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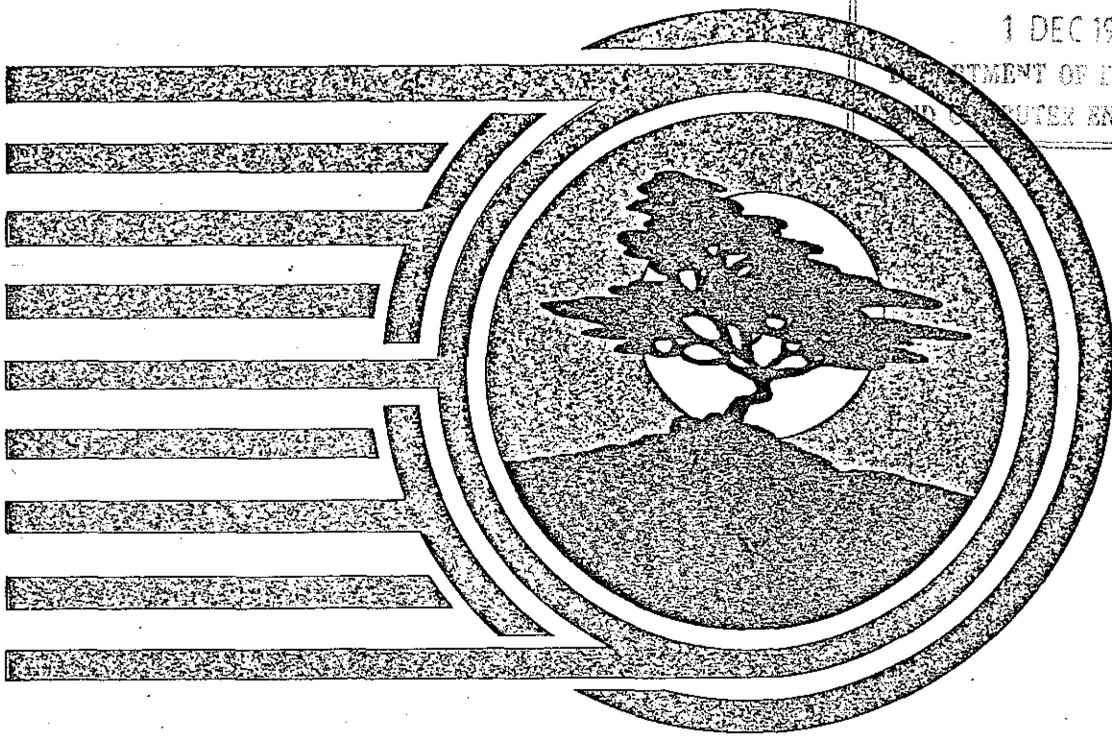

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DIGITAL LATTICE AND LADDER BLOCK STRUCTURES

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Abstract

Several techniques are available for implementing a scalar digital transfer function using lattice and ladder structures. The lattice and ladder structure proposed by Gray and Markel have the advantage of being internally scaled and usually lead to low-noise realizations.

There are some advantages in implementing a scalar digital filter in block form. There is a technique available for obtaining a lattice and ladder structure for implementing a block digital filter. In this paper new algorithms to realize block digital filters in the form of cascaded lattice or ladder two-pairs are proposed. Block lattice and ladder structures similar to Gray and Markel scalar structures are also developed. Also a technique for obtaining block lattice and ladder structures with internal scaling is developed.

I. Introduction

Block implementation of a scalar digital filter has some advantages such as lesser number of computations, the possibility of using FFT techniques for intermediate computations, implementation by array processors, reduced roundoff noise and high speed computation. A scalar digital filter can be implemented in block form in many ways. Block state structures have been studied by Gnanasekaran and Mitra [1], Barnes and Shinnaka [2], Ananthakrishna and Mitra [3]. In this paper, some techniques for realizing a block digital filter in block lattice and ladder form are presented.

Lattice and ladder realization of scalar digital filters have been proposed by Mitra, Kamat and Huey [4] and Gray and Markel [5]. Gray and Markel's structure has the advantage of being internally scaled. Gnanasekaran has proposed a technique for realizing the block lattice and ladder realization of a digital filter [6]. However, in his realization, the design equations become very complicated when the order of the filter is large.

In this paper new algorithms to realize block digital filters in the form of cascaded lattice or ladder two-pairs are proposed. Block lattice and ladder structures similar to Gray and Markel scalar structure are also developed. Finally, a technique for obtaining block lattice and ladder structures with internal scaling is developed.

The general block equation for an N-th order scalar digital transfer function

$$h(z) = \left(\sum_{i=0}^{N} a_i z^{-i}\right) \Big/ \left(\sum_{i=0}^{N} b_i z^{-i}\right)$$

is given by [3]:

$$B_0 Y_k + B_1 Y_{k-1} + \dots + B_p Y_{k-p} = A_0 X_k + A_1 X_{k-1} + \dots + A_p X_{k-p} \quad (1)$$

where p is the smallest integer such that $pL \geq N$ and Y_k and X_k are the k-th output and input blocks of length L;

$$Y_k = [y_{kL}\ y_{kL+1} \cdots y_{kL+L-1}]^t$$

$$X_k = [x_{kL}\ x_{kL+1} \cdots x_{kL+L-1}]^t$$

and, A_k and B_k are $L \times L$ matrices given by

$$A_k = \begin{bmatrix} a_{kL} & \cdots & a_{kL-L+1} \\ \vdots & & \vdots \\ a_{kL+L-1} & \cdots & a_{kL} \end{bmatrix} \quad B_k = \begin{bmatrix} b_{kL} & \cdots & b_{kL-L+1} \\ \vdots & & \vdots \\ b_{kL+L-1} & \cdots & b_{kL} \end{bmatrix}$$

where $a_i = b_i = 0$ for $i < 0$ and for $i > N$. p is defined to be the block order of the transfer function H such that $Y = HX$, where Y and X are the $\mathcal{Z}$-transforms of the output and input blocks, respectively.

The basic building block in the realization scheme is the block digital two pair with $L \times 1$ input vectors X_1 and Y_2, and $L \times 1$ output vectors Y_1 and X_2. The inputs and the outputs are related by the chain matrix:

$$\begin{bmatrix} X_1 \\ Y_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} X_2 \\ Y_2 \end{bmatrix} \quad (2)$$

or by the transfer matrix:

$$\begin{bmatrix} Y_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ Y_2 \end{bmatrix} \quad (3)$$

The transfer parameters are related to the chain parameters by

$$T_{11} = CA^{-1},\ T_{12} = (D - CA^{-1}B),\ T_{21} = A^{-1},\ T_{22} = -A^{-1}B$$

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The block digital filter (1) will be realized by *cascading* p such blocks.

II. Cascaded Block Lattice and Ladder Structure

The main idea in the realization scheme is to realize the p-th order block transfer function $G_p(Z)$ such that

$$Y = G_p(Z)X = Y^{(p)} = G_p(Z)X^{(p)} \tag{4}$$

as a constrained two-pair where the constraining transfer function $G_{p-1}(Z)$ such that $Y^{(p-1)} = G_{p-1}(Z)X^{(p-1)}$ is of lower block order than $G_p(Z)$. $G_{p-1}(Z)$ can then be realized as another constrained two-pair where the constraining transfer function $G_{p-2}(Z)$ is of lower block order than $G_{p-1}(Z)$. This process is continued until the constraining transfer function reduces to a constant matrix. In equation (4) $Z = z^L$.

Let $X^{(i)}$ and $Y^{(i-1)}$ be the input vectors and $Y^{(i)}$ and $X^{(i-1)}$ be the output vectors of i-th stage. The block filter will be realized by cascading p stages. From (2) and (4) the transfer function $G_i(Z)$ such that $Y^{(i)} = G_i(Z)X^{(i)}$ is obtained as

$$G_i(Z) = (C_i + D_iG_{i-1})(A_i + B_iG_{i-1})^{-1} \tag{5}$$

where A_i, B_i, C_i and D_i are the chain parameters of the i-th stage and G_{i-1} is the constraining transfer function such that $Y^{(i-1)} = G_{i-1}(Z)X^{(i-1)}$.

From (5) we can obtain $G_{i-1}(Z)$ as

$$G_{i-1}(Z) = (D_i - G_iB_i)^{-1}\,(G_iA_i - C_i) \tag{6}$$

The general procedure for synthesizing a block p-th order transfer function

$$G_p(Z) = \left(\sum_{i=0}^{p} A_{i,p}Z^{-i}\right)\left(\sum_{i=0}^{p} B_{i,p}Z^{-i}\right)^{-1} \tag{7}$$

is given below. Without loss of generality $B_{0,p}$ can be assumed to be an identity matrix.

Let the chain parameters for the p-th stage be chosen to be

$$A_p = I,\ B_p = B_{p,p}Z^{-1},\ C_p = A_{0,p},\ D_p = A_{p,p}Z^{-1} \tag{8}$$

Let the constraining transfer function G_{p-1} be given by

$$G_{p-1}(Z) = \left(\sum_{i=0}^{p-1} A_{i,p-1}Z^{-i}\right)\left(\sum_{i=0}^{p-1} B_{i,p-1}Z^{-i}\right)^{-1} \tag{9}$$

where $B_{0,p-1}$ is assumed to be an identity matrix. From (5), (8) and (9) we have

$$\begin{aligned} G_p &= (C_p + D_pG_{p-1})(A_p + B_pG_{p-1})^{-1} \\ &= (A_{0,p} + A_{p,p}Z^{-1}G_{p-1})(I + B_{p,p}Z^{-1}G_{p-1})^{-1} \\ &= \left[\left(A_{0,p}\sum_{i=0}^{p-1} B_{i,p-1}Z^{-i}\right) + \left(A_{p,p}Z^{-1}\sum_{i=0}^{p-1} A_{i,p-1}Z^{-i}\right)\right] \\ &\quad\cdot\left[\left(\sum_{i=0}^{p-1} B_{i,p-1}Z^{-i}\right) + \left(B_{p,p}Z^{-1}\sum_{i=0}^{p-1} A_{i,p-1}Z^{-i}\right)\right]^{-1} \end{aligned} \tag{10}$$

From (7) and (10) by comparing the coefficients of the powers of Z^{-1} we get

$$A_{0,p}B_{k,p-1} + A_{p,p}A_{k-1,p-1} = A_{k,p}$$
$$B_{k,p-1} + B_{p,p}A_{k-1,p-1} = B_{k,p}$$

where $k=1,\ldots,(p-1)$ and

$$A_{p-1,p-1} = I \tag{11}$$

Solving equation (11) for $k=1,\ldots,(p-1)$ we get

$$\begin{aligned} A_{k-1,p-1} &= (A_{p,p}-A_{0,p}B_{p,p})^{-1}(A_{k,p}-A_{0,p}B_{k,p}) \\ B_{k,p-1} &= B_{k,p}-B_{p,p}A_{k-1,p-1} \end{aligned} \tag{12}$$

and this defines $G_{p-1}(Z)$ in terms of $G_p(Z)$ provided the inverse exists. This process is repeated for $n=p,(p-1),\ldots,1$ giving the equations

$$\begin{aligned} &A_n = I, B_n = B_{n,n}Z^{-1}, C_n = A_{0,n}, D_n = A_{n,n}Z^{-1} \\ &A_{k-1,n-1} = (A_{n,n}-A_{0,n}B_{n,n})^{-1}(A_{k,n}-A_{0,n}B_{k,n}) \\ &B_{k,n-1} = B_{k,n} - B_{n,n}A_{k-1,n-1} \\ &A_{n-1,n-1} = I \end{aligned} \tag{13}$$

where $k=1,\ldots,(n-1)$. The i-th stage will then be given by

$$\begin{bmatrix} X^{(i)} \\ Y^{(i)} \end{bmatrix} = \begin{bmatrix} I & B_i \\ C_i & D_i \end{bmatrix} \begin{bmatrix} X^{(i-1)} \\ Y^{(i-1)} \end{bmatrix} = \begin{bmatrix} I & \hat{B}_i \\ C_i & \hat{D}_i \end{bmatrix} \begin{bmatrix} X^{(i-1)} \\ Z^{-1}Y^{(i-1)} \end{bmatrix}$$

where $\hat{B}_i = ZB_i$, $\hat{D}_i = ZD_i$ are constant matrices. This gives us the lattice structure. Using the transfer parameters we can get the ladder structure.

In this section we have presented a procedure for synthesizing any given p-th order block transfer function as a cascaded lattice or ladder structure. This lattice structure can be obtained provided the inverses in equation (13) exist.

III. A Structure Similar to Gray and Markel Structure

The transfer function (7) can also be synthesized in a way similar to the Gray and Markel procedure for synthesizing a scalar digital filter as a cascaded lattice and ladder structure [5]. In this section this procedure will be outlined for a block transfer function

$$G_p(Z) = \left(\sum_{i=0}^{p} A_{i,p} Z^{-i}\right)\left(\sum_{i=0}^{p} B_{i,p} Z^{-i}\right)^{-1}$$

In [5] Gray and Markel have outlined a procedure for synthesizing a scalar digital transfer function as a cascaded lattice and ladder structure. They have proposed two procedures, one of which yields two multipliers per stage and the other one, a single multiplier per stage. These structures have the advantage of an internal scaling property.

We wish to design a block digital filter that will implement the transfer function (7), which is given above for convenience. The block lattice and ladder filter will be formulated as follows.

Let $A_p(Z)$ and $B_p(Z)$ denote the matrix polynomials

$$A_p(Z) = \sum_{n=0}^{p} A_{n,p} Z^{-n} \tag{14}$$

$$B_p(Z) = \sum_{n=0}^{p} B_{n,p} Z^{-n} \tag{15}$$

where $B_{0,p} = I$. From $B_p(Z)$ we can define a block p-th order matrix polynomial $P_p(Z)$ as

$$P_p(Z) = Z^{-p} B_p(Z^{-1}) = \sum_{n=0}^{p} P_{n,p} Z^{-n}$$

$$= \sum_{n=0}^{p} B_{p-n,p} Z^{-n} \tag{16}$$

Let $K_p = B_{p,p}$; then $B_p(Z) - K_p P_p(Z)$ is of block order (p-1).

Defining $B_{p-1}(Z)$, a matrix polynomial of block order (p-1), by the equation

$$(I-K_p^{\,2})B_{p-1} = B_p - K_p P_p \tag{17}$$

where for simplicity the Z-dependence of the variables is not explicitly shown and $P_{p-1}(Z)$ by

$$P_{p-1}(Z) = Z^{-(p-1)} B_{p-1}(Z^{-1}) \tag{18}$$

we get

$$(I-K_p^{\,2})P_{p-1}Z^{-1} = -K_p B_p + P_p \tag{19}$$

Solving the simultaneous matrix equations (17) and (19) gives us,

$$B_p = B_{p-1} + K_p Z^{-1} P_{p-1} \tag{20}$$

$$P_p = K_p B_{p-1} + Z^{-1} P_{p-1} \tag{21}$$

provided $(I-K_p^{\,2})$ is invertible.

Also from (17) it can be easily seen that

$$B_{0,p-1} = I$$

Hence, the same process can be repeated to get B_{p-1} and P_{p-1} from B_{p-2} and P_{p-2} and so on.

From (7), (14) and (15) we have

$$G_p(Z) = A_p(Z)[B_p(Z)]^{-1} \tag{22}$$

as the transfer function to be implemented. Using (14) and (16) and letting

$$H_p = A_{p,p}$$

$[A_p(Z) - H_p P_p(Z)]$ will be a matrix polynomial of block order (p-1). This will be defined to be the polynomial $A_{p-1}(Z)$. This process can also be repeated. Hence, we have

$$A_{p-1} = A_p - H_p P_p \tag{23}$$

Therefore

$$A_p = A_{p-1} + H_p P_p$$

$$= A_{p-2} + H_{p-1}P_{p-1} + H_p P_p$$

$$= \sum_{i=0}^{p} H_i P_i \tag{24}$$

where $H_0 = A_{0,0} = A_0(Z)$ and $P_0 = I$.

The transfer function $G_p(Z)$ will now be given by

$$G_p(Z) = \sum_{i=0}^{p} H_i P_i B_p^{-1} \tag{25}$$

and the output block Y is given in terms of input block X as

$$Y = \sum_{i=0}^{p} H_i P_i B_p^{-1} X \tag{26}$$

Hence the output is obtained as a weighted sum of vectors $P_i B_p^{-1} X$, $i=1,2,\ldots,p$.

The tap parameters H_i and K_i can be obtained recursively from $A_p(Z)$ and $B_p(Z)$ using the following equations for $m=p,p-1,\ldots,1$:

$$P_m(Z) = Z^{-m} B_m(Z^{-1}) \tag{27}$$

$$K_m = B_{m,m} \tag{28}$$

$$B_{m-1}(Z) = (I-K_m^{\,2})^{-1}[B_m(Z) - K_m P_m(Z)] \tag{29}$$

$$H_m = A_{m,m} \tag{30}$$

$$A_{m-1}(Z) = A_m(Z) - H_m P_m(Z) \tag{31}$$

provided $(I-K_m^{\,2})^{-1}$ exists. From (27)-(29), it can easily be shown that

$$\begin{bmatrix} B_m \\ P_m \end{bmatrix} = \begin{bmatrix} I & K_m \\ K_m & I \end{bmatrix} \begin{bmatrix} B_{m-1} \\ Z^{-1}P_{m-1} \end{bmatrix} \tag{32}$$

Hence

$$\begin{bmatrix} B_m B_p^{-1} X \\ P_m B_p^{-1} \end{bmatrix} = \begin{bmatrix} I & K_m \\ K_m & I \end{bmatrix} \begin{bmatrix} B_{m-1} B_p^{-1} X \\ Z^{-1} P_{m-1} B_p^{-1} X \end{bmatrix} \tag{33}$$

Once $P_m B_p^{-1} X$ is obtained for $m=1,\ldots,p$, the output Y can be obtained as a weighted sum of these vectors. From (33) it can clearly be seen that the structure is obtained as a cascaded lattice or ladder.

The conditions on $B_p(Z)$ which will guarantee that $(I-K_m^2)$ is invertible for all m (thus permitting execution of the algorithm) have yet to be established. Preliminary indications are that stability of $G_p(Z)$ will suffice.

This formulation of the block lattice and ladder does not have the advantage of internal scaling property, the scalar formulation has. But this formulation does reduce to Gray and Markel's form with the advantage of internal scaling when the block length is unity.

IV. Block Lattice and Ladder Structure with Internal Scaling

Our aim is to generate block lattice structures which exhibit an internal scaling property as in the scalar structure proposed by Gray and Markel. One way of getting the block lattice structure with scaling is to obtain block versions of each of the polynomials obtained in the scalar case, which will be the same as those in the previous section for a block length of unity.

The block transfer function we would like to implement is given by

$$G_p(Z) = \left(\sum_{i=0}^{p} A_{i,p} Z^{-i}\right)\left(\sum_{i=0}^{p} B_{i,p} Z^{-i}\right)^{-1} \qquad (34)$$

where $A_{i,p}$ and $B_{i,p}$ are of the form of A_i and B_i of (1). For simplicity we assume that $pL = N$. If this is not the case zero coefficients can be added to the scalar transfer function.

$$\text{Let } A_p(Z) = \sum_{i=0}^{p} A_{i,p} Z^{-i} \qquad (35)$$

$$B_p(Z) = \sum_{i=0}^{p} B_{i,p} Z^{-i} \qquad (36)$$

If the corresponding scalar transfer function is

$$h_N(z) = \left(\sum_{i=0}^{N} a_{i,N} z^{-i}\right) \Big/ \left(\sum_{i=0}^{N} b_{i,N} z^{-i}\right) \qquad (37)$$

where $b_{0,N}$ is assumed to be unity, then $B_p(Z)$ is given by (36) where

$$B_{0,p} = \begin{bmatrix} 1 & \cdots & 0 \\ \vdots & & \vdots \\ b_{L-1,N} & \cdots & 1 \end{bmatrix} \quad B_{i,p} = \begin{bmatrix} b_{iL,N} & \cdots & b_{iL-L+1,N} \\ \vdots & & \vdots \\ b_{iL+L-1,N} & \cdots & b_{iL,N} \end{bmatrix}$$

for $i=1,\ldots,p$. (38)

Let $a_N(z)$ and $b_N(z)$ be the numerator and denominator polynomials of $h_N(z)$. Then $B_p(Z)$ is the block version of the polynomial $b_N(z)$.

If $P_p(Z)$ is obtained from $B_p(Z)$ as

$$P_p(Z) = Z^{-p} B_p^{t}(Z^{-1}) \qquad (39)$$

then $P_p(Z)$ will be the block version of $p_N(z)$ obtained from $b_N(z)$ by the relation

$$p_N(z) = z^{-N} b_N(z^{-1}) \qquad (40)$$

Defining $\gamma_1 = (P_{0,p})_{11}$, $\alpha_1 = \dfrac{1}{1-\gamma_1^2}$, $\beta_1 = -\gamma_1\alpha_1$ and

$$\begin{bmatrix} B_p^{(1)} \\ P_p^{(1)} \end{bmatrix} = \begin{bmatrix} \alpha_1 I & \beta_1 I \\ \beta_1 J & \alpha_1 J \end{bmatrix} \begin{bmatrix} B_p \\ P_p \end{bmatrix} \qquad (41)$$

$$\text{where } J = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \ldots & 1 \\ Z & 0 & 0 & \ldots & 0 \end{bmatrix} \text{ and } J^L = ZI \qquad (42)$$

we find that $B_p^{(1)}$ and $P_p^{(1)}$ will be the block versions of the scalar polynomials $b_{N-1}(z)$ and $p_{N-1}(z)$ obtained using the scalar Gray and Markel lattice formulation. Repeating this process L times we get

$$\begin{bmatrix} B_p^{(L)} \\ P_p^{(L)} \end{bmatrix} = \begin{bmatrix} \alpha_L I & \beta_L I \\ \beta_L J & \alpha_L J \end{bmatrix} \cdots \begin{bmatrix} \alpha_1 I & \beta_1 I \\ \beta_1 J & \alpha_1 J \end{bmatrix} \begin{bmatrix} B_p \\ P_p \end{bmatrix}$$

$$= \begin{bmatrix} M_{1p} & M_{2p} \\ M_{3p} & M_{4p} \end{bmatrix} \begin{bmatrix} B_p \\ P_p \end{bmatrix} \qquad (43)$$

$B_p^{(L)}(Z)$ and $P_p^{(L)}(Z)$ will be the block versions of $b_{N-L}(z)$ and $p_{N-L}(z)$, respectively. These will be denoted by $B_{p-1}(Z)$ and $P_{p-1}(Z)$. Further, it can be shown that

$$P_{p-1}(Z) = Z^{-(p-1)} B_{p-1}^{t}(Z^{-1}) \qquad (44)$$

and

$$M_{4p} = ZM_{1p}^{t}(Z^{-1}) = ZM_{1p}^{*}$$

$$M_{3p} = ZM_{2p}^{t}(Z^{-1}) = ZM_{2p}^{*} \qquad (45)$$

where $M^* = M^t(Z^{-1})$. Hence

$$\begin{bmatrix} B_{p-1} \\ P_{p-1} \end{bmatrix} = \begin{bmatrix} M_{1p} & M_{2p} \\ ZM_{2p}^{*} & ZM_{1p}^{*} \end{bmatrix} \begin{bmatrix} B_p \\ P_p \end{bmatrix} \qquad (46)$$

Equation (46) can be rewritten as

$$\begin{bmatrix} B_p \\ P_p \end{bmatrix} = \frac{1}{\nu_p} \begin{bmatrix} M_{1p}^{*} & -M_{2p}Z^{-1} \\ -M_{2p}^{*} & M_{1p}Z^{-1} \end{bmatrix} \begin{bmatrix} B_{p-1} \\ P_{p-1} \end{bmatrix} \qquad (47)$$

where ν_p is a constant such that

$$M_{1p}M_{1p}^{*} - M_{2p}M_{2p}^{*} = \nu_p I$$

Now we can obtain A_{p-1} from A_p by the relation

$$A_{p-1} = A_p - H_p P_p \tag{48}$$

where

$$H_p = P_{p,p}^{-1} A_{p,p} \tag{49}$$

It can be proved that $P_{p,p}$ is invertible. This process can be repeated giving the relation

$$A_p = \sum_{i=0}^{p} H_i P_i \tag{50}$$

The output block Y will therefore be obtained from the input block X as

$$Y = \sum_{i=0}^{p} H_i P_i B_p^{-1} X \tag{51}$$

where $P_i B_p^{-1} X$ is obtained from the equation

$$\begin{bmatrix} B_i B_p^{-1} X \\ P_i B_p^{-1} X \end{bmatrix} = \frac{1}{\nu_i} \begin{bmatrix} M_{1i}^* & -M_{2i} Z^{-1} \\ -M_{2i}^* & M_{1i} Z^{-1} \end{bmatrix} \begin{bmatrix} B_{i-1} B_p^{-1} X \\ P_{i-1} B_p^{-1} X \end{bmatrix} \tag{52}$$

for $i=1,\ldots,p$, where M_{1i}^* and M_{2i}^* are matrix polynomials of order 1 and $B_0 = P_0 = I$. Equation (52) gives the lattice structure and the output is obtained as a weighted sum of $P_i B_p^{-1} X$, $i=1,\ldots,p$.

Equation (41) can also be written as

$$\begin{bmatrix} B_p \\ P_p \end{bmatrix} = \begin{bmatrix} I & \gamma_1 J^{-1} \\ \gamma_1 I & J^{-1} \end{bmatrix} \begin{bmatrix} B_p^{(1)} \\ P_p^{(1)} \end{bmatrix} \tag{53}$$

where

$$J^{-1} = \begin{bmatrix} 0 & 0 & \ldots & 0 & Z^{-1} \\ 1 & 0 & \ldots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \ldots & 1 & 0 \end{bmatrix} \tag{54}$$

This process can be repeated giving us

$$\begin{bmatrix} B_p \\ P_p \end{bmatrix} = \begin{bmatrix} I & \gamma_1 J^{-1} \\ \gamma_1 I & J^{-1} \end{bmatrix} \cdots \begin{bmatrix} I & \gamma_L J^{-1} \\ \gamma_L I & J^{-1} \end{bmatrix} \begin{bmatrix} B_{p-1} \\ P_{p-1} \end{bmatrix} \tag{55}$$

If, instead of multiplying these matrices we implement (55) as a cascaded section, each of the subsections will be a lattice.

This formulation will give scaled internal variables as these are block versions of the internal variables in the scalar structure. This can be shown to give a stable filter provided $|\gamma_i| < 1$.

V. Conclusion

In this paper we have presented some new algorithms to obtain lattice and ladder structures of a block digital filter. Section II gives a general method of synthesizing a block digital filter in lattice and ladder form. In section III lattice and ladder structures similar to the scalar Gray and Markel structures are discussed. Both of these structures are unscaled structures. But the formulation is easy. Finally, in section IV we have developed an algorithm to obtain a structure in which the internal variables are scaled.

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