

# ADAPTIVE CONTROL RESEARCH - SOME TACTICAL GOALS<sup>†</sup>

Brian D.O. Anderson  
 Department of Electrical Engineering  
 University of Newcastle  
 New South Wales, 2308, Australia

## ABSTRACT

A collection of desirable results is suggested which represent some suitable midway goals in the overall study of adaptive control systems. These are suggestions of several helpful directions for research rather than an explicit list of open problems but should indicate some areas of current interest.

We aim to suggest several directions in which it would be helpful to have some results. The suggestions are not meant to be exhaustive, but rather representative of the author's personal biases.

1. It is a very natural idea to imagine that an adaptive control problem should be tackled by a two stage procedure - identification, plus control using the current estimate of the plant. However, the direct translation of this idea into reality has not proved easy. Difficulties arise as to the existence of differential equation solutions, or their boundedness, and these need to be resolved before the problem can be considered solved. Extra signals sometimes are required. What we need to know is whether the identification-followed-by-control strategy is intrinsically bad, or whether the technical difficulties are simply artifacts of our present way of looking at the problem. At least the issue of extra signals is beginning to be resolved in an intuitively appealing way, e.g., in the work of Egardt at Lund.

2. There is a great need for some robustness-type of results. More precisely, we need to know what happens if the model order is less than the plant order, what happens if there is a small amount of nonlinearity present, what happens if the plant parameters are slowly varying. In the noiseless case where the above sorts of difficulties do not occur, the problem is generally reduced to one of establishing the asymptotic stability, or global asymptotic stability, of a set of unforced, time-varying equations. Robustness-type results will generally be results then which require some form of adjustment or perturbation to these equations. Now differential equations which are most tolerant of adjustments, in the sense that qualitative properties of solutions are maintained, are those

with an exponential stability property. So a first step towards the development of robust results may be the establishing of exponential, rather than merely asymptotic, stability in the usually considered situations. Exponential stability may also prove a helpful tool in dealing with noise.

3. Just as with observer design, it has been common in adaptive identification and control to consider two cases, deterministic and stochastic. In the deterministic case, there is no noise present and system inputs are deterministic signals. In the stochastic case, there is noise present, and the system inputs may not be deterministic. But there is an inbetween situation, at least for adaptive identification: no output noise, but stochastic inputs. This requires one in the first instance to examine equations like

$$\dot{x} = -A(t)x$$

where  $A(t) = u(t)u'(t)$ ,  $u(\cdot)$  a vector random process. If  $u(\cdot)$  is gaussian, it is impossible to guarantee that for some positive  $\alpha_1, \alpha_2$  and  $T$  and all  $t$ ,

$$\alpha_1 I \leq \int_t^{t+T} u(\tau)u'(\tau) d\tau \leq \alpha_2 I$$

(This condition plays a crucial role in the deterministic case). New theoretical techniques are needed to analyze the equation - one of the simplest in the deterministic theory - when a stochastic  $A(t)$  is involved. Of course, when positive real functions appear in addition, the problem is significantly more difficult.

4. We should be flexible in seeking, in the first instance, results in discrete time if continuous time is too hard, or vice versa. In discrete-time, we have the advantage of the existence of tapped delay line models, we have the property that

$$\sum_{i=0}^{\infty} x_i^2 < \infty \text{ implies } x_i \rightarrow 0 \text{ as } i \rightarrow \infty \text{ (whereas}$$

$$\int_0^{\infty} x^2 dt < \infty \text{ need not imply } x(t) \rightarrow 0 \text{ as } t \rightarrow \infty),$$

many stochastic process issues are reduced to random variable issues, and the possibility of adaptive control with delay becomes a theoretically attractive technique for coping with nonminimum phase and/or awkward relative order difficulties.

On the other hand, the fact that a discrete-time positive real function inevitably contains direct feedthrough (which is not so in continuous time) apparently can cause technical difficulties.

5. We should not downplay the importance of local, as opposed to global, stability results. Telephone channel equalizers, while not adaptive control elements, are certainly in a sense adaptive identification elements, and there is an important class (decision feedback) which may be locally, but not globally, stable.

6. The problem of polynomial root conditioning is well known. In adaptive identification/control, this suggests that a reliance on canonical forms or their equivalent may be dangerous (though more so in continuous than discrete-time). The advantage with canonical forms is that we are often easily able to have a 1-1 onto relationship between parameter values and nominal transfer function, and perhaps more importantly parameters enter linearly into the relevant equations. This would not be the case if, for example we thought of a degree 6 proper transfer function as a product of 4 degree 2 proper transfer functions. There seems to be a challenge in constructing representations that will avoid the numerical difficulties of those currently used, yet preserve as far as possible their advantages.

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