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STABILITY OF CONTROL SYSTEMS WITH MULTIPLE NONLINEARITIES

by

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ABSTRACT

The paper generalizes the Popov criterion for the stability of a system containing a simple memoryless nonlinearity to the case of a system containing an arbitrary number of memoryless nonlinearities.

1. Introduction

One of the more successful control systems investigations of recent years has concerned the stability of automatic control systems which contain a simple memoryless nonlinearity; normally the nonlinearity is of the type shown in Fig. 1, that is, it is confined to a sector of the first and third quadrants.

The study of this problem was initiated by Lur'e [1] and continued by others. In 1961 Popov [2] made a most important contribution to the problem by giving a solution in frequency domain terms, and then in 1963 Kalman [3] gave a treatment unifying the various approaches used up till that time. This treatment relied on a theorem relating the (network theory) concept of a positive real function to the (control theory) concept of a minimal realization of a transfer function.

The aim of this paper is to extend the theory to include the case of multiple memoryless nonlinearities. To do so we shall use a recently developed theorem [4] generalizing that used in [3]; this theorem relates the concept of a positive real *matrix* to the concept of a minimal realization of a *matrix* of transfer functions.

In section 2 we state the problem formally; we point out two apparently distinct occurrences of nonlinearities which are really not different, so that both are in the ambit of the problem considered. Section 3 is devoted to establishing the main result, while we indicate an application of the result in section 4 to deriving a subsidiary result, already discovered by other techniques, on the stability of second order nonlinear systems.

2. Statement of the Problem

Our examination is of lyapunov stability, and accordingly we neglect the inputs of the systems considered. Figure 2 shows the prototype system. The matrix $W(s)$ is an $n \times n$ matrix of stable rational transfer functions, assumed to be such that

$$W(\infty) = 0 \tag{1}$$

(which means that there is at least one integration between any one input and any one output of the linear part of the system). The block of nonlinearities may be described by a function of a vector which is itself a vector. If σ is the (vector) input to the nonlinearities, the (vector) output is $\psi(\sigma)$. We shall assume that for all σ

$$\psi'(\sigma)\,\sigma \geq \psi'(\sigma)\,K\,\psi(\sigma) \tag{2}$$

where K is a nonnegative definite matrix. Note that (2) includes the case shown in Figure 3, where each ψ_i satisfies

$$0 \leq \psi_i(\sigma_i)\sigma_i \leq k_i\,\sigma_i^2 \qquad (i = 1, 2, \ldots n) \tag{3}$$

Equation (3) when $n = 1$ reduces to the usual constraint imposed in the treatment of the single nonlinearity problem.

A transfer function $W(s)$ satisfying (1) possesses a minimal realization $\{F, G, H\}$, which is a set of three constant matrices, F being $p \times p$, G and H being $p \times n$, p of minimal dimension, and for which [5]

$$W(s) = H'(sI - F)^{-1} G \tag{4}$$

This is equivalent to saying that $W(s)$ is the transfer function relating the input u to output y of the system whose state space representation is

$$\dot{x} = Fx + Gu \tag{5a}$$

$$y = H'x \tag{5b}$$

Since W is stable, all eigenvalues of F have negative real parts.

For the system of Figure 2 we must identify y with σ and u with $\psi(\sigma)$. However to conform with more usual notation, we shall assume an element of gain equal to -1 to be inserted in the loop, so that we have

$$\dot{x} = Fx - G\,\psi\,(H'x) \tag{6a}$$

$$\psi'(H'x)\,H'x \geq \psi'(H'x)\,K\,\psi(H'x) \tag{6b}$$

Our aim is to study the stability properties of Eqs. (6).

Before proceeding directly with this task however, we point out that the form of Figure 2 for the nonlinear system is not as restrictive as might appear. For example the system of Figure 4 can be represented as a system of the form of Figure 2, as is shown by Figure 5. Clearly this sort of equivalence is extendable, ad infinitum.

3. Main Result

The statement of the main result is as follows:

Suppose for the system (6), there exists $\alpha \geq 0$, $\beta \geq 0$, $\alpha + \beta > 0$ such that $\alpha K + (\alpha + \beta s)W(s)$ is positive real. Then the system is stable.

The proof of this theorem rests on the following lemma, proved in [4]:

Lemma Let $Z(s)$ be a positive real $n \times n$ matrix, with $Z(\infty)$ finite. Let $\{F_1, G_1, H_1\}$ be a minimal realization for $Z(s) - Z(\infty)$. Then there exists a positive definite symmetric P and a matrix L such that

$$P F_1 + F_1' P = - LL' \tag{7a}$$

$$P G_1 = H_1 - L W_o \tag{7b}$$

$$W_o' W_o = Z(\infty) + Z'(\infty) \tag{7c}$$

Proof of theorem. We shall initially relate a triple $\{F_1, G_1, H_1\}$ associated with $\alpha K + (\alpha + \beta s) W(s)$ to the triple $\{F, G, H\}$ associated with W. We have

$$\alpha K + (\alpha + \beta s) W(s) = \alpha K + \alpha H'(sI - F)^{-1} G + \beta H'(sI - F)(sI - F)^{-1} G$$
$$+ \beta H' F(sI - F)^{-1} G$$

$$= (\alpha K + \beta H'G) + (\alpha H' + \beta H'F)(sI - F)^{-1}G \tag{8}$$

Thus we identify

$$Z(\infty) = \alpha K + \beta H'G \tag{9a}$$

and

$$W_o'W_o = 2\alpha K + \beta H'G + \beta G'H \tag{9b}$$

$$H_1 = \alpha H + \beta F'H \tag{9c}$$

$$F_1 = F \tag{9d}$$

$$G_1 = G \tag{9e}$$

Because $\{F, G, H\}$ is minimal, $\{F, G\}$ is completely controllable and $\{F, H\}$ completely observable [5]. This implies $\{F_1, G_1\}$ is completely controllable and $\{F_1, H_1\}$ is completely observable, the latter because $\{H_1', H_1'F_1, H_1'F_1^2,\ldots\}$ has clearly the same rank as $\{H', H'F, H'F^2,\ldots\}$. See [5].

Using the lemma, we select P and L such that

$$PG = \alpha H + \beta F'H - L W_o \tag{10a}$$

$$PF + F'P = - LL' \tag{10b}$$

We adopt as a tentative lyapunov function

$$V(x) = x'Px + 2\beta \int_0^{H'x} \psi'(\sigma)\, d\sigma \tag{11}$$

which is positive for all nonzero x by the positive definiteness of P (see the lemma) and the restriction on ψ (see Eq. 2).

Differentiating and using Eq. (6a) for $\dot{x}$, one has

$$
\begin{aligned}
V(x) &= x'(F'P + PF)x - x'PG\,\psi(H'x) - \psi'(H'x)G'Px \\
&\quad + 2\beta\,\psi'(H'x)\,H'\,[Fx - G\,\psi(H'x)] \\
&= -x'LL'x - x'(\alpha H + \beta F'H)\,\psi(H'x) + x'LW_o\,\psi(H'x) \\
&\quad - \psi'(H'x)\,(\alpha H' + \beta H'F)x - \psi'(H'x)\,W_o'\,L'x \\
&\quad + 2\beta\,\psi'(H'x)\,H'Fx - 2\beta\,\psi'(H'x)\,H'G\,\psi(H'x)
\end{aligned}
$$

(using Eqs. (10))

$$
\begin{aligned}
&= -x'LL'x + 2x'L\,W_o\,\psi(H'x) - \psi'(H'x)\,W_oW_o'\,\psi(H'x) \\
&\quad -\alpha\,\{2x'H\,\psi(H'x) - 2\,\psi'(H'x)K\,\psi(H'x)\}
\end{aligned}
$$

(using Eq. (9b) on the last term, and rearranging)

$$
\begin{aligned}
&= -\,[x'L - \psi'(H'x)\,W_o']\,[L'x - W_o\,\psi(H'x)] \\
&\quad - 2\alpha\,[\psi'(H'x)\,H'x - \psi'(H'x)K\,\psi(H'x)] \qquad (12)
\end{aligned}
$$

The first term is clearly nonpositive, while the second is also, by the restriction on ψ, Eq. (6b), and the fact that $\alpha \geq 0$. This proves the theorem.

We comment that conditions for asymptotic stability ($\dot{V} < 0$ for all $x \neq 0$, with x sufficiently close to zero) or global asymptotic stability are awkward to formulate in terms of ψ, K etc. for the general case, although they are easy to check for a specific case. If for example α is nonzero, then it is clear from (12) that if Eq. (6b) is satisfied with an inequality sign for $x \neq 0$, asymptotic stability prevails.

4. Example

To illustrate the preceding theory we shall examine the stability of the second order nonlinear system

$$\ddot{y} + a(\dot{y}) + b(y) = 0 \tag{13}$$

We assume that a and b are continuous functions of their arguments.

Setting $x' = [y, \dot{y}]$ it is easy to see that (13) is equivalent to

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x - \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} b(x_1) \\ a(x_2) \end{bmatrix} \tag{14}$$

We make the identifications

$$F = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad G = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \qquad H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{15}$$

and

$$\psi(H'x) = \begin{bmatrix} b(x_1) \\ a(x_2) \end{bmatrix} \tag{16}$$

Then $W(s) = H'(sI - F)^{-1} G$ is given by

$$W(s) = \frac{1}{s^2} \begin{bmatrix} 1 & 1 \\ s & s \end{bmatrix} \tag{17}$$

The nonlinearity ψ must satisfy

$$\psi'(\sigma)\,\sigma \geq \psi'(\sigma)\,K\,\psi(\sigma) \tag{6b}$$

or

$$b(x_1)x_1 + a(x_2)x_2 \geq [b(x_1)]^2 k_1 + [a(x_2)]^2 k_2 \qquad (18)$$

where we arbitrarily select K to be diagonal; the scalars k_1 and k_2 are positive. Then from (18) it is clear that separately

$$b(x_1)x_1 \geq [b(x_1)]^2 k_1 \qquad (19a)$$

and

$$a(x_2)x_2 \geq [a(x_2)^2] k_2 \qquad (19b)$$

Equations (19) require that a and b pass through the origin, and lie in sectors in the first and third quadrants.

To examine stability, we check to see if there exist α and β such that $\alpha K + (\alpha + \beta s) W(s)$ is positive real. Indeed there do, for with $\alpha = 0$, $\beta = 1$,

$$sW(s) = \begin{bmatrix} \frac{1}{s} & \frac{1}{s} \\ 1 & 1 \end{bmatrix} \qquad (20)$$

which is easily established to be positive real.

Thus we conclude that Eqs. (19) guarantee stability of the solution of (13).

5. Conclusions

The main result we have given is especially similar to the Popov criterion known for the single nonlinearity case, and, naturally, reduces to the Popov criterion in the single nonlinearity case. Because of its matrix nature however, it is not so convenient for graphical use as is the scalar criterion.

The result, with the aid of an earlier lemma, permits the writing down of a lyapunov function and its derivative in a form which allows examination of asymptotic and other types of stability. It also allows consideration of the stability of nonlinear differential equations, as for example Eq. (13), the technique used being perhaps not so "ad hoc" as other methods of tackling the same problem.

Problems where the nonlinearities are restricted to sectors which are the n-dimensional generalization of that shown in Figure 1, rather than sectors which include the σ axes, can be attacked using the standard pole shifting theory, see e.g. [6].

Future extensions of the theory included herein could attempt n-dimensional generalizations of the results of Desoer [7] where $W(s)$ is not rational, and Brockett and Willems [8] where the nonlinearity is monotonic.

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Figure Captions

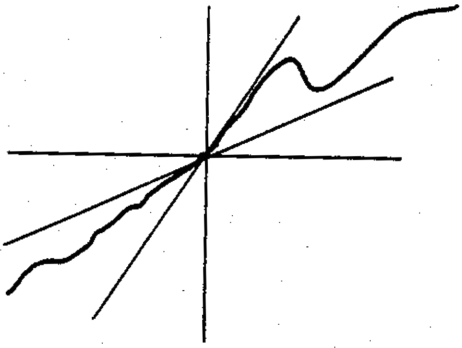
1. Bounds indicating allowable nonlinearities

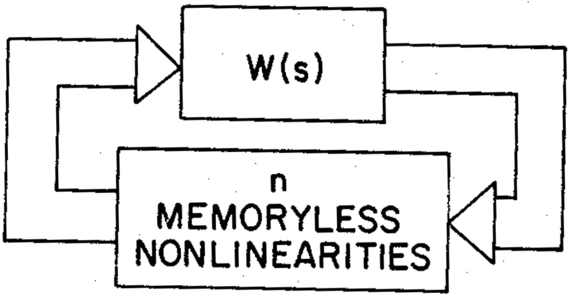
2. Prototype system

3. Specific type of nonlinearity

4. System which is of apparently different nature to that of Figure 2.

5. Alternative representation of system of Figure 4.




W(s)
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MEMORYLESS
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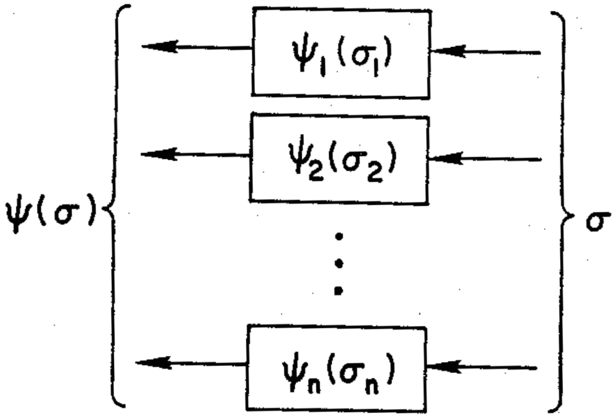
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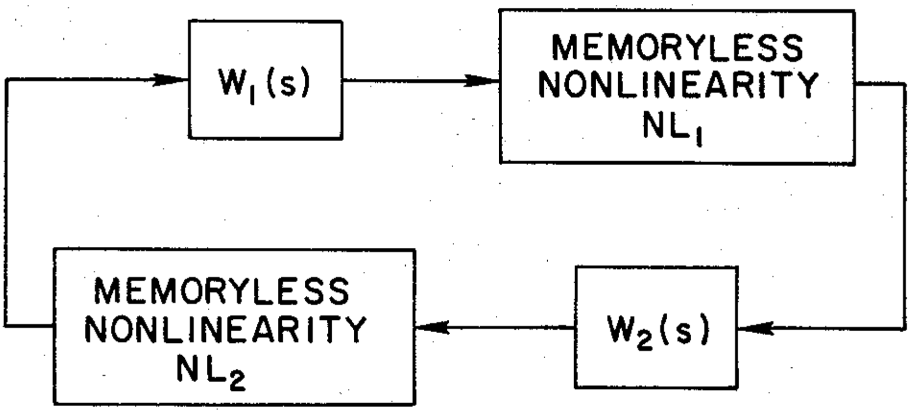
σ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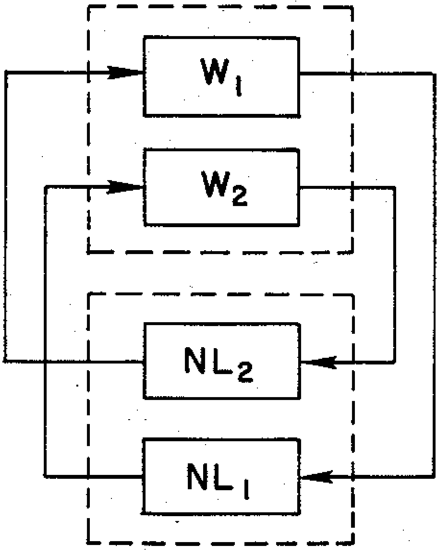
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