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SYSTEMS THEORY LABORATORY

STANFORD ELECTRONICS LABORATORIES

STANFORD UNIVERSITY · STANFORD, CALIFORNIA

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ABSTRACT

Using the control theory concept of minimal state-space realizations an algebraic synthesis of positive-real impedance matrices is obtained through an appropriate basis change in the state-space.

I. INTRODUCTION

The disciplines of network theory and control systems theory have much in common; for example, network functions (or matrices) are generally particular cases of transfer functions (or matrices); again, networks may profitably be examined from the state space point of view, which is essentially a control systems concept.

It is surprising therefore to find that there are not more links between the two disciplines. It is possible to point to only isolated examples, for instance [1], [2], [3] which are concerned with developing a state space description of a network, or [4], [5] which discuss positive-real functions and matrices from a control viewpoint. Groundwork for a control viewpoint of the scattering matrix synthesis problem is discussed in [6].

This paper is an attempt to lay another bridge across the gap. It is concerned with giving an impedance matrix synthesis via control theory concepts.

The early work of Cauer [7], Brune [8], Darlington [9] and others, and later Bott and Duffin [10], represented some of the first successful attempts to establish synthesis procedures for one-port networks. Generally the problem they considered was that of synthesising a network given a mathematical description of it, usually a positive-real function.

The desire to extend network theory to multiport situations led to the study of positive-real matrices. An n x n positive-real matrix $A(s)$ fulfills the following conditions [11, p. 217], [12], (the superscript star denotes complex conjugation and the prime matrix transposition):

(1) $A(s)$ is analytic in the strict right half plane

(2) $A^*(s) = A(s^*)$ in the strict right half plane

(3) $A(s) + A'(s^*)$ is a nonnegative definite matrix in the strict right half plane.

This definition is a natural extension of the definition of a positive-real function [13, p. 67].

It is not difficult to show that, if it exists, the impedance matrix of a multiport network which is linear, finite, time-invariant and passive is a positive-real matrix of rational functions [14, p. 153]. It is however considerably harder to establish the converse, namely, that to a rational positive-real matrix there corresponds a linear, finite, time-invariant, passive network with the given matrix as the impedance matrix of the network.

This impedance matrix synthesis problem, or, what amounts to a variant of it, the scattering matrix synthesis problem, has been solved in various ways by a number of workers, [11, Part II], [14], [15], [16], [17], [18]. Both reciprocal syntheses (those using resistors, inductors, capacitors and transformers, but no gyrators) and nonreciprocal syntheses (those using also gyrators) have been considered. None of these syntheses could be construed as depending on control theory techniques for its establishment.

Our approach in this paper is to express the network synthesis problem in control theory terms, to solve the resulting control problem, and then reinterpret this solution in network theoretic terms.

In Section II we outline briefly, but it is hoped fully, the necessary control systems preliminaries. The principal idea is that of a realization of a matrix of rational transfer functions, which is essentially a collection of four constant matrices describing the transfer function matrix. The theory of minimal realizations (where exactly what is minimal will be explained in Section II) is also considered. Section III poses the impedance synthesis problem in control theory language, reducing it to a search for a realization possessing certain properties (corresponding to the passivity of a resistive coupling network).

Section IV is concerned with explaining an interesting lemma, which characterizes the concept of positive reality in terms of the matrices of a minimal realization. Section V shows that this characterization allows ready selection of a realization possessing the properties mentioned above as being sought after in Section III, so that a passive synthesis can then be given. In this section the details of a synthesis procedure are also discussed, and it is shown that the synthesis uses the minimal number of reactive and resistive elements.

Examples of the synthesis procedure are discussed in Section VII, while Section VI discusses reciprocal synthesis with special emphasis on

RL networks.

Section VIII discusses some of the remaining problems of the control-network theory interface.

II. CONTROL SYSTEMS PRELIMINARIES

Before turning attention to the main problem in hand, we digress in this section to point out some pertinent results of a control systems nature. Linear, time-invariant, multivariable, finite dimensional control systems can be characterized by an $m \times n$ transfer function matrix $W(s)$ whose elements are rational functions of the variable s [19]. The matrix $W(s)$ relates the Laplace transform of the input n-vector, $U(s)$, to the Laplace transform of the output m-vector, $Y(s)$, through

$$Y(s) = W(s)U(s) \tag{1}$$

It will be sufficient for most of the material following to restrict consideration to the case where $W(s)$ has no pole at infinity, that is, $W(\infty)$ is finite.

Under these conditions it is possible to describe the control system via a state space representation. In this representation, the input u and output y are mathematically related via an intermediate variable, the state x. The relevant equations are

$$\dot{x} = Fx + Gu \tag{2a}$$

$$y = H'x + Ju \tag{2b}$$

In these equations, x, u, and y are vector functions of time rather than Laplace transforms as in (1); $\dot{x}$ is the time derivative of x. The vector x has dimension p (which we shall not specify for the moment), while the matrices F, G, H, J are all constant, and of appropriate dimension, respectively $p \times p$, $p \times n$, $p \times m$, $m \times n$.

By taking the Laplace transform of (2) and eliminating $X(s)$, it is possible to obtain, with I_p the $p \times p$ identity,

$$Y(s) = [J + H'(sI_p - F)^{-1}G]U(s) \tag{3}$$

and it follows by comparing (1) and (3) that the matrix $W(s)$ of rational functions of s is related to the four constant matrices F, G, H and J by

$$W(s) = J + H'(sI_p - F)^{-1}G \tag{4}$$

Note that many authors use H where we use H'.

It is clear that any quadruple $\{F, G, H, J\}$ determines a $W(s)$ which is a matrix of rational functions of s, having $W(\infty)$ finite. The converse, however, that $W(s)$ determines a quadruple $\{F, G, H, J\}$, is not obvious immediately. From (4) it follows that J is determined as $W(\infty)$, but otherwise the existence of F, G, H is not a priori guaranteed.

Nonetheless, as is discussed for example in [19], [20], any $W(s)$ does determine an infinity of triples $\{F, G, H\}$ such that (4) is satisfied with $J = W(\infty)$. These references discuss methods of determining the triples, and consider in particular the question of determining all triples when one is known.

Any quadruple $\{F, G, H, J\}$ satisfying (4) is termed a <u>realization</u> of $W(s)$, while the triple $\{F, G, H\}$ is termed a realization for $W(s) - W(\infty)$ *since* J *in the quadruple is zero.*

The dimensions of the various possible F matrices which can occur in the triples are not the same; but it is true that there is a minimal dimension for the set of all matrices F appearing in the realizations of a prescribed $W(s)$. For example if $W(s)$ is a constant matrix, it is clear from (4) that this dimension is zero, or if $W(s)$ is a scalar of the form $\frac{a}{s}$ it is clear that this dimension is one.

A realization $\{F, G, H, J\}$ for which F has minimal dimension is termed a <u>minimal realization</u>.

A most important feature of minimal realizations is that they are uniquely determined by $W(s)$ except for arbitrary prescription of the basis vectors of the state space [19]. What concerns us more however is the way this arbitrary prescription affects $\{F, G, H\}$. Reference [19, p. 157] shows that if $\{F, G, H\}$ is a minimal realization of $W(s) - W(\infty)$, any other minimal realization is of the form $\{T^{-1}F\,T, T^{-1}G, T'H\}$ where T is an arbitrary nonsingular matrix. Thus if $\{F_1, G_1, H_1\}$ and

$\{F_2, G_2, H_2\}$ are both minimal, then the existence is guaranteed of a nonsingular T such that

$$F_2 = T^{-1}F_1T \tag{5a}$$

$$G_2 = T^{-1}G_1 \tag{5b}$$

$$H_2 = T'H_1 \tag{5c}$$

The <u>dimension of a minimal realization</u>, that is, the dimension of the associated state space or the order of the square matrix F, is termed the <u>degree</u> of $W(s)$, written $\delta[W]$.

The history of the concept of degree in network and control theory is an interesting one. Tellegen's definition of the order of a network [21, p. 322] proceeds on physical grounds by defining the order as the maximum number of natural frequencies obtainable by embedding the given network in an arbitrary passive network. This order definition agrees with the mathematical definition of McMillan [11, pp. 543, 592] of the degree of a square matrix $Z(s)$, which is shown to imply that $\delta[Z]$ is the minimal number of reactive elements in any passive synthesis of $Z(s)$ when $Z(s)$ is a positive-real impedance matrix. Since we can conceive of deriving a state space representation of $Z(s)$ by associating a state variable with each reactive element in a network synthesizing $Z(s)$, see [1], [2], [3], it is not surprising to find that McMillan's definition is essentially the same as the one we give above. Still another mathematical definition of degree, motivated by a different set of physical concepts, is given in [22]. Because of the corresponding physical meanings of these it is therefore fortunate to find [23, p. 542] that these definitions are mathematically the same thing, provided poles at infinity are suitably dealt with.

We shall be especially interested in the fact that <u>the minimal number of reactive elements in a synthesis of an impedance matrix $Z(s)$, i.e., McMillan's $\delta[Z(s)]$, is the same thing as the dimension of a minimal (control systems) realization, provided $Z(\infty)$ is finite</u>.

III. THE IMPEDANCE SYNTHESIS PROBLEM IN CONTROL THEORY LANGUAGE

Our solution of the synthesis problem is a control theoretic one, and to achieve the solution it is necessary to express the synthesis problem in control theoretic language.

Formally the synthesis problem is: given a positive-real $n \times n$ matrix $Z(s)$ (whose elements are rational, functions of s), find a finite circuit connection of passive network elements synthesising $Z(s)$.

To motivate the synthesis procedure presented, it will be necessary to make some apparently restrictive assumptions concerning the final form of the synthesis. These assumptions include more than merely the assumption of existence of a synthesis; they will however be shown to be valid as a result of the synthesis technique presented.

A synthesis may contain any of the following types of linear, passive, time-invariant network elements: resistors, gyrators, (ideal) transformers, inductors, capacitors. The first three classes are nondynamic, or memoryless. The last two classes are dynamic, and thus not memoryless; the behaviour of an individual element can, if desired, be specified with the aid of state variables.

It is possible at one stroke to entirely eliminate one of these classes, namely the capacitors. It is now reasonably well known that if a unit gyrator of impedance matrix

$$Z_g = \begin{bmatrix} 0 & +1 \\ -1 & 0 \end{bmatrix} \qquad (6)$$

is terminated at one port in a unit inductance, then the impedance viewed at the other port is that of a unit capacitance. See Fig. 1. Consequently, all capacitors in a circuit may be replaced by gyrators and inductors.

Therefore, in any n-port network N synthesizing $Z(s)$ it is possible to always assume that the only dynamic elements used are inductors, and positive unit inductors at that, since transformers may be used to provide the normalization.

By dividing the elements of N into two classes, the nondynamic elements and the unit inductors, assumed to be p in number, it is possible to regard N as an interconnection of two networks N_1 and N_2, where N_1 is an $(n+p)$-port, consisting of the nondynamic elements of N, and N_2 is simply p unit inductors, uncoupled from one another. One of these inductors loads each of the last p ports of N_1, as shown in Fig. 2.

Although N possesses an impedance matrix $Z(s)$ by assumption, there is no guarantee that N_1 will possess an impedance matrix. For our purposes it will suffice to simply assume that an impedance matrix does exist for N_1; such will indeed be the case for the synthesis to be considered. Because N_1 consists of purely nondynamic elements, this impedance matrix is constant; it is also positive-real, when N_1 consists of purely passive elements. The port partition of N_1 determines a corresponding partition of its impedance matrix, which we write as

$$M = \left[\begin{array}{c|c} z_{11} & z_{12} \\ \hline z_{21} & z_{22} \end{array}\right] \tag{7a}$$

Here the matrices z_{11}, z_{12}, z_{21} and z_{22} have dimensions respectively $n \times n$, $n \times p$, $p \times n$ and $p \times p$.

It is now possible to express the input impedance at the first n ports of N_1 (when the latter is terminated in the unit inductors) in terms of the z_{ij} and the impedance matrix of N_2, viz., sI_p. The result, which may be derived by straightforward calculation, is

$$Z(s) = z_{11} - z_{12}(sI_p + z_{22})^{-1}z_{21} \tag{8}$$

Equation (8) bears a striking similarity to Equation (4); in fact we observe that one possible realization of $Z(s)$, in the sense of Section 2, is given by

$$\{F, G, H, J\} = \{-z_{22}, z_{21}, -z_{12}', z_{11}\} \tag{9}$$

This appears to have been first recognized by Youla, [24, p. 30].

Let us review the significance of (9). If $Z(\infty)$ is finite, there are many quadruples $\{F, G, H, J\}$ constituting a realization in the sense of Section 2. If we possess a synthesis of $Z(s)$, and the non-dynamic part of this synthesis possesses a constant impedance matrix M, this impedance matrix determines one particular realization through (9). Drawing further on the material of Section 2, if the synthesis uses a minimal number of reactive elements, the realization (9) is a minimal one. Thus each minimal reactive element synthesis yields via M a minimal realization. This fact is not especially significant for our purposes here; we know how to construct minimal realizations without the necessity of synthesizing a network first.

What is significant however is that (9) implies that <u>each minimal realization yields a minimal reactive element synthesis</u>. Thus given an impedance matrix $Z(s)$ with $Z(\infty)$ finite, we can determine a minimal realization by the known methods, see e.g. [19], [23], [24]. This minimal realization determines the impedance matrix of a network N_1, through (9), such that if N_1 is synthesized and its last p ports terminated in unit inductors, then the resulting n-port has impedance matrix $Z(s)$. The difficulty arises however in that given an arbitrary minimal realization, <u>the impedance matrix of N_1</u>

$$M = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} J & -H' \\ G & -F \end{bmatrix} \tag{7b}$$

<u>may not be positive-real</u>. If it is not, then we cannot synthesize the corresponding N_1 using only passive elements even though the given $Z(s)$ is positive-real. If M is positive-real, then the synthesis problem is easy [25, pp. 255-261], being that of synthesizing a purely resistive network. Further, we achieve thereby a synthesis of $Z(s)$ with the minimum number of reactive elements.

A second apparent difficulty, that of requiring $Z(\infty)$ to be finite, is easily resolved It is well known (see e.g., [11] for the reciprocal case, [26, p. 3] for the general case) that a positive-real $Z(s)$ can be written as

$$Z(s) = s\hat{L} + \hat{Z}(s) \tag{10}$$

where $\hat{L}$ is a nonnegative definite constant symmetric matrix and $\hat{Z}(s)$ is positive-real with $\hat{Z}(\infty)$ finite. The matrix $Z(s)$ can be synthesized as the series connection of transformer-coupled inductors (of impedance matrix $s\hat{L}$) and a network $\hat{N}$ (of impedance matrix $\hat{Z}(s)$). It is moreover true that [26, p. 4]

$$\delta[Z(s)] = \delta[s\hat{L}] + \delta[\hat{Z}(s)] \tag{11}$$

where we are using the degree definition of McMillan; in other words (11) says that we can achieve a minimal reactive element synthesis of $Z(s)$ by series connecting two minimal reactive element syntheses, one of $s\hat{L}$, one of $\hat{Z}(s)$. We note also that since $\hat{Z}(s)$ is free of poles at infinity, $\delta[\hat{Z}(s)]$ is also the dimension of a minimal (control systems) realization of $\hat{Z}(s)$.

In the case where $Z(s)$ (or $\hat{Z}(s)$) has finite poles on the $j\omega$-axis, it is possible to further simplify the synthesis problem by writing [26, p. 3]

$$\hat{Z}(s) = \hat{Z}_1(s) + \hat{Z}_2(s) \tag{12}$$

where $\hat{Z}_1(s)$ and $\hat{Z}_2(s)$ are both positive-real, $\hat{Z}_1(s)$ has poles only on the $j\omega$-axis and $\hat{Z}_2(s)$ has poles in the strict left half plane. The matrix $\hat{Z}_1(s)$ can be synthesized by known methods [14, p. 155], [27, p. 27], as a series connection of transformer-coupled tuned circuits, possibly in conjunction with gyrators.

It is moreover true that

$$\delta[\hat{Z}(s)] = \delta[\hat{Z}_1(s)] + \delta[\hat{Z}_2(s)] \tag{13}$$

implying that a minimal reactive element synthesis of $\hat{Z}(s)$ derives from a minimal reactive element synthesis of $\hat{Z}_1(s)$ and $\hat{Z}_2(s)$. A minimal reactive element synthesis of $\hat{Z}_1(s)$ is a result of the mentioned procedures.

As a consequence we shall feel free to restrict attention to the problem of synthesising a positive-real $Z(s)$ which is finite at $s = \infty$, and has poles only in the strict left half plane. Moreover, the minimal number of reactive elements in a synthesis of $Z(s)$ is the dimension of a minimal realization of $Z(s)$.

Returning now to the main stream of the argument, we note that the problem of giving a minimal reactive element synthesis for a rational positive-real $Z(s)$ reduces to the following problem: Given a minimal realization $\{F, G, H, J\}$ of $Z(s)$, assumed to be positive-real with $Z(\infty)$ finite, and to have all poles in the strict left half plane, find a nonsingular T such that the realization $\{T^{-1}F\,T, T^{-1}G, T'H, J\}$ has

$$M = \begin{bmatrix} J & -H'T \\ T^{-1}G & -T^{-1}F\,T \end{bmatrix} = (I_n \dot{+} T^{-1}) \begin{bmatrix} J & -H' \\ G & -F \end{bmatrix} (1_n \dot{+} T) \qquad (14)$$

positive-real (where $\dot{+}$ denotes the direct sum), or alternatively such that

$$M + M' \geq 0 \qquad (15)$$

The notation ≥ 0 is shorthand for nonnegative definite.

(Note that, by (5)), all minimal realizations will be of the form $\{T^{-1}F\,T, T^{-1}G, T'H, J\}$ for some T).

We remark that if $Z(s)$ is not positive-real, there is certainly no possibility that a suitable T will exist. Even if $Z(s)$ is positive-real, the existence of T is not guaranteed a priori; this is because the existence of T is equivalent to the existence of an impedance matrix M for N_1. In the next sections we shall show how to find such a T.

IV. THE POSITIVE-REAL CONSTRAINT AS A CONTROL THEORY CONCEPT

The existence of T in (14) such that (15) is satisfied is hopefully a consequence of $\{F, G, H, J\}$ satisfying some set of conditions, and hopefully this set of conditions will be satisfied if $Z(s)$ is positive-real. Accordingly, we ask: What constraint is placed on the matrices in a minimal realization $\{F, G, H, J\}$ of a transfer function $Z(s)$ if the transfer function is constrained to being positive-real?

The answer to this question is contained in the following lemma [5].

Lemma 1. Let $Z(s)$ be an $n \times n$ matrix of rational transfer functions with $Z(\infty)$ finite. Let $\{F, G, H, J\}$ be a minimal realization for $Z(s)$. Let all poles of $Z(s)$ either be in the left half plane, or be simple on the $j\omega$-axis. Then necessary and sufficient conditions for $Z(s)$ to be positive-real are: there exist a symmetric positive definite matrix P, and matrices W_o, L such that

$$PF + F'P = -LL' \tag{16a}$$

$$PG = H - LW_o \tag{16b}$$

$$W_o'W_o = J + J' \tag{16c}$$

While we shall not attempt to prove this result here, we shall make several remarks about it by way of giving a partial outline of the proof. The result was first established for the case $n = 1$ in [28], and for the case of arbitrary n in [5]. Reference [4] states, but does not prove, a less general theorem applying for arbitrary n.

The fact that Eqs. (16) imply $Z(s)$ is positive-real is not hard to establish; the converse is considerably more difficult, however, and depends for its proof on a decomposition valid for positive-real $Z(s)$ which is established in [29]. For positive-real $Z(s)$ there exists a matrix $W(s)$, unique to within multiplication by a constant orthogonal matrix; such that

$$Z(s) + Z'(-s) = W'(-s)W(s) \tag{17}$$

with $W(s)$ having several additional properties.

The first additional property concerns the size of W, which is $r \times n$, where r is the <u>normal rank</u> of $Z(s) + Z'(-s)$. The normal rank of a matrix of rational transfer functions is the rank of that matrix almost everywhere, that is, throughout the s-plane except perhaps at a finite number of isolated points which result in certain minors of $Z(s) + Z'(-s)$ being zero, or infinite, at these points only. Note that $r \leq n$.

The second and third additional properties are that $W(s)$ is analytic in the right half plane, and that there exists at least one <u>right inverse</u> of W (that is, a matrix W^{-1} such that $WW^{-1} = I_r$) with W^{-1} also analytic everywhere in the right half plane. Equivalently, W has (strict) rank r in the right half plane. These additional conditions then ensure that W is unique to within multiplication by an arbitrary orthogonal matrix.

As pointed out earlier we can restrict consideration to those $Z(s)$ which have poles with negative real part. Then it is possible to show that the particular $W(s)$ above has a minimal realization $\{F, G, L, W_o\}$, two of the matrices of this realization being identical to two of the minimal realization of $Z(s)$. This property will not in general be possessed by other $W(s)$ satisfying (17). The matrix L in this realization of W is the matrix L of (16), while, naturally $W_o = W(\infty)$.

The proof of the lemma now requires the exhibition of P, and a demonstration that Eqs. (16) are satisfied. Equation (16c) is readily checked, by putting $s = \infty$ in (17). To define P, we start with any minimal realization of the $r \times n$ $W(s)$, and then transform it so that its system and input matrices, F and G, are identical with the corresponding matrices of the minimal realization of Z, thus obtaining L in the quadruple $\{F, G, L, W_o\}$. Equation (16a) may then be solved for P, since it can be shown to have a unique symmetric positive definite solution. The proof of the lemma concludes by showing that (16b) is automatically satisfied. Details of the preceding can be found in [5].

If the minimal realization $\{T_1^{-1} F T_1, T_1^{-1}G, T_1' H\}$ of $Z(s) - Z(\infty)$ is employed instead of $\{F, G, H\}$ a different P and L will be required to satisfy the equations corresponding to (16). The new P and L in terms of the old P and L may be readily verified to be $T_1'P T_1$ and $T'L_1$. In other words, as a consequence of (16), there results

$$(T_1'P T_1)(T_1^{-1}F T_1) + (T_1^{-1}F T_1)'(T_1'P T_1) = -(T_1'L)(T_1'L)' \tag{18a}$$

$$(T_1'P T_1)(T_1^{-1}G) = (T_1'H) - (T_1'L)W_o \tag{18b}$$

$$W_o'W_o = J + J' \tag{18c}$$

V. SYNTHESIS PROCEDURE

We recall, see (14) and (15), that if $\{F, G, H, J\}$ is a minimal realization of $Z(s)$, then the problem of finding a passive structure synthesizing $Z(s)$ reduces to finding a T such that

$$M = \begin{bmatrix} J & -H'T \\ T^{-1}G & -T^{-1}F\,T \end{bmatrix} \tag{14}$$

has

$$M + M' \geq 0 \tag{15}$$

Lemma 1 sets out conditons satisfied by F, G, H and J for $Z(s)$ to be positive-real. In particular lemma 1 guarantees the existence of a symmetric positive definite matrix P satisfying (16). For such a matrix, one may define a square root, $P^{1/2}$ which is also symmetric and positive definite [30, p. 76].

Theorem If $T = P^{-1/2}$, (15) is satisfied.

Proof: By direct calculation,

$$M + M' = \begin{bmatrix} J + J' & G'P^{1/2} - H'P^{-1/2} \\ P^{1/2}G - P^{-1/2}H & -P^{1/2}F\,P^{-1/2} - P^{-1/2}F'P^{1/2} \end{bmatrix} \tag{19}$$

From (16), there obtains

$$P^{1/2}F\,P^{-1/2} + P^{-1/2}F'P^{1/2} = -P^{-1/2}L\,L'P^{-1/2} \tag{20a}$$

and

$$P^{1/2}G = P^{-1/2}H - P^{-1/2}L\,W_o \tag{20b}$$

Using these relations in (19),

$$M + M' = \begin{bmatrix} W_o'W_o & (-P^{-1/2}L\,W_o)' \\ -P^{-1/2}L\,W_o & P^{-1/2}L\,L'P^{-1/2} \end{bmatrix}$$

$$= \begin{bmatrix} W_o' & 0 \\ 0 & -P^{-1/2}L \end{bmatrix} \begin{bmatrix} I_r & I_r \\ I_r & I_r \end{bmatrix} \begin{bmatrix} W_o & 0 \\ 0 & -L'P^{-1/2} \end{bmatrix} \tag{21}$$

The latter equality may be verified by direct calculation. From (21) it is evident that

$$M + M' \geq 0 \tag{15}$$

since the right hand side of (21) is of the form $A'BA$ where B is non-negative definite. This proves the theorem.

Having shown that M is the impedance of a passive network, the question arises as to how to synthesize M. This is discussed in [14, p. 156] and [25, p. 261]. We use the fact that $M = \frac{1}{2}(M + M') + \frac{1}{2}(M - M')$, and the fact that $\frac{1}{2}(M + M')$ and $\frac{1}{2}(M - M')$ are both positive-real impedances (the first because $M + M' \geq 0$, the second because it is skew). Then it can be seen that a synthesis of M is obtained by series connecting transformer-coupled resistors (corresponding to $\frac{1}{2}(M + M')$) and transformer-coupled gyrators (corresponding to $\frac{1}{2}(M - M')$).

By way of example, we consider in detail the synthesis of $\frac{1}{2}(M + M')$ and show that it uses r resistors. The synthesis of $\frac{1}{2}(M - M')$ will use no resistors, and thus we shall be able to conclude that $Z(s)$ can be synthesized with r resistors. Since r is the normal rank of $Z'(-s) + Z(s)$, this means we have achieved a synthesis of $Z(s)$ using the minimal number of resistors [14, p. 132], [17, p. 305] as well as a synthesis using the minimal number of reactive elements.

From (21) it follows, as may be checked by direct multiplication, that

$$\frac{1}{2}(M + M') = \frac{1}{2}\begin{bmatrix} W_o' \\ -P^{-1/2}L \end{bmatrix} I_r \begin{bmatrix} W_o & -L'P^{-1/2} \end{bmatrix} \tag{22}$$

This equation says that $\frac{1}{2}(M + M')$ may be synthesized by terminating a multiport transformer of turns ratio $\frac{1}{\sqrt{2}}[W_o' \quad -L'P^{-1/2}]$ in r unit resistors [25, p. 256].

The procedure for synthesizing an arbitrary positive-real impedance can now be stated:

(A) Separate out the pole at infinity (if any), corresponding to a series extraction of transformer—coupled inductors. The remaining positive-real $Z(s)$ has $Z(\infty)$ finite.

(B) (Actually optional) Separate out poles on the $j\omega$-axis, corresponding to a series extraction of tuned circuits (also transformer coupled in general). The effect of this is to leave a positive-real $Z(s)$ to be synthesized which is of lower degree than before performing this extraction. Further, this $Z(s)$ has strictly left half plane poles.

(C) Find the four matrices comprising any minimal realization $\{F, G, H, J\}$ for the impedance $Z(s)$ which remains to be synthesized, using any of the techniques outlined for instance in [19], [23], or [24].

(D) Find $W(s)$, using [29], such that $Z(s) + Z'(-s) = W'(-s)W(s)$ with $W(s)$ analytic in the right half plane, and there possessing rank equal to the normal rank of $Z(s) + Z'(-s)$.

(E) Find a realization of W of the form $\{F, G, L, W_o\}$ which will be minimal if step (B) has been carried out. Thus L is determined.

(F) Calculate P as the unique solution of the equation $P F + F'P = -LL'$. This matrix equation can be regarded as $p(p+1)/2$ linear simultaneous equations for the elements of P, p being the order of F or $p = \delta[Z(s)]$. Alternatively, P may be found from, [5], $P = \int_0^\infty \exp(F't)L L' \exp(F t)\, dt$.

(G) Using this P form a new minimal realization of Z given by $\{P^{1/2}F P^{-1/2}, P^{1/2}G, P^{-1/2}H, J\}$.

(H) Synthesize the nonreactive (constant) positive-real impedance

$$M = \begin{bmatrix} J & -H'P^{-1/2} \\ P^{1/2}G & -P^{1/2}F P^{-1/2} \end{bmatrix} \tag{23}$$

by a series connection of a transformer-resistor network and a transformer-gyrator network, both of $(n + \delta[Z])$ ports.

(I) Terminate the last $p = \delta[Z(s)]$ ports of this network in unit inductors to obtain a synthesis of $Z(s)$.

Examples of this procedure will be given in Section 7, for which Section 6 is not a prerequisite.

VI. RECIPROCAL RL SYNTHESIS

In this section we apply similar techniques to obtain passive reciprocal coupling networks for RL (transformer) circuits.

As a preliminary consider the more general situation where capacitors, but no gyrators, are also present, as illustrated in Fig. 3a. The resistive coupling network N_c is described by the symmetric impedance matrix

$$Z_c = Z_c' = \begin{array}{c} \left[\begin{array}{ccc} z_{11} & z_{12} & z_{13} \\ z_{12}' & z_{22} & z_{23} \\ z_{13}' & z_{23}' & z_{33} \end{array}\right] \begin{array}{l} \}\, n \\ \}\, k_1 \\ \}\, k_2 \end{array} \\ \begin{array}{ccc} \;\; n & \;\; k_1 & \;\; k_2 \end{array} \end{array} \tag{24}$$

Here the matrix is partitioned such that the last k_2 rows and columns correspond to the capacitors. By connecting unit gyrators in cascade with each of these final k_2 ports, Fig. 3a is seen,by Fig. 1, to be equivalent to Fig. 3b; the resulting network N_1 is of the form considered earlier, and has [31, pp. 4 & 28]

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{12}' & m_{22} & m_{23} \\ -m_{13}' & -m_{23}' & m_{33} \end{bmatrix} ; \; m_{ii} = m_{ii}' \qquad i = 1,2,3 \tag{25a}$$

$$= \begin{bmatrix} z_{11}-z_{13}z_{33}^{-1}z_{13}' & z_{12}-z_{13}z_{33}^{-1}z_{23}' & z_{13}z_{33}^{-1} \\ z_{12}'-z_{23}z_{33}^{-1}z_{13}' & z_{22}-z_{23}z_{33}^{-1}z_{23}' & z_{23}z_{33}^{-1} \\ -z_{33}^{-1}z_{13}' & -z_{33}^{-1}z_{23}' & z_{33}^{-1} \end{bmatrix} \tag{25b}$$

It is then important to note that $[I_{n+k_1} \dotplus (-I_{k_2})]M$ is symmetric and that repeating the gyrator extraction on N_1 yields Z_c from M by equations identical to those, (25b), giving M in terms of Z_c. One also observes, since passivity is unaffected by a gyrator extraction, that Z_c of (24) will be positive-real when (and only when) M is positive-real.

One can synthesize N_c, given M of (25a), by synthesizing Z_c through a (reciprocal) resistor-transformer network [25, pp. 255, 261], at least when m_{33} is nonsingular. If m_{33} is singular and a scattering matrix S_M exists for M (as when M is positive-real) then a reciprocal synthesis results through a gyrator extraction from the network N_1 which synthesizes S_M. From these arguments we conclude that a gyratorless minimal synthesis exists (when m_{33} is nonsingular or S_M exists) for a given M [as in (14)] if and only if there exists a nonnegative integer k_2, a permutation matrix P_1 (corresponding to a relabeling of inductor-capacitor ports) and a sign matrix $\Sigma = 1_{k_1} \dot{+} (-1_{k_2})$, such that $[I_n \dot{+} \Sigma][I_n \dot{+} P_1]M[I_n \dot{+} P_1']$ is symmetric. It is convenient to call such an M reciprocal, even though M itself is not symmetric.

At this point we apply some of the ideas developed for scattering matrices by Youla and Tissi [6], referring to their work for omitted proofs. Thus, consider any minimal realization $\hat{M}$ of a symmetric $Z(s)$; then there exists a symmetric T such that [6, p. 9]

$$\hat{M}' = (I_n \dot{+} T^{-1})\hat{M}(I_n \dot{+} T) \tag{26}$$

Since T is symmetric it can be diagonalized to plus and minus ones via a congruency transformation [30, p. 56]

$$T = T_o \, \Sigma \, T_o' \tag{27a}$$

$$\Sigma = I_{k_1} \dot{+} (-I_{k_2}) \; , \; p = k_1 + k_2 \tag{27b}$$

from which we can form

$$M = (I_n \dot{+} T_o^{-1})\hat{M}(I_n \dot{+} T_o) \tag{28}$$

On substituting (26) into (28) we find that $[I_n \dot{+} \Sigma]M$ is symmetric;

further k_1 and k_2 are unique [6, p. 7]. Thus, when $Z(s)$ is symmetric there exists a reciprocal M, from which a reciprocal synthesis results, at least when m_{33} is nonsingular or S_M exists (certainly when M is positive-real). Unfortunately there seems no guarantee that M is positive-real. Nevertheless, every other reciprocal M_R results from M of (28) by

$$M_R = (I_n \dotplus T_R^{-1})M(I_n \dotplus T_R) \tag{29}$$

with T_R satisfying [6, p. 7, lemma 6]

$$\Sigma = T_R \, \Sigma \, T_R' \tag{30}$$

In the RL case, then, since $k_2 = 0$, and $\Sigma = I_p$ we require T of (27) positive-definite and T_R of (30) orthogonal.

Finally, consider a given symmetric positive-real $Z(s)$ with $Z(\infty)$ finite, for which $x'Z(s)x$ satisfies the standard RL 1-port realizability conditions [13, p. 149] for all real n-vectors x. By standard n-port synthesis techniques [25, p. 270] a structure using transformers and passive resistors and inductors exists, using in fact the minimum number of inductors. By performing this synthesis in continued fraction form, one can demonstrate the existence of an impedance matrix M, [32], of the positive-real type under discussion. From this, or any other reciprocal M, all reciprocal M_R then result from (29) with T_R orthogonal, or

$$M_R = (I_n \dotplus T_R')M(I_n \dotplus T_R) \tag{31}$$

Since such an M_R is positive-real with M, being derived through a congruency transformation, we conclude that every minimal reciprocal M_R realizing a positive-real inductor-resistor $Z(s)$ must itself be positive-real. This result is in line with a similar one based upon scattering matrix arguments [6, p. 14]. Of course, by duality, an identical result holds for RC networks.

VII. SYNTHESIS EXAMPLES

In this section we present two moderately easy examples, different parts of the theory being highlighted by each.

Example 1. Synthesis of the (positive-real) impedance

$$Z(s) = \begin{bmatrix} s + \dfrac{2s}{s^2+1} + 2 & 4\,\dfrac{s-1}{s+1} \\ 0 & \dfrac{2s}{s^2+1} + 2 \end{bmatrix} \tag{32}$$

The first step is to separate out the term corresponding to the pole at infinity, and then to carry out the (optional) step of removing $j\omega$-axis pole terms Thus

$$Z(s) = \begin{bmatrix} s & 0 \\ 0 & 0 \end{bmatrix} + \frac{s}{s^2+1}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 4\,\dfrac{s-1}{s+1} \\ 0 & 2 \end{bmatrix} \tag{33}$$

The first two terms are readily synthesized, see Fig. 4a and Fig. 4b for the separate syntheses. Thus we now consider the positive-real

$$Z(s) = \begin{bmatrix} 2 & 4\,\dfrac{s-1}{s+1} \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -\dfrac{8}{s+1} \\ 0 & 0 \end{bmatrix} \tag{34}$$

A minimal realization for Z is given by

$$F = \begin{bmatrix} -1 \end{bmatrix} \qquad G = \begin{bmatrix} 0, & 1 \end{bmatrix}$$
$$H = \begin{bmatrix} -8, & 0 \end{bmatrix} \qquad J = \begin{bmatrix} 2 & 4 \\ 0 & 2 \end{bmatrix} \tag{35}$$

This may be derived by the techniques described in for example [19], or may be found by inspection, since the F matrix is simple. Observe that

$$Z(s) = J + H'(sI-F)^{-1}G \tag{36}$$

is, naturally, satisfied.

We also compute, by inspection or using [29], that

$$Z(s) + Z'(-s) = 4\begin{bmatrix} 1 & \frac{s-1}{s+1} \\ \frac{s+1}{s-1} & 1 \end{bmatrix} = 2\begin{bmatrix} 1 \\ \frac{s+1}{s-1} \end{bmatrix} 2\begin{bmatrix} 1 & \frac{s-1}{s+1} \end{bmatrix} \tag{37}$$

Hence

$$W(s) = 2\begin{bmatrix} 1 & \frac{s-1}{s+1} \end{bmatrix} \tag{38}$$

Further, a realization for $W(s)$ is given by using F and G as for $Z(s)$, and

$$L = \begin{bmatrix} -4, & 0 \end{bmatrix} \quad W_o = \begin{bmatrix} 2 & 2 \end{bmatrix} \tag{39}$$

Note that in the right half plane W has strict rank equal to the normal rank of $Z(s) + Z'(-s)$, that is, unity, and a right inverse is $[\frac{1}{2}, \quad 0]'$; W is moreover analytic in the right half plane.

The next step is to form P through

$$P F + F'P = -L L'$$

from which one readily determines

$$P = \begin{bmatrix} 8 \end{bmatrix} \tag{40}$$

and thus

$$P^{1/2} = \begin{bmatrix} 2\sqrt{2} \end{bmatrix} \tag{41}$$

Then although

$$\begin{bmatrix} J & -H' \\ G & -F \end{bmatrix} = \begin{bmatrix} 2 & 4 & 8 \\ 0 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix} \tag{42}$$

is not positive-real, it is true that

$$M = \begin{bmatrix} J & -H'P^{-1/2} \\ \\ P^{1/2}G & -P^{1/2}F\,P^{-1/2} \end{bmatrix} = \begin{bmatrix} 2 & 4 & 2\sqrt{2} \\ 0 & 2 & 0 \\ 0 & 2\sqrt{2} & 1 \end{bmatrix} \tag{43}$$

is positive-real. We note that

$$\frac{M+M'}{2} = \begin{bmatrix} 2 & 2 & \sqrt{2} \\ 2 & 2 & \sqrt{2} \\ \sqrt{2} & \sqrt{2} & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2} \\ \sqrt{2} \\ 1 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} \sqrt{2}, & \sqrt{2}, & 1 \end{bmatrix} \tag{44a}$$

and

$$\frac{M-M'}{2} = \begin{bmatrix} 0 & 2 & -\sqrt{2} \\ -2 & 0 & -\sqrt{2} \\ \sqrt{2} & \sqrt{2} & 0 \end{bmatrix} = \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 0 & 1 \\ 0 & \sqrt{2} & -1 \end{bmatrix} \tag{44b}$$

The network N_1 of impedance matrix M thus has the synthesis of Fig. 5.

The network synthesizing $Z(s)$ of (34) is found by terminating port 3 of N_1 in a unit inductor, while the original $Z(s)$ has the synthesis of Fig. 5, where the networks shown in Fig. 4 have been included. It is interesting to compare the terminated N_1 with the similar result using two reactive elements obtained by the Bayard synthesis [18, p. 88].

Example 2. Synthesis of the (positive-real) impedance

$$Z(s) = \frac{s^2 + 2s + 4}{s^2 + s + 1} \tag{45}$$

Having no poles on the $j\omega$-axis or at infinity to remove, we write

$$Z(s) = 1 + \frac{s + 3}{s^2 + s + 1}$$

Transfer functions of the form $\dfrac{\sum_{i=0}^{n-1} b_i s^i}{\sum_{i=0}^{n} a_i s^i}$ with $a_n = 1$ have a convenient canonical minimal realization (which does not extend in a straightforward way to the matrix situation). This is given by [19]

$$F = \begin{bmatrix} 0 & 1 & & & & \\ 0 & 0 & 1 & \cdot & & \\ \vdots & & & & \cdot & \\ 0 & \cdot & \cdot & \cdot & 0 & 1 \\ -a_0 & \cdot & \cdot & \cdot & \cdot & -a_{n-1} \end{bmatrix} \quad G = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix} \quad H = \begin{bmatrix} b_0 \\ b_1 \\ \cdot \\ \cdot \\ \cdot \\ b_{n-1} \end{bmatrix} \qquad (46)$$

Thus for the $Z(s)$ under consideration, we have

$$F = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \quad G = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad H = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \quad J = \begin{bmatrix} 1 \end{bmatrix} \qquad (47)$$

Direct calculation yields

$$Z(s) + Z'(-s) = 2\,\frac{(s^2+s+2)(s^2-s+2)}{(s^2+s+1)(s^2-s+1)} \qquad (48)$$

and then we take

$$W(s) = \sqrt{2}\,\frac{s^2+s+2}{s^2+s+1} = \sqrt{2} + \frac{\sqrt{2}}{s^2+s+1} \qquad (49)$$

A minimal realization for W is then

$$F = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \quad G = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad L = \begin{bmatrix} \sqrt{2} \\ 0 \end{bmatrix} \quad W_o = \begin{bmatrix} \sqrt{2} \end{bmatrix}$$

Forming the equation

$$P\,F + F'P = -L\,L'$$

there obtains

$$P = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \tag{50}$$

which has

$$P^{1/2} = \begin{bmatrix} \frac{3}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{bmatrix} \tag{51}$$

The network N_1 has the positive-real impedance

$$M = \begin{bmatrix} J & -H'P^{-1/2} \\ P^{1/2}G & -P^{1/2}F\,P^{-1/2} \end{bmatrix} = \begin{bmatrix} 1 & -\frac{5}{\sqrt{5}} & 0 \\ \frac{1}{\sqrt{5}} & \frac{4}{5} & -\frac{7}{5} \\ \frac{2}{\sqrt{5}} & \frac{3}{5} & \frac{1}{5} \end{bmatrix}$$

We have then

$$\frac{M+M'}{2} = \begin{bmatrix} 1 & -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} & \frac{4}{5} & -\frac{2}{5} \\ \frac{1}{\sqrt{5}} & -\frac{2}{5} & \frac{1}{5} \end{bmatrix} \tag{53a}$$

$$= \begin{bmatrix} 1 \\ -\frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} 1 & -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{bmatrix} \tag{53b}$$

while

$$\frac{M-M'}{2} = \begin{bmatrix} 0 & -\frac{3}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \\ +\frac{3}{\sqrt{5}} & 0 & -1 \\ \frac{1}{\sqrt{5}} & +1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{\sqrt{5}} & -\frac{3}{\sqrt{5}} \\ -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{5}} & -1 & 0 \\ -\frac{3}{\sqrt{5}} & 0 & 1 \end{bmatrix} \tag{54b}$$

Figure 7 shows a synthesis for the nonreactive network N_1, derived by series connecting networks of impedance matrices $\frac{M+M'}{2}$. Terminating the final two ports in unit inductors yields Figure 8 for the complete synthesis of the original $Z(s)$ of (45).

Observe that one of the penalties of obtaining a synthesis using simultaneously the minimum number of reactances and resistances is the presence of a gyrator in the realization of the positive-real function $Z(s)$. However, by extracting the resistor and then transforming the resulting lossless structure (after adjoining another port for further resistive termination) the somewhat complicated procedures of Oono and

Yasuura [14, pp. 149-153, 168] yield a gyratorless circuit with the minimum possible number of elements.

VIII. CONCLUSION

The material presented highlights the strong interrelation between network theory and control theory in an elegant manner. One of the classical problems of network theory has been solved by an investigation in terms of the state using not especially advanced control theory concepts.

An interesting and important feature of the synthesis is that it is primarily algebraic in character, rather than analytic, as for example the Brune synthesis. This is quite proper, for the synthesis problem is evidently in some sense a finite-dimensional one, and thus is a priori more reasonably attacked by algebra than analysis.

The key point of the synthesis is the translation of the analytical concept of positive reality into algebraic properties of the matrices of a minimal realization of $Z(s)$. From this point on, the development of the synthesis becomes algebraic.

There are still a number of open problems however. The present theory must certainly be regarded as incomplete when the synthesis of positive-real functions leads to a network containing gyrators. In Section 6 we have attempted to outline some of the difficulties which arise when a reciprocal or, by extension, a minimal gyrator synthesis is sought. Very possibly satisfactory results will be achieved by using the algebraic characterization of reciprocity in [6]. Since however reciprocal synthesis may often have to use more than the minimum number of resistors [14, p. 148], further investigations of the effect of positive reality and reciprocity on realizations is in order.

Another pertinent problem is the development of a scattering matrix synthesis procedure which uses, in a simple manner, some hitherto unestablished property of minimal realizations of scattering matrices. A very positive step has been made in this direction in [6]; reference [20] discusses the statement of the network problem in control systems terms. Nevertheless, the method given here allows the synthesis of any rational bounded-real scattering matrix $S(s)$ since one can form the positive-real impedance matrix $Z = 2(I_n - S)^{-1} - I_n$ if $I_n - S$ is nonsingular. If $I_n - S$ is singular of rank ρ, then one forms $T_o S T_o' = S_o \dotplus I_{n-\rho}$ with T_o a constant orthogonal matrix [14, p. 155] (representing transformers) with $I_\rho - S_o$ nonsingular. This yields a realization through $Z_o = 2(I_\rho - S_o)^{-1} - I_\rho$, which is a positive-real impedance matrix.

The question naturally arises as to how to obtain all passive minimal realizations. From Section 2 we know that every minimal realization results from applying the transformation of (5) to a fixed one. In particular this procedure yields all passive minimal realizations. Nevertheless, except for the RL (or RC) case treated in the sixth section, the restrictions on the transformation T needed to retain passivity can not as yet be specifically stated.

In a different, but somewhat related, manner one can obtain all nonminimal realizations by the use of a previous theory [20].

Some remarks are in order on the computation difficulties of the synthesis described. The major problem is to determine $W(s)$ from $Z(s) + Z'(-s)$. Certainly [29] outlines the procedure, but the actual calculations are long, and considered by Youla to be somewhat inappropriate for programming. The other calculations required in the synthesis are refreshingly easy, and in the one-port case lead to a fairly simple synthesis through use of the canonical minimal realization described by equation (46).

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REFERENCES

1. Bashkow, T. R , "The A Matrix, A New Network Description," IRE Trans. on Circuit Theory, vol. CT-4, no. 3, September 1957, pp. 117-120.
2. Byrant, P. R., "The Explicit Form of Bashkow's A Matrix," IRE Trans. on Circuit Theory, vol. CT-9, no. 3, September 1962, pp. 303-306.

3. Kuh, E. S. and R. A. Rohrer, "The State-Variable Approach to Network Analysis," Proc. of the IEEE, vol. 53, no. 7, July 1965, pp. 672-686.
4. Kalman, R. E., "On a New Characterization of Linear Passive Systems," Proc. of the First Allerton Conference on Circuit and System Theory, University of Illinois, November 1963, pp. 456-470.
5. Anderson, B. D. O., "A Control Theory Viewpoint of Positive Reality," Rept. No. SEL-66-009 (TR No. 6558-4), Stanford Electronics Laboratories, Stanford, California, February 1966.
6. Youla, D. C. and P. Tissi, "N-Port Synthesis via Reactance Extraction - Part I," Electrophysics Memo PIBMRI-1309-66, January 1966, Polytechnic Institute of Brooklyn, Brooklyn, N. Y.
7. Cauer, W., "Die Verwirklichung von Wechselstromwiderstanden vorgeschriebener Frequenzabhängigkeit," Archiv für Elektrotechnik, vol. 17, no. 4, December 5, 1926, pp. 355-388.
8. Brune, O., "Synthesis of a Finite Two-Terminal Network Whose Driving Point Impedance is a Prescribed Function of Frequency," Journal of Mathematics and Physics, vol. 10, no. 3, August 1931, pp. 191-236.
9. Darlington, S., "Synthesis of Reactance Four-Poles which Produce Prescribed Insertion Loss Characteristics," Journal of Mathematics and Physics, vol. 18, September 1939, pp. 257-353.
10. Bott, R. and R. J. Duffin, "Impedance Synthesis Without Use of Transformers," Journal of Applied Physics, vol. 20, no. 8, August 1949, p. 816.
11. McMillan, B., "Introduction to Formal Realizability Theory," Bell System Technical Journal, vol. 31, no. 2, March 1952, pp. 217-279 and vol. 31, no. 3, May 1952, pp. 541-600.
12. Newcomb, R. W., "On Network Realizability Conditions," Proc. of the IRE, vol. 50, no. 9, September 1962, p. 1995.
13. Van Valkenburg, M. E., Introduction to Modern Network Synthesis, John Wiley & Sons, Inc., N. Y., 1960.
14. Oono, Y. and K. Yasuura, "Synthesis of Finite Passive 2n-Terminal Networks with Prescribed Scattering Matrices," Memoirs of the Faculty of Engineering, Kyushu University, vol. 14, May 1954, pp. 125-177.

15. Bayard, M., "Résolution du Problème de la Synthèse des Réseaux de Kirchhoff par la Détermination de RéseauxPurement Reactifs," Câbles et Transmission, vol. 4, no. 4, October 1950, pp. 281-296.
16. Belevitch, V., "Factorization of Scattering Matrices with Applications to Passive Network Synthesis," Philips Research Reports, vol. 18, no. 4, August 1963, pp. 275-317.
17. Belevitch, V., "Synthèse des Reseaux Électrique Passifs à n Paires de Bornes de Matrice de Répartition Prédeterminée," Annales de Télécommunications, vol. 6, no. 11, November 1951, pp. 302-312.
18. Newcomb, R. W., "A Bayard-Type Nonreciprocal n-Port Synthesis," IEEE Trans. on Circuit Theory, vol. CT-10, no. 1, March 1963, pp. 85-90.
19. Kalman, R. E., "Mathematical Description of Linear Dynamical Systems," Journal of the Society for Industrial and Applied Mathematics Control, Series A, vol. 1, no. 2, 1963, pp. 152-192.
20. Anderson, B. D. O., R. W. Newcomb, R. E. Kalman, and D. C. Youla, "On the Equivalence of Linear Time-Invariant Dynamical Systems," Journal of the Franklin Institute, to be published.
21. Tellegen, B. D. H., "Synthesis of Passive Resistanceless Four-Poles that May Violate the Reciprocity Condition," Philips Research Reports, vol. 3, no. 5, October 1948, pp. 321-337.
22. Duffin, R. J. and D. Hazony, "The Degree of a Rational Matrix Function," Journal of the Society for Industrial and Applied Mathematics, vol. 11, no. 3, September 1963, pp. 645-658.
23. Kalman, R. E., "Irreducible Realizations and the Degree of a Matrix of Rational Functions," Journal of the Society for Industrial and Applied Mathematics, vol. 13, no. 2, June 1965, pp. 520-544.
24. Youla, D. C., "The Synthesis of Linear Dynamical Systems from Prescribed Weighting Functions," PIB Report No. PIBMRI-1271-65, June 1965, Polytechnic Institute of Brooklyn, Brooklyn, N. Y.
25. Hazony, D., Elements of Network Synthesis, Reinhold, New York, 1963.
26. Newcomb, R. W., "A Nonreciprocal n-Port Brune Synthesis," Report No. SEL-62-125 (TR No. 2254-5), Stanford Electronics Laboratories, Stanford, California, November 1962.

27. Carlin, H. J., "Synthesis of Nonreciprocal Networks," Proc. of the Polytechnic Institute of Brooklyn Symposium on Modern Network Synthesis, vol. 5, 1955, pp. 11-44.

28. Kalman, R. E., "Lyapunov Functions for the Problem of Lur'e in Automatic Control," Proc. of the National Academy of Sciences, vol. 49, no. 2, February 1963, pp. 201-205.

29. Youla, D. C., "On the Factorization of Rational Matrices," IRE Transactions on Information Theory, vol. IT-7, no. 3, July 1961, pp. 172-189.

30. Macduffee, C. C., The Theory of Matrices, Springer, Berlin, 1933.

31. Duffin, R. J., D. Hazony and N. Morrison, "Network Synthesis Through Hybrid Matrices," Scientific Report No. 4, AFCRL 63-568, Case Institute of Technology, Ohio, August 1964.

32. Rao, T. N., R. W. Newcomb and B. D. O. Anderson, "A Cauer Synthesis of RL n-Ports," in preparation.

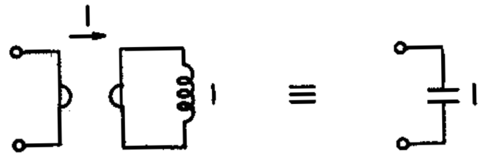


FIG. 1

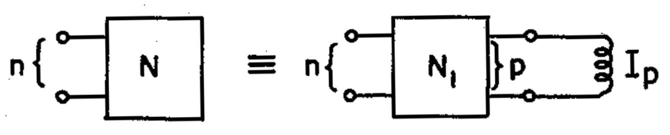
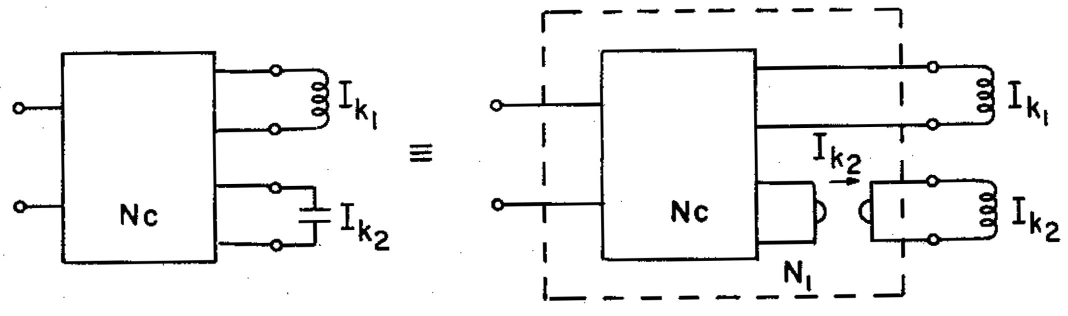
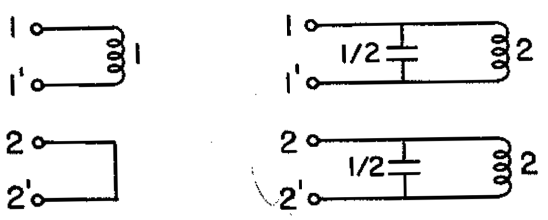


FIG. 2



(a) (b)

FIG. 3



(a) (b)

FIG. 4

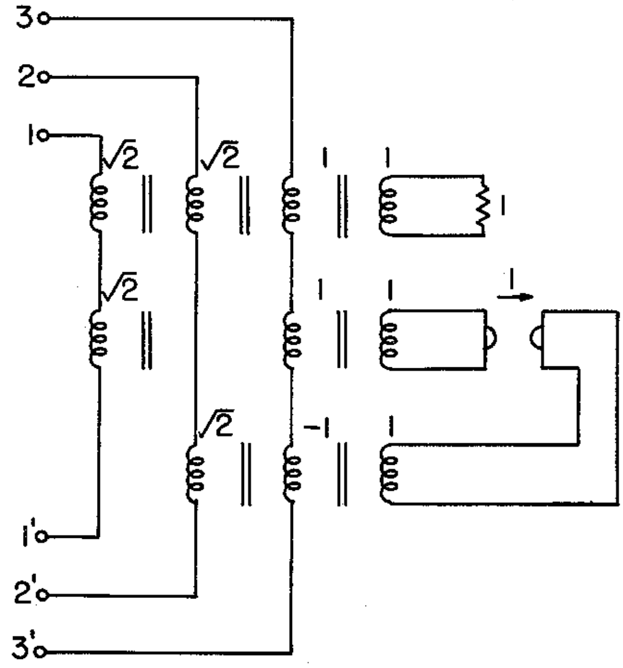


FIG. 5

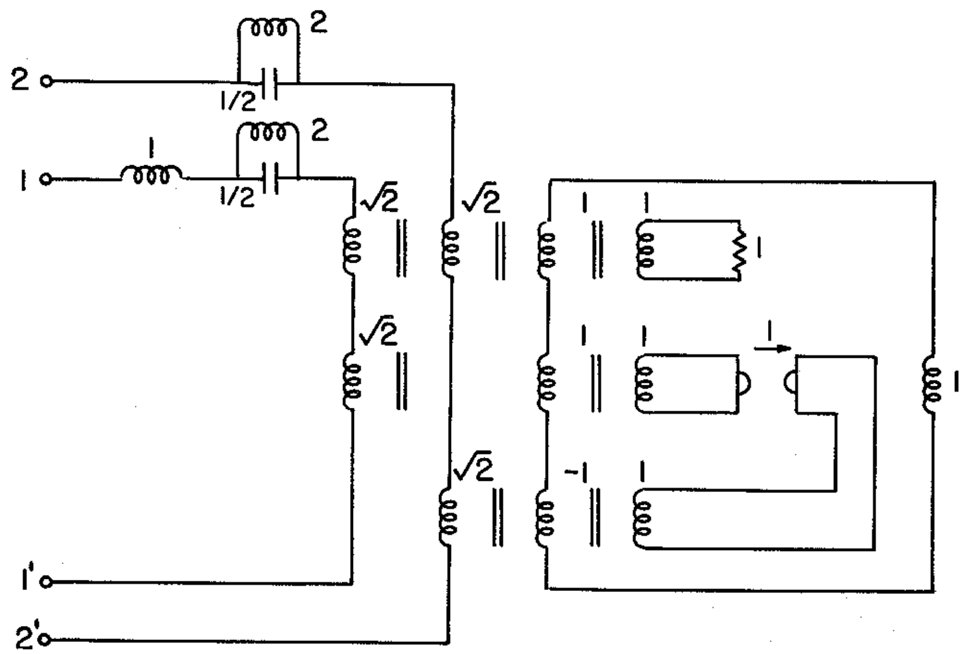
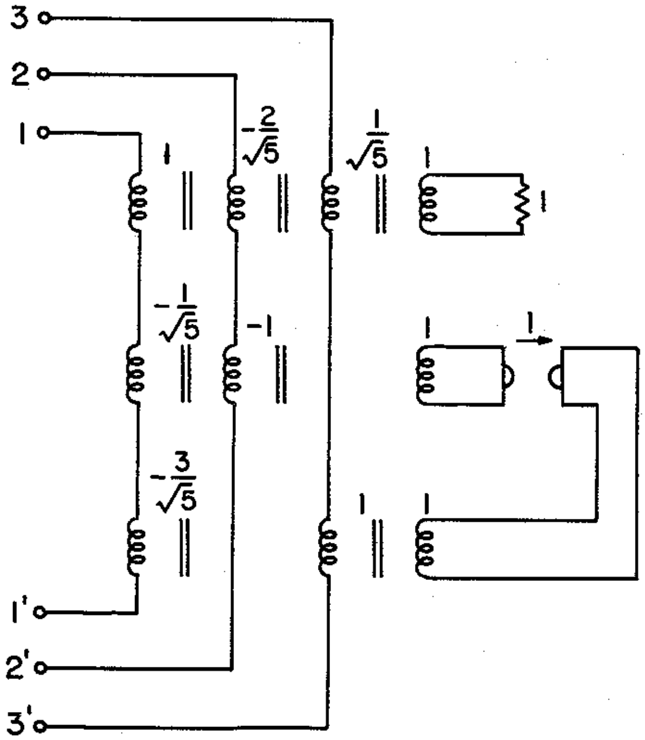
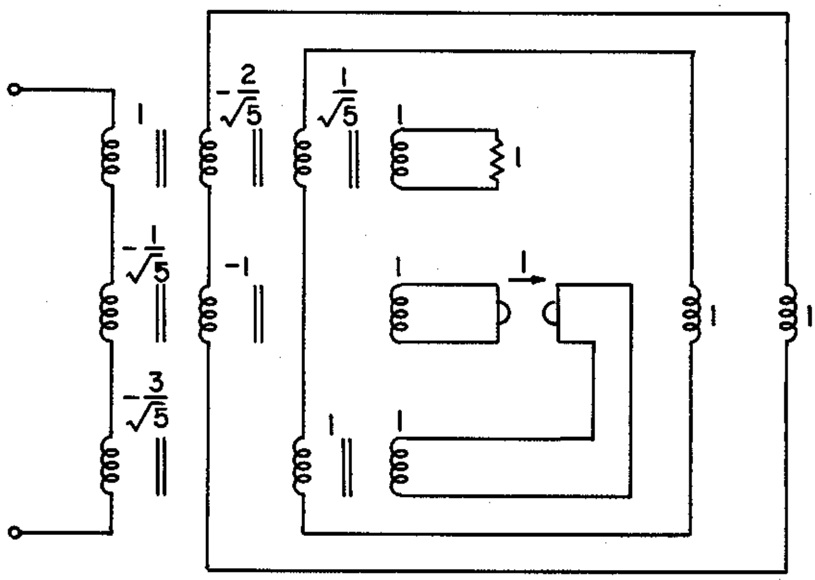


FIG. 6



3
2
1
1'
2'
3'
1
$-\frac{2}{\sqrt{5}}$
$\frac{1}{\sqrt{5}}$
$-\frac{1}{\sqrt{5}}$
-1
$-\frac{3}{\sqrt{5}}$

FIG. 7



1
$-\frac{2}{\sqrt{5}}$
$\frac{1}{\sqrt{5}}$
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FIG. 8

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