline{x}^T(k) \triangleq [z(k-1) \;\ldots\; z(k-n) \;\; u(k-1) \;\ldots\; u(k-m)] \tag{2.18}$$

$$\alpha \triangleq 1$$

$$\underline{\Gamma} \triangleq \mathrm{diag}[\mu_1 \;\ldots\; \mu_n \;\; \rho_1 \;\ldots\; \rho_m]. \tag{2.20}$$

Similarly the adaptive output error identifier combining (1)-(6) in [6] and (1)-(4) in [7] can also be written in the form of (2.7) through (2.10). As shown in [10] and [11], (2.7)-(2.10) can be combined as

$$\begin{bmatrix} \underline{e}(k+1) \\ \\ \underline{\phi}(k+1) \end{bmatrix} = \underline{F}(k) \begin{bmatrix} \underline{e}(k) \\ \\ \underline{\phi}(k) \end{bmatrix} \tag{2.21}$$

with

$$\underline{F}(k) = \begin{bmatrix} \underline{A} - \dfrac{\alpha\underline{x}^T(k)\underline{\Gamma}\underline{x}(k)\underline{b}\underline{h}^T}{1 + \alpha d\underline{x}^T(k)\underline{\Gamma}\underline{x}(k)} & \dfrac{\underline{b}\underline{x}^T(k)}{1 + \alpha d\underline{x}^T(k)\underline{\Gamma}\underline{x}(k)} \\ \dfrac{-\underline{\Gamma}\underline{x}(k)\underline{h}^T}{1 + \alpha d\underline{x}^T(k)\underline{\Gamma}\underline{x}(k)} & \underline{I} - \dfrac{d\underline{\Gamma}\underline{x}(k)\underline{x}^T(k)}{1 + \alpha d\underline{x}^T(k)\underline{\Gamma}\underline{x}(k)} \end{bmatrix} \tag{2.22}$$

The main result for (2.1)-(2.21) is that if

$$H(z) = d + \underline{h}^T(z\underline{I} - A)^{-1}\underline{b} = \frac{1 + \sum_{i=1}^{n} c_i z^{-i}}{1 - \sum_{i=1}^{n} a_i z^{-i}} \tag{2.23}$$

is strictly positive real (SPR), then the algorithm is convergent in the sense that $y(k) - z(k) \to 0$ and $\underline{\phi}(k)$ remains bounded.

Now for reduced order identification, we shall assume that (2.6) is replaced by a higher order model, and we week to identify an n-th order part of this model. Thus as in [15] (2.6) is replaced by

$$y(k) = y_M(k) + y_U(k), \tag{2.24}$$

where $y_M$ is the modeled portion of y and has the same order as the identifier filter, so that,

$$y_M(k) = \sum_{i=1}^{n} a_i y_M(k-i) + \sum_{j=1}^{m} b_j u(k-j) \tag{2.25}$$

and $y_U(k)$ is the unmodeled portion of y. Note that $y_U(k)$ could also represent a noise signal. The first concern addressed in [15] is whether or not z(k) and subsequently $\hat{y}(k)$ will remain bounded with (2.24) and (2.25) rather than (2.6) used in (2.1) through (2.5), if $y_U(k)$ remains bounded. The desired implication that

$$\|y_U(k)\|^2 \le \delta^2 < \infty\,\forall k \Rightarrow \|y(k) - z(k)\|^2 \le \varepsilon^2 < \infty\,\forall k > \bar{k} \tag{2.26}$$

was shown to be true in [15]. The second concern, i.e. whether or not $\underline{\phi}(k)$ remains bounded, was argued heuristically in [15] to be true given sufficient excitation, i.e. small $|\underline{\phi}^T\underline{x}| \Rightarrow$ small $\|\underline{\phi}\|$ or conversely large $\|\underline{\phi}\| \Rightarrow$ large $|\underline{\phi}^T\underline{x}|$.

These results were obtained by examination of the error model resulting from the alterations of (2.7)-(2.10) reflecting the use of (2.24) and (2.25) for (2.6). The altered error model is obtained by first noting the change in (2.4)

$$v(k) = \sum_{i=1}^{n} [c_i + a_i][y_M(k-i) - z(k-i)] + w(k) + p(k), \quad (2.27)$$

where w is defined in (2.16) and the perturbation signal p is defined by

$$p(k) \triangleq y_U(k) + \sum_{i=1}^{n} c_i y_U(k-i). \quad (2.28)$$

With the definition of $\underline{e}$ in (2.11) altered to

$$\underline{e}^T(k) \triangleq [y_M(k-1) - z(k-1) \ldots y_M(k-n) - z(k-n)] \quad (2.29)$$

(2.8) becomes

$$v(k) = \underline{h}^T \underline{e}(k) + dw(k) + p(k) \quad (2.30)$$

in order to match (2.27). The remainder of the error system, i.e. (2.7), (2.9), (2.10), and (2.12)-(2.20) remains unaltered in accommodating the effects of reduced order modeling. The modification required for (2.21) is

$$\begin{bmatrix} \underline{e}(k+1) \\ \underline{\phi}(k+1) \end{bmatrix} = \underline{F}(k) \begin{bmatrix} \underline{e}(k) \\ \underline{\phi}(k) \end{bmatrix} + \underline{G}(k)p(k) \quad (2.31)$$

with $\underline{F}(k)$ in (2.22) as before and

$$\underline{G}(k) = \begin{bmatrix} \dfrac{-\alpha b \underline{x}^T(k)\Gamma \underline{x}(k)}{1 + \alpha d \underline{x}^T(k)\Gamma \underline{x}(k)} \\ \dfrac{-\Gamma \underline{x}(k)}{1 + \alpha d \underline{x}^T(k)\Gamma \underline{x}(k)} \end{bmatrix} \quad (2.32)$$

The perturbed error system of (2.31) given (2.22) and (2.32) now compactly describes the reduced-order AIF behavior.

## 3. QUALITATIVE DISCUSSION OF SUFFICIENT EXCITATION AND BOUNDEDNESS PROOF

To consider why a persistently (or sufficiently) exciting condition on the driving sequence u(k) is important suppose first that such a condition does not hold. It is easily established that in the full or nonreduced order model case, i.e. when $y_U(k) \equiv 0$, certain linear functionals of the parameter error vector $\underline{\phi}(k)$ neither converge nor diverge. (The analysis neglects the effect of noise. If this is allowed for, divergence of the functions is to be expected.) Now it may then be that with $y_U(k)$ not identically zero, i.e. with a driving term appearing in the system equations, this "neutral stability" of the linear functionals becomes unboundedness in $\|\underline{\phi}\|$; it is possible to conceive at the same time of having $\underline{\phi}^T(k)\underline{x}(k)$ small, which implies w(k), e(k) and v(k) are small [15]. This opportunity for instability will be avoided by the requirement of sufficient excitation.

The key to proving boundedness is embodied in the following remarks. It is known that under a persistently exciting condition on u(k), see [4], in the nonreduced-order model case, (2.21), the homogeneous equivalent of (2.31), is exponentially convergent in a sense made more precise subsequently. It would apparently follow that if in (2.31) $\underline{G}(k)$ and p(k) were bounded, then, due to this exponential stability of the unperturbed system and the acknowledged retention of stability of such systems in the presence of bounded additive perturbations [10], e(k) and $\underline{\phi}(k)$ would be bounded with the addition of a suitably bounded perturbation G(k)p(k). (It is easy to affirm from (2.32) that $\underline{G}(k)$ is bounded). What is the flaw in this argument? The difficulty is that $\underline{F}(k)$ in (2.32) changes when $p(k) \not\equiv 0$ from its value when $p(k) \equiv 0$. This is because some entries of x(k) involve z(k-i), i = 1, ..., n, and these quantities depend on $\underline{e}(k)$, in a manner made precise subsequently. Thus, the apparently linear, abbreviated notation in (2.31) masks a situation that is actually nonlinear. Therefore the effect of the perturbation signal on (2.31) is not included solely in the driving term G(k)p(k). Also note at this juncture the unguided and therefore arbitrary designation of $y_u$ within y and the resulting effect on $\underline{e}$, $\underline{\phi}$, and p. Choosing $y_u$ such that $\underline{e}(0) = \underline{\phi}(0) = 0$ is one absurd choice that increases $\max_k |y_u(k)|$ and therefore $\max_k |p(k)|$ relative to the choice for the parameterization of $y_u$ centering (k) and $\underline{\phi}(k)$ in the stability region reached after the adaptive transients decay. Nevertheless, in the next section by building on the tolerance to slight perturbations and slight nonlinearities of the underlying exponentially stable, linearized homogeneous version of (2.31), we show that if $|p(k)|$ is bounded by a suitably small quantity, and u(k) is persistently exciting, (2.31) has a bounded solution. Then we comment on the meaning of that bound.

## 4. OUTPUT AND PARAMETER ERROR BOUNDEDNESS

For notational ease define

$$\tilde{\underline{x}}(k) = [y_M(k-1) \ldots y_M(k-n) u(k-1) \ldots u(k-m)]^T. \quad (4.1)$$

Notice that $\tilde{\underline{x}}(k)$ depends only on the driving sequence and the initial conditions in the modeled portion of y(k). (Since the latter dependence is exponentially decaying, it will be neglected.) Also note that

$$\tilde{\underline{x}}(k) = x(k) + [\underline{e}^T(k) \quad \underline{0}]^T. \quad (4.2)$$

Next, define $\tilde{\underline{F}}(k)$ to be $\underline{F}(k)$ with $\underline{x}(k)$ replaced by $\tilde{\underline{x}}(k)$ in (2.22) and set $\underline{\Psi}[\underline{e}(k), k] = \underline{F}(k) - \tilde{\underline{F}}(k)$. Notice that the matrix sequence $\tilde{\underline{F}}(k)$ is truly independent of $\underline{e}(k)$, where $\underline{F}(k)$ is not; accordingly, the arguments of $\underline{\Psi}$ display the dependence on $\underline{e}(k)$ as well as k. Now write (2.31) as

$$\begin{bmatrix} \underline{e}(k+1) \\ \underline{\phi}(k+1) \end{bmatrix} = \tilde{F}(k) \begin{bmatrix} \underline{e}(k) \\ \underline{\phi}(k) \end{bmatrix} + \underline{\Psi}[\underline{e}(k), \ k] \begin{bmatrix} \underline{e}(k) \\ \underline{\phi}(k) \end{bmatrix} + \underline{G}(k)p(k). \quad (4.3)$$

Adopt the abbreviation

$$\underline{q}(k) = \begin{bmatrix} \underline{e}(k) \\ \underline{\phi}(k) \end{bmatrix} \quad (4.4)$$

and regard $\underline{\Psi}$ as a function of $\underline{q}(k)$ and k.

To obtain the first result for (4.3), recall

Lemma 1 [4]: With u(k) satisfying a persistently exciting condition,

$$\underline{r}(k+1) = \tilde{\underline{F}}(k)\underline{r}(k) \quad (4.5)$$

is exponentially stable.□

This allows statement of the

First Main Theorem: With $\|q(0)\|$ and $\max_k |p(k)|$ suitably small in (4.3), $\|q(k)\|$ remains bounded. □

(For statement of the proofs of this theorem and the following lemmas and theorem the interested reader is referred to [16].)

This result, though clearly important, is easy to obtain. The main interest in the rest of this section is to establish a result which says no matter what $\|q(0)\|$ is, there is an appropriately small bounded $\max_k |p(k)|$ which implies a bound on $\|q(k)\|$. This result, stated below as the Second Main Theorem, will be developed from two additional lemmas. These lemmas develop in precise terms the nature of the exponential stability of the homogeneous version of (4.3).

Lemma 2 [6-8]: With $p(k) \equiv 0$ in (4.3) and $q(0)$ arbitrary, $q(k)$ is bounded and

$$\sum_{k=k_0}^{\infty} \|\underline{e}(k)\|^2 \le \gamma_1 V[q(k_0)] \qquad (4.6)$$

$$\sum_{k=k_0}^{\infty} \|\underline{\gamma}(q(k), k)\|^2 \le \gamma_2 V[q(k_0)], \qquad (4.7)$$

where $V[q(k)]$ is a positive definite quadratic form in $q(k)$ monotonically decreasing along $q(\cdot)$ trajectories and $\gamma_1$, $\gamma_2$ are constants. □

Lemma 3: Under the persistently exciting condition which guarantees stability of (4.5) (the linearized, homogeneous version of (4.3)), the homogeneous ($p(k) \equiv 0$) version of (4.3) yields

$$\|q(k)\| \le \gamma_3 \alpha^{k-k_0} \|q(k)_0\| \qquad (4.8)$$

for some $\alpha < 1$, all $\|q(k_0)\|$ with $V[q(k_0)] \le R$, the constant $\gamma_3$ depending on R, which can be arbitrary. □

Remark: A standard definition of exponential stability for the homogeneous version of (4.3) would require that there exists a positive $\gamma_3 > \alpha$ with $\alpha < 1$ and with $\gamma_3$ independent of $q(k_0)$ or of R. Thus in the strictest sense (4.8) is not the same as exponential stability.

Now the main result can be stated as the

Second Main Theorem: Consider (4.3) with $q(k_0)$ such that $V[q(k_0)] \le R$, where R is arbitrary. There exists a constant T(R) such that, when $|p(k)| < T(R)$ for all k, $\|q(k)\|$ is bounded and a constant S(R) exists such that

$$\lim_{k\to\infty} \sup \|q(k)\| < S(R) \lim_{k\to\infty} \sup |p(k)|. \; □ \qquad (4.9)$$

Remarks:

(i) The interpretation is as follows. The theorem guarantees that the algorithm will not blow up provided that $\max_k |p(k)|$ is small enough.

The question of how small is small is initial condition dependent; thus the larger the initial error, the smaller the bound on $|p(k)|$. Provided that $\max_k |p(k)|$ is within such a limit, then the smaller the actual value of $\max_k |p(k)|$, the smaller will be $\lim_{k\to\infty} \sup \|q(k)\|$, and thus the smaller will be the tracking error.

(ii) A previously mentioned ambiguity of this result is in the designation of $y_M$ in (2.25) and the subsequent error state space origin $q = \underline{0}$ from (4.4). Clearly the designation of $y_M$ effects $\max_k |p(k)|$ and V(0) and therefore R,T(R). Consider the possibility that $y_M$ is parameterized such that after the adaptive transients decay the trajectory of $q(k)$ as far as possible is evenly distributed within a ball of radius $\xi_1$. Now if $y_M$ were reselected such that the new $q(0)$ were shifted by a distance $\xi_2$ outside the original $\xi_1$-ball, then the new bound on $\|q(k)\|$ would be $\xi_1 + \xi_2$ with no real change in the adaptive identifier. This coordinate specification problem arises due to the lack of a formula for a nominal (or mean) convergent reduced-order AIF parameterization. The results of this Second Main Theorem suggest that this nominal convergent parameterization could be designated as that $y_M$ which minimizes the bounding constant in (4.9) with the other factors influencing this bound assumed fixed.

(iii) Since (4.9) clearly suggests that the size of p(k) influences the size of $\|q(k)\|$, consider (2.28). If $y_u$ is the response of the high frequency component of the plant being identified, then $y_u$ and p can be reduced by limiting the frequency content of u to low frequency components. Similarly the error smoothing coefficients $c_i$ may be selected to minimize p while retaining the SPR of (2.23).

(iv) A faster convergence rate for the homogeneous, linearized (around $q_0$) version of (4.3) yields a smaller S(R). Therefore, a faster convergence rate results in a closer approximation of the reduced-order AIF error time history to that of the asymptotically convergent full-order AIF. The convergence rate is influenced in a complex fashion [4] by the selection of $\underline{\Gamma}$ and the frequency content of $\underline{x}(k)$.

## 5. CONCLUSIONS

The key result of the paper is that reduced order models can on occasions (i.e. given sufficient excitation and limited mismodelling) be satisfactory for adaptive output error identification and adaptive infinite impulse response filtering. This result plays a key role in justifying the practical use of many proposed algorithms. For as is well known, the "true plant" being identified can virtually never be regarded as being able to be described by a finite ARMA model, so that reduced order modelling is not the atypical but normal practical situation.

Note that the permissible amount of "mismodelling" is bounded, the bound being a complex function of many quantities, including the initial parameter estimation error and the convergence rate which would be encountered where there is no modelling error, the latter in turn being dependent on the excitation signal. Thus it is hard, if not impossible, to lay down precise guidelines on the amount of mismodelling which can be tolerated without exploitation of the exact character of the neglected dynamics.

ACKNOWLEDGEMENT

This work supported by the National Science Foundation (USA) under Grant ECS-79-08695 and the Australian Research Grants Committee.

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