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ADAPTIVE IDENTIFICATION OF MULTIPLE-INPUT MULTIPLE-OUTPUT PLANTS

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Abstract

Situations are described in which adaptive identification of a linear plant is possible. These prototype situations are then applied to exhibit procedures for the identification of multivariable plants. Adaptive observers are also discussed.

1. Introduction

Early in the application of state-variable techniques to linear, finite-dimensional, control systems problems, it was recognized that some device excluding differentiators was required to generate the state of a dynamical system, when input and output, but not state, measurements were available. The state estimation problem is solved by the use of Luenberger observers or Kalman-Bucy filters [1-3]; both classes of estimators have the appealing feature that they themselves are linear finite-dimensional systems. Thus a device conceptually no more complicated than the plant under consideration can be built into a black box, can be fed with the plant input and output, and can be made to produce an estimate of the plant state.

Adaptive identifiers in some ways are analogous devices. They are used to identify the parameters of the transfer function or transfer function matrix of a plant; they are fed with the plant input and output; they are finite-dimensional, like the plant, but generally nonlinear, unlike the plant; their outputs converge asymptotically to the correct plant parameters, and they do not contain differentiators. We might even claim that the adaptive identifiers considered in this paper parallel Luenberger, as opposed to Kalman-Bucy estimators, for the reason that no account is taken of noise performance (other than in the disallowing of differentiators in the identifiers).

In this paper, we attempt basically to do two things. First, we describe two prototype situations in which identification is possible. The situations may not look to be general, or of much application. Our second contribution is to show that they are in truth of wide applicability, and this we demonstrate by showing how an adaptive identification can be carried out of a transfer function matrix with unknown denominator and numerator, but subject to a dimensionality constraint. (Virtually all work on adaptive identifiers to this point has concentrated on scalar plants).

The earlier work which has most influenced our thinking is that of Lion [4], Narendra and coworkers, e.g. [5-7], and Carroll and Lindorff, [8], all for the scalar transfer function identification problem. The connection between our results and those of other authors is quite important; Lion's results in essence are recovered when our procedure for identifying transfer function matrices via the first prototype is specialized to the scalar case, while Narendra's and Carroll and Lindorff's result by specializing our second prototype.

The reader familiar with Narendra's work might argue that the coefficient matrices in his state-variable equations look nothing like those appearing in our equations. Indeed this is true, but *the point is that in the identification of input-output quantities*, the particular state-variable realizations used are of little consequence. Indeed, it is for this reason that we have tried to play down the role of state-variables as much as possible and tried to emphasise input-output descriptions.

In our second prototype situation, positive real matrices make their appearance. As far as we know, Parks [9] was the first to observe that when a transfer function appearing in a certain way in an adaptive control problem was positive real, the adaptive control problem could be readily tackled. (The idea has also been exploited in the adaptive control context by Landau, see e.g. [10]). Parks' idea is in effect translated to the adaptive identification problem by Narendra, and Carroll and Lindorff, and in this paper, we indicate the multivariable extension.

2. Prototype Situations for Identification

In this section, we indicate without proof situations in which identification is possible. Proofs will appear elsewhere.

The first situation is depicted in Figure 1, which should be considered as the plant. We assume

(a) $W(s)$, $W_1(s)$ are known real rational transfer function matrices with poles of all elements in $\mathrm{Re}[s] < 0$

(b) Measurements of the plant input $v_p(\cdot)$ and output $y_p(\cdot)$ are available

(c) The plant is asymptotically stable.

The task is to use the measurements $v_p(\cdot)$ and $y_p(\cdot)$ to identify the overall plant transfer function matrix, viz. $K_p[I-W_1(s)K_p]^{-1}W(s)$. This may or may not imply that K_p also is identified.

The scheme for doing this is depicted in Figure 2, which in a rough sense constitutes a model of the plant. All signals in the model are available for measurement, and the model gain matrix is both known and adjustable. We have the following important result:

Theorem 1 Suppose the adjustment law is

$$(\dot{K}_m)_{ij} = -\lambda_{ij}(y_m-y_p)_i(u_m)_j \tag{1}$$

where the subscripts outside parentheses denote a matrix or vector entry and λ_{ij} is an arbitrary positive constant. Then $\lim_{t\to\infty}[y_m(t)-y_p(t)] = 0$; moreover if $v_p(\cdot)$ is totally exciting (as explained below),

$$\lim_{t\to\infty} K_m(t)[I-W_1(j\omega)K_m(t)]^{-1}W(j\omega) = K_p[I-W_1(j\omega)K_p]^{-1}W(j\omega) \tag{2}$$

for all real ω with the convergence being exponential.

We term $v_p(\cdot)$ totally exciting if

$$v_p(t) = \mathrm{Re}[\sum_{\ell} \exp(j\omega_\ell t)V_\ell]$$

and the ω_ℓ and v_ℓ are such that if

$$\beta' K[I-W_1(j\omega_\ell)K_1]^{-1}W(j\omega_\ell)V_\ell = 0$$

for all ℓ and any fixed β, K and K_1 then

$$\beta' K[I-W_1(j\omega)K_1]^{-1}W(j\omega) = 0 \quad \text{for all } \omega$$

In effect, this requires entries of $v_p(\cdot)$ to be linearly independent, to be almost-periodic, and to contain a certain minimum number of frequencies depending on $W_1(s)$ and $W(s)$. Note that $v_p(\cdot)$ is not necessarily periodic (but may be).

A special case of the above result which we shall show is relevant to the problem of identifying the numerator of a transfer function matrix (when the denominator is known) is provided by taking $W_1(s) = 0$.

The second situation to be considered is depicted in Figure 3, which actually shows plant and model in the one diagram. The same assumptions are in force as before, [save that the plant output is now $w_p(\cdot)$, not $y_p(\cdot)$]. We also require

(d) $V(s-\sigma)$ is a positive real matrix [11] for some real $\sigma > 0$, $V(s)$ is nonsingular except for isolated values of s, and $\lim_{s\to\infty} V(s) = 0$.

Actually, we can obtain a result when $\lim_{s\to\infty} V(s)$ is nonzero, but it is not especially helpful.

We then have a result very similar to that of Theorem 1:

Theorem 2 For the arrangement of Figure 3 with assumptions as listed, suppose the adjustment law is

$$(\dot{K}_m)_{ij} = -\lambda_{ij}(w_m-w_p)_i(u_m)_j \tag{3}$$

Then $\lim_{t\to\infty}(w_m(t)-w_p(t)) = 0$; moreover, if $v_p(\cdot)$ is totally exciting, then

$$\lim_{t\to\infty} V(j\omega)K_m(t)[I-W_1(j\omega)K_m(t)V(j\omega)]^{-1}W(j\omega) = V(j\omega)K_p[I-W_1(j\omega)K_pV(j\omega)]^{-1}W(j\omega) \tag{4}$$

for all real ω with the convergence being exponential.

3. Application to Realistic Identification Problems

In the preceding section, we studied two prototype identification problems. Here we shall exhibit their relevance along the following lines:

(a) Identification with input dynamics only, i.e. $W_1(s) \equiv 0$, tackles the problem of identifying the numerator of a transfer function, or transfer function matrix, of a plant knowing the denominator.

(b) Adding the possibility of feedback allows identification of numerator and denominator (both of a scalar and a matrix).

(c) Using the results relating to the insertion of $V(s)$ with positive real $V(s-\sigma)$ allows reduction in the complexity of an identifier of an entire scalar or matrix transfer function.

Numerator identification - scalar transfer function

Let a plant transfer function be

$$\Pi(s) = \frac{\sum_{i=1}^{n} b_i s^{i-1}}{s^n + \sum_{i=1}^{n} a_i s^{i-1}} \tag{5}$$

where the a_i and the integer n are known, and the poles of $\Pi(s)$ are in $\mathrm{Re}[s] < 0$. The task is to identify the b_i Define, with reference to Figures 1 and 2

$$W(s) = (sI-F)^{-1}g \qquad W_1(s) = 0$$

where

$$F = \begin{bmatrix} 0 & 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & 1 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & & \cdot \\ \cdot & & & & & \cdot & \cdot \\ \cdot & & & & & & 1 \\ -a_1 & -a_2 & -a_3 & \cdot & \cdot & \cdot & -a_n \end{bmatrix} \quad g = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix} \tag{6}$$

$$K_p = [b_1 \;\; b_2 \;\; \cdot \;\; \cdot \;\; \cdot \;\; b_n]$$

(Thus $K_pW(s) = \Pi(s)$). Then $\Pi(s)$ (and accordingly its numerator) can be uniquely identified.

Some other relevant points follow.

1. The physical object with transfer function $\Pi(s)$, i.e. the real plant, need not have the precise structure depicted in Figure 1. The crucial issue is that the real plant with transfer function (5) is indistinguishable (no matter what its internal structure) from the input-output point of view with the mentally constructed object depicted in Figure 1 and defined via (6); then any input-output description of the mentally constructed plant derived by adaptive identification (which uses input-output data) must be equally valid as an input-output description of the real plant. In particular, this is true of the transfer function numerator.

2. Should the denominator of $\Pi(s)$ be available in factored form, so that

$$\Pi(s) = \sum \frac{b_i}{s+a_i} + \sum \frac{c_j s + d_j}{s^2 + e_j s + f_j}$$

for known a_i, e_j, f_j and unknown b_i, c_j, d_j, a trivial variation on the argument above allows determination of the b_i, c_j and d_j as entries of K_p.

3. More generally, if the plant is known to be described by equations $\dot{x} = Fx + gv_p$, $y_p = h'x$ with F and g known, and h unknown, then the plant transfer function $h'(sI-F)^{-1}g$ can be identified, as can h if $[F, g]$ is completely controllable.

Numerator identification - transfer function matrix

Let a plant transfer function matrix be

$$\Pi(s) = \frac{\sum_{i=1}^{n} B_i s^{i-1}}{s^n + \sum_{i=1}^{n} a_i s^{i-1}} \quad (a_i \text{ scalar, } B_i \text{ matrix}) \tag{7}$$

where again, the a_i and integer n are known, and the zeros of $s^n + \sum_{i=1}^{n} a_i s^{i-1}$ lie in $\mathrm{Re}[s] < 0$. The task is to identify the matrices B_i, assumed $q \times r$.

One way to do this is to define

$$W(s) = (sI-F)^{-1}G \qquad W_1(s) = 0$$

where

$$F = \begin{bmatrix} 0_r & I_r & 0_r & \cdot\cdot\cdot & 0_r \\ 0_r & 0_r & I_r & \cdot\cdot\cdot & 0_r \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ -a_1 I_r & -a_2 I_r & -a_3 I_r & \cdot\cdot\cdot & -a_n I_r \end{bmatrix} \quad G = \begin{bmatrix} 0_r \\ 0_r \\ \cdot \\ \cdot \\ 0_r \\ I_r \end{bmatrix} \tag{8}$$

$$K_p = [\, B_1 \;\; B_2 \;\; \cdot\cdot\cdot \;\; B_n \,] \tag{9}$$

Again, $K_pW(s) = \Pi(s)$. Unique identification of $\Pi(s)$ coincides with unique identification of K_p, and is possible using the scheme of Figure 2.

In some ways, use of this particular $W(s)$ is counterintuitive: the triple $\{F, G, K_p\}$ constitutes a completely controllable realization of $\Pi(s)$ which is not completely observable in general. Preliminary thinking suggests that, at the least, this is inefficient, since the state-space dimension of the model would seem to be nonminimal. More seriously, one might wonder how it is that parameters of a nonminimal realization can possibly be identified uniquely from input-output data only.

The explanation of the former point in some ways is trickier. The problem is that there is no single canonical minimal realization $\{F, G, K_p\}$ with F, G depending only on the known a_i and K_p on the unknown B_i and possibly the a_i as well. The nearest one can get are finite collections of canonical forms, in which the G matrix consists of all zeros except for ones in a few locations, and the F matrix consists of a direct sum of companion matrices with a few other nonzero elements thrown in for good measure, see e.g. [12, 13]. It has in fact been pointed out [14] that an identification procedure could possibly be based on trying to identify the unknown parameters in each of these possible canonical forms; the attempt would only be successful in the case of one. The earliest recognition of the importance of state-variable realizations with F and G as in (8) in the context of the identification problem appear to be due to Spain [15], who termed such realizations structure-independent, a name which the efflux of time has demonstrated to be most apposite.

The second issue is resolved by inspection, essentially. It is a fact that the possibly unobservable realization $\{F, G, K_p\}$ of $\Pi(s)$ defined in (8) and (9) is sufficiently structured that literals in the realization are in bijective correspondence with those in $\Pi(s)$, and since $\Pi(s)$ is an input-output quantity, one can therefore expect to identify literals in it. Thus, while one could not expect that parameters in an arbitrary nonminimal realization could be identified using input-output data, the same conclusion is not true for every single member of the class of all nonminimal realizations.

Having said all this, we hasten to add that if we know in advance that $\Pi(s)$ has a minimal realization $\{F, G, K_p\}$ with F, G known and

K_p unknown, one should work with this [taking $W(s) = (sI-F)^{-1}G$] rather than the realization of (8) and (9).

Numerator and denominator identification - scalar transfer function

Let the plant transfer function be as in (5) where now the a_i and b_i are unknown, but the integer n is known. Suppose that the numerator and denominator have no common zeros, so that n is actually the dimension of a minimal realization of $\Pi(s)$. We aim to use a scheme like that of Figure 2 for identification.

With F $n\times n$, g $n\times 1$, let $[F, g]$ be a completely controllable pair with $\operatorname{Re}\lambda_i(F) < 0$. Set

$$W_1(s) = \begin{bmatrix} (sI-F)^{-1}g \\ 0 \end{bmatrix} \qquad W(s) = \begin{bmatrix} 0 \\ (sI-F)^{-1}g \end{bmatrix}$$
$$K_p = [k_1' \quad k_2'] \tag{10}$$

where k_1 and k_2 are both n-vectors. The closed-loop transfer function is identifiable; it is given by

$$\begin{aligned}\Pi(s) &= K_p[I - W_1K_p]^{-1}W \\ &= [I - K_pW_1]^{-1}K_pW \\ &= \frac{k_2'(sI-F)^{-1}g}{1 - k_1'(sI-F)^{-1}g} \\ &= \frac{\text{numerator of } k_2'(sI-F)^{-1}g}{\text{Characteristic polynomial of } F+gk_1'}\end{aligned} \tag{11}$$

With $[F, g]$ completely controllable, the coefficients of the numerator of $k_2'(sI-F)^{-1}g$ are in *bijective correspondence* with the entries of k_2 [In fact, if F and g have the form of (6), the entries of k_2 are precisely the coefficients of the numerator of $k_2'(sI-F)^{-1}g$]. Further, again on account of the complete controllability, the coefficients of the characteristic polynomials of $F+gk_1'$ are in bijective correspondence with the entries of k_1. Hence the entries of K_p are in *bijective* correspondence with the unknown a_i and b_i; identification of the transfer function is equivalent to identification of the entries of K_p, and the a_i and b_i.

Several miscellaneous points follow.

1. Suppose that a value for n was assumed that was too large, i.e. in any supposed expression of $\Pi(s)$ in the form (5), cancellation between numerator and denominator polynomial would occur. Clearly, no unique identification of the a_i and b_i would be possible. Nevertheless, the *adaptive* algorithm would lead to values of K_m - possibly nonstationary - satisfying (11), nonunique as they might be. From them, nonunique numerator and denominator *polynomials for* $\Pi(s)$ *would follow.*

2. Notice that

$$\Pi(s) = \frac{k_2'(sI-F)^{-1}g}{1 - k_1'(sI-F)^{-1}g} = k_2'(sI - \overline{F+gk_1'})^{-1}g \tag{12}$$

This formula also suggests the connection of k_2 with the numerator coefficients and k_1 with the denominator coefficients.

3. It is the introduction of the feedback which allows rephrasing of the denominator and numerator *identification* problem as simply a numerator identification problem. As suggested by Figure 5, and (10), we are identifying the numerator matrix of

$$\begin{bmatrix} k_1'(sI-F)^{-1}g \\ k_2'(sI-F)^{-1}g \end{bmatrix}$$

4. Reference to Section 2 will show that the rate of adaption is in part governed by the pole positions of entries of $W(s)$ and $W_1(s)$, i.e. by the eigenvalues of F. By choosing F to have eigenvalues with large negative real parts, we can ensure that the primary influences on the adaption rate are the selection of the gain constants in the adaptive laws, and *the nature of the* exciting input.

5. As *discussed in connection* with numerator identification, the physical plant may have a structure quite unlike that of the mentally constructed plant of Figure 1, though both have the same input-output performance, and, therefore, *transfer* function.

6. The model can be viewed as providing an adaptive observer of the state vector of the mentally constructed plant. Referring to Figure 2, denote the state vectors of $W_1(s)$ and $W(s)$ by η_m and μ_m and in *Figure 1*, by η_p and μ_p. Then

$$\begin{aligned}\dot{\eta}_q &= F\eta_q + gy_p \qquad q = p \text{ or } m \\ \dot{\mu}_q &= F\mu_q + gv_p\end{aligned} \tag{13}$$

and in view of the asymptotic stability of F, $\eta_p - \eta_m$ and $\mu_p - \mu_m$ must be exponentially convergent to zero. Now we know also that

$$y_p = k_1'\eta_p + k_2'\mu_p$$

so that

$$\begin{bmatrix} \dot{\eta}_p \\ \dot{\mu}_p \end{bmatrix} = \begin{bmatrix} F+gk_1' & gk_2' \\ 0 & F \end{bmatrix}\begin{bmatrix} \eta_p \\ \mu_p \end{bmatrix} + \begin{bmatrix} 0 \\ g \end{bmatrix} v_p \tag{14}$$

It is this state-variable realization of the transfer function $\Pi(s)$ for which the model provides an estimator. In the sense that (14) is a state-variable realization of the physical plant (though presumably not one which mirrors accurately the internal structure of the physical plant), the model can be said to provide an *estimator for* the physical plant, although this is rather loose, and certainly unhelpful, phraseology.

A state-estimator of dimension n of a certain n-dimensional state-variable realization of $\Pi(s)$ can be obtained in the following way. Since $\Pi(s) = k_2'(sI-F-gk_1')^{-1}g = g'(sI-F'-k_1g')^{-1}k_2$, an n-dimensional state-variable representation of the plant is provided by

$$\dot{x} = (F'+k_1g')x + k_2v_p, \qquad y_p = g'x \quad (15)$$

Set $K_m(t) = [k_{m1}'(t) \quad k_{m2}'(t)]$. Then an estimator using k_{m1}, k_{m2} of x is provided by

$$\dot{x}_e = (F'+kg')x_e - (k-k_{m1}(t))y_p + k_{m2}v_p \quad (16)$$

Here, k is a vector chosen so that $F'+kg'$ has eigenvalues suitably far in the left half plane: it is these eigenvalues, together with the rate of adaption, which determine the speed of convergence of x_e to x. To check the estimation property, observe that

$$\dot{x}_e - \dot{x} = (F'+kg')(x_e-x) + (k_{m1}-k_1)y_p + (k_{m2}-k_2)v_p$$

with $k_{m1}-k_1$ and $k_{m2}-k_2$ approaching zero exponentially fast.

A further reduction of estimator dimension by 1 is possible using the fact that y_p is a known linear functional of x, and then applying the Luenberger ideas, [1].

It is this style of estimator (with some modification as noted in a later subsection) which is the topic of much of [5] and [8]. In our view it is possible to overemphasise its importance. It would seem that the value is dubious of an estimate of the state of the particular plant realization (15) before k_1 and k_2 are properly identified, since it is very hard to imagine a situation when for example one might want to implement a feedback law using the estimate without knowledge of k_1 and k_2.

It is not even as if the entries of x_e estimated a known linear transformation of $[y_p \ \dot{y}_p \ \cdots \ y_p^{(n-1)}]$. Now if one retreats from the position that observers of this type are valuable in the adaptive phase and rests the case on the value of the observer after adaption is complete, i.e. $k_{m1}(t)-k_1$ and $k_{m2}(t)-k_2$ have effectively shrunk to zero, another criticism enters the picture. One can construct the estimate via a memoryless transformation from η_p and μ_p; in fact, for any minimal realization of the plant, $\dot{x} = F_1x + g_1v$, $y_p = h_1'x$, it will be true that $x_e = T[\eta_p' \ \mu_p']'$ for some T, computable from F_1, g_1, h_1, k_1 and k_2. Accordingly, $x_e = T[\eta_m' \ \mu_m']'$, and the extra dynamics (over and above the model dynamics) of (16) are seen to be superfluous.

In summary, (16) provides an adaptive estimate for the state of a particular minimal realization of the plant, viz. (15) at the expense of requiring dynamics, and with perhaps little application during the adaptive process, while after adaptation has concluded, the model state-variable can be used to construct an estimate of the state of any minimal realization of the plant.

Numerator and denominator identification - matrix transfer function

Now suppose that $\Pi(s)$ has the form of (7), with the scalar a_i and matrix B_i unknown, with $s^n + \sum_{i=1}^{n} a_i s^{i-1}$ the lowest common denominator of all entries of $\Pi(s)$, and with n known. We assume $s^n + \sum_{i=1}^{n} a_i s^{i-1}$ has all zeros in $\mathrm{Re}[s] < 0$. Suppose B_i is $q \times r$.

Let us take F and G as in (8), save that the a_i are replaced by constants α_i chosen such that $s^n + \sum_{i=1}^{n} \alpha_i s^{i-1}$ has all zeros in the left-half plane. In fact, these zeros should be well in the left-half plane to avoid slowing up the adaption, the argument being as for the scalar case. The reader may feel that the choice of F and g for the scalar transfer function identification problem was much freer. This is only superficially true. In the scalar case, any controllable pair [F, g] is such that $[TFT^{-1}, Tg]$ have the form of (6), and the introduction of the T contributes an inessential change of state co-ordinate basis to an input-output problem. In the multiple input case, it is of course not true that any controllable pair [F, G] is such that $[TFT^{-1}, TG]$ has a canonical form depending only on $|sI-F|$ and the size of F, G only. (Of course, one can introduce the inessential basis change to the F and G of the style of (8) if desired). Now take

$$W_1(s) = \begin{bmatrix} (sI-F)^{-1}G \\ 0 \end{bmatrix} \qquad W(s) = \begin{bmatrix} 0 \\ (sI-F)^{-1}G \end{bmatrix} \quad (17)$$

$$K_p = [k_1I_q \quad k_2I_q \quad \cdots \quad k_nI_q \quad K_1 \quad K_2 \quad \cdots \quad K_n]$$

Here, $W_1(s)$ and $W(s)$ are of dimension $2nr \times r$, and the K_i are of dimension $q \times r$. Notice that

$$(sI-F)^{-1}G = \frac{1}{s^n + \sum_{i=1}^{n} \alpha_i s^{i-1}} \begin{bmatrix} I \\ sI \\ \vdots \\ s^{n-1}I \end{bmatrix}$$

The transfer function matrix which the model identifies is

$$\begin{aligned} & K_p[I - W_1K_p]^{-1}W \\ &= [I - K_pW_1]^{-1}K_pW \\ &= \left(I - \frac{\Sigma k_i s^{i-1}}{s^n + \sum_{i=1}^{n} \alpha_i s^{i-1}} I \right)^{-1} \frac{\sum_{i=1}^{n} K_i s^{i-1}}{s^n + \sum_{i=1}^{n} \alpha_i s^{i-1}} \\ &= \frac{\sum_{i=1}^{n} K_i s^{i}}{s^n + \sum_{i=1}^{n} (\alpha_i - k_i) s^{i-1}} \end{aligned} \quad (18)$$

Since the transfer function matrix is uniquely identified, it follows that the K_i are uniquely identified, and correspond to the B_i; likewise, the k_i and a_i are uniquely identified.

Similar remarks can be made as for the scalar case (see points 1-4 in that discussion). The adaptive observer situation is a mildly more complicated. We have the following observations:

a) the state vectors η_m and μ_m of the $W(s)$ and $W_1(s)$ blocks of the model approach the corresponding η_p and μ_p of the plant, which define a realization of the plant as

$$\begin{bmatrix}\dot{\eta}_p\\ \dot{\mu}_p\end{bmatrix} = \begin{bmatrix} F+G[K_1' \;\; K_2' \;\; \dots \;\; K_n'] & G[k_1 I \;\; k_2 I \; \dots \; k_n I] \\ 0 & F \end{bmatrix} \times \begin{bmatrix}\eta_p\\ \mu_p\end{bmatrix} + \begin{bmatrix}0\\ G\end{bmatrix} v_p$$

$$y_p = K_p \begin{bmatrix}\eta_p\\ \mu_p\end{bmatrix} \tag{19}$$

b) A state-variable realization of the plant transfer function matrix is provided by

$$\dot{x} = \begin{bmatrix} 0_q & 0_q & \cdots & -(\alpha_1-k_1)I_q \\ I_q & 0_m & \cdots & -(\alpha_2-k_2)I_q \\ 0_q & I_q & & -(\alpha_3-k_3)I_q \\ \vdots & \vdots & & \vdots \\ 0_q & 0_q & \cdots & -(\alpha_n-k_n)I_q \end{bmatrix} x + \begin{bmatrix} K_1\\ K_2\\ \vdots\\ K_n\end{bmatrix} v_p \tag{20}$$

$$y_p = [0_q \;\; 0_q \;\; \cdots \;\; 0_q \;\; I_q]x$$

The first equation may be written as

$$\dot{x} = \begin{bmatrix} 0_q & 0_q & \cdots & -\alpha_1 I_q \\ I_q & 0_q & \cdots & -\alpha_2 I_q \\ 0_q & I_q & \cdots & \alpha_3 I_q \\ \vdots & \vdots & & \vdots \\ 0_q & 0_q & \cdots I_q & -\alpha_n I_q \end{bmatrix} x + \begin{bmatrix} K_1\\ K_2\\ \vdots\\ K_n\end{bmatrix} v_p + \begin{bmatrix} k_1 I_q\\ k_2 I_q\\ \vdots\\ k_n I_q\end{bmatrix} v_p \tag{21}$$

An adaptive estimator for this particular state-variable realization is provided by

$$\dot{x}_e = \begin{bmatrix} 0_q & 0_q & \cdots & -\alpha_1 I_q \\ I_q & 0_q & \cdots & -\alpha_2 I_q \\ 0_q & I_q & \cdots & -\alpha_3 I_q \\ \vdots & \vdots & & \vdots \\ 0_q & 0_q & \cdots I_q & -\alpha_n I_q \end{bmatrix} x_e + K[0_q \;\; 0_q \;\; \cdots \;\; 0_q \;\; I_q]x_e$$

$$-\left(K - \begin{bmatrix} k_{m1}(t)I_q \\ k_{m2}(t)I_q \\ \vdots \\ k_{mn}(t)I_q \end{bmatrix}\right) y_p + \begin{bmatrix} K_{m1}(t) \\ K_{m2}(t) \\ \vdots \\ K_{mn}(t) \end{bmatrix} v_p \tag{22}$$

where K is suitably chosen. Arguments as for the scalar problem justify the claim that this is indeed an estimator. Again, the Luenberger ideas [2] can lower the dimension somewhat.

c) As before, once identification has been achieved, we can estimate the state x in any minimal realization via $x_e = T[\eta_m' \;\; \mu_m']$ where T is computed using the parameter of the minimal realization as well as the estimate of K_p.

Introduction of Output Dynamics Defined by Positive Real Matrix

As foreshadowed at the start of this section, we shall show that in transfer function matrix identification, a saving in model complexity is achievable by adopting a set up like that of Figure 3. Complexity is measured, quite precisely, by the sum of the dimensions of the state-variable realization of the blocks $W_1(s)$, $W(s)$ and $V(s)$ in Figure 3.

The following ideas naturally apply to the case of scalar transfer function identification; we shall confine attention to the matrix case.

As earlier, our aim is to identify the scalars a_i and matrices B_i defining the $q \times r$ plant transfer function matrix of (7), with n assumed known.

Choose F and G as in (8), save that the constants a_i are replaced by constants α_i with $s^n + \sum_{i=1}^{n} \alpha_i s^{i-1}$ having all zeros in $Re[s] < 0$, with at least one negative real zero. We shall choose $V(s) = (s+\lambda)^{-1}I$ where $-\lambda$ is a zero of $s^n + \sum_{i=1}^{n} \alpha_i s^{i-1}$. Then $V(s-\sigma)$ is certainly positive real for any σ with $0 < \sigma \leq \lambda$.

Now observe the equivalences of Figure 4, in which $E(s)$ is shorthand for $[I_r \;\; sI_r \;\; \dots \;\; s^{n-1}I_r]'$. The three block diagrams all depict systems with the same transfer function matrix relating $L[v_p(\cdot)]$ to $L[y_p(\cdot)]$, as a rapid examination will show.

The first block diagram corresponds to the scheme of Figure 1 for adaptive identification of a transfer function matrix, i.e. we know from the earlier analysis that, first, the transfer function matrix $\Pi(s)$ of the arrangement of Figure 4a can be identified, and second, given the fact that n is the smallest integer for which $\Pi(s)$ has th the form (7), we know that the k_i and K_i can be uniquely identified, being in bijective correspondence with the a_i and B_i.

The second block diagram is merely included to make clear the link between the first and the third, while the third block diagram has the general form of the plant part of Figure 3. Accordingly, again the k_i and K_i can be adaptively identified.

It remains to substantiate the claim that a saving in complexity can result using the Figure 4c arrangement. First, notice that the arrangement of Figure 4a involves state vectors for the two dynamic blocks of dimensions totalling 2nr.

In Figure 4c, we have, using first the fact that $s^n + \sum_{i=1}^{n} \alpha_i s^{i-1} = (s+\lambda)(s^{n-1} + \sum_{i=1}^{n-1} \beta_i s^{i-1}$ for some β_i

$$\frac{s+\lambda}{s^n + \sum_{i=1}^{n} \alpha_i s^{i-1}} [I_r \;\; sI_r \;\; \ldots \;\; s^{n-1}I_r]'$$

$$= \frac{1}{s^{n-1} + \sum_{i=1}^{n-1} \beta_i s^{i-1}} [I_r \;\; sI_r \;\; \ldots \;\; s^{n-1}I_r]'$$

$$= [0_r \;\; 0_r \ldots \;\; 0_r \;\; I_r]' + \frac{1}{s^{n-1} + \sum_{i=1}^{n-1} \beta_i s^{i-1}}$$

$$\times [I_r \;\; sI_r \;\; \ldots \;\; s^{n-2}I_r \;\; - \sum_{i=1}^{n-1} \beta_i s^{i-1} I_r]' \quad (23)$$

and it is readily seen that the dimension of a state vector realizing this block is $(n-1)r$. The sum of the state vector dimensions of the linear blocks of the Figure 4c arrangement is $2(n-1)r+q$. Accordingly, if $q < 2r$, there will be a saving in the complexity of the linear part of the model. In the case of a scalar transfer function, the saving in the model dimension is precisely one. [Note that it is the model which has to be physically realized with the structure shown in Figure 3, not the plant. The physical plant may have any structure at all, so long as its transfer function matrix is $\Pi(s)$. As earlier the plant structure of Figure 3 is simply a mental vehicle used in developing the model structure].

Once again, the supplementary remarks originally made regarding the numerator and denominator identification of a scalar transfer function apply, mutatis mutandis, with the possible exception of that concerning adaptive observers. The story here is as follows.

(a) The state vectors of the $W_1(s)$, $W(s)$ and $V(s)$ blocks of the model in Figure 3 are estimates of the corresponding quantities in a state-variable realization of the plant based on the structure depicted in Figure 3; the $V(s)$ state vector can be regarded as an adaptive estimate, with the time constant governing the convergence of the estimate to the true value in part determined by the time constant of the identification process.

(b) A lower order adaptive estimator can still be provided as described in the earlier discussion of transfer function matrix identification.

(c) Again, once identification has been achieved, the state x in any minimal realization of the plant can be estimated by a certain linear transformation of the states of the $W_1(s)$, $W(s)$ and $V(s)$ blocks of the model.

Another application (apparently limited in scope) of the fourth prototype identification situation can be found in work of Lüders and Narendra [16]. One has measurements of the <u>state</u> and input of a plant, which is assumed to have an equation of the form

$$\dot{x}_p = Ax_p + Bu_p$$

with $\text{Re}\,\lambda_i[A] < 0$, and A and B otherwise unknown. Suppose x_p has dimension n and u_p has dimension m. Then with arbitrary $\sigma > 0$, we can take

$$V(s) = (s+\sigma)^{-1}I_n \qquad W_1(s) \begin{bmatrix} I_n \\ 0_{m\times n} \end{bmatrix} \qquad W(s) = \begin{bmatrix} 0_{n\times m} \\ I_m \end{bmatrix}$$

$$K_p = [A+\sigma I \quad B]$$

It is not hard to check that a plant of the form depicted in Figure 3 has transfer function matrix $(sI-A)^{-1}B$. Accordingly, A and B can be found with a model of particularly low dimension.

4. Discussion and Conclusions

We begin by summarizing departures from earlier work which have been described in the earlier sections. First, we have constructed two prototype situations in which identification has been possible. Second, we have indicated that identification can occur for signals which are not necessarily periodic and that the adaption process is exponentially convergent. Third, we have used the prototype identification schemes to provide solutions to unsolved problems, centering round transfer function matrix identification; of interest is the appearance of nonminimal state variable realizations, and the notion that transfer function matrix identification may occur even without, necessarily, identification of certain parameters. Fourth, we have more fully analyzed conclusions of the existing literature on adaptive observers, and suggested alternative approaches to this problem.

Several avenues remain to be pursued. We have given little or no discussion of <u>rate of adaption</u>. One could seek to optimize the gains in the adaptive gain adjustment laws, to maximize the rate of adaption, or one could develop the ideas of Lion [4] (described in [4] for scalar plants) for the matrix case. These ideas involve the use of more complex models in return for securing the ability to have a rate of adaption as high as desired. Given the insight into the matrix identification problem of the paper, extension of the Lion ideas would be straightforward.

An issue in part related to the question of rate of adaption is that of performance in the presence of noise. Analytical attempts to handle noise lead to equations in which noise appears multiplicatively in an otherwise linear equation. As far as is known, no necessary and sufficient conditions for stochastic stability for this type of arrangement are known (though partial results are known). Evidently then, quantitative results concerning noise performance will be difficult to attain.

Finally, we note that some of the ideas of this paper may prove helpful in multivariable adaptive control problems, and further work is in progress along these lines.

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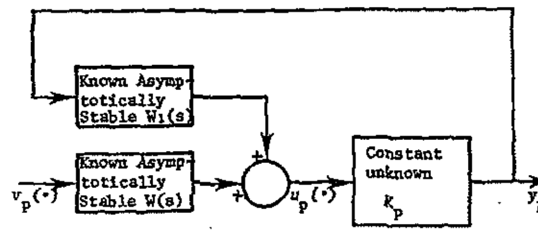

Figure 1.
Plant for first prototype situation.

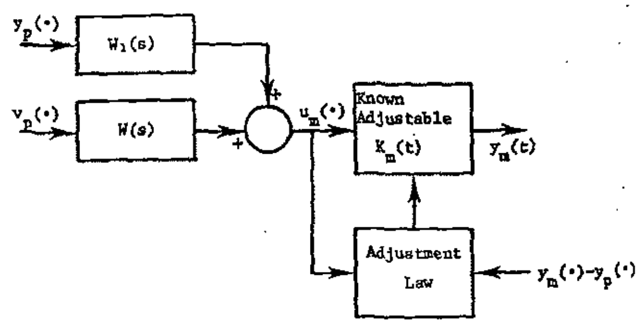

Figure 2.
Model for first prototype situation.

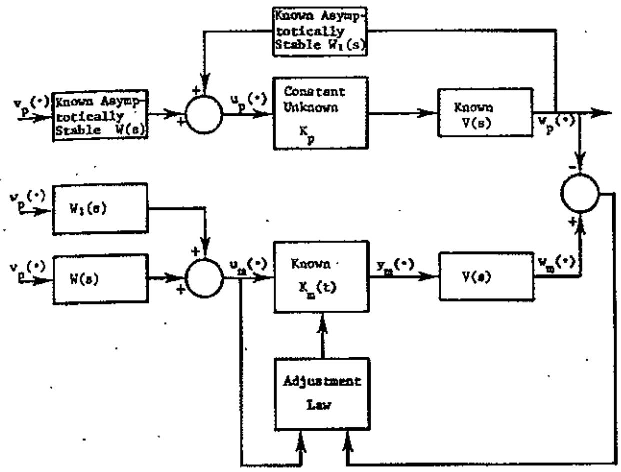

Figure 3.

Plant and model for second prototype situation

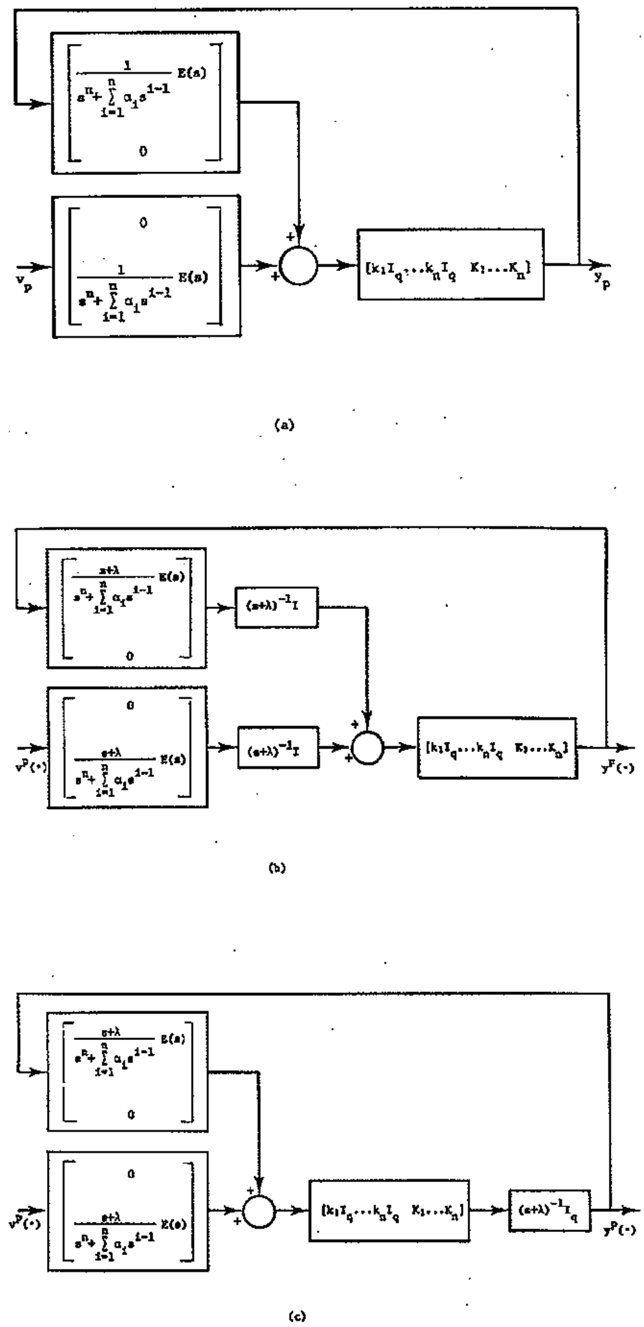

Figure 4.

Various structures for the plant which are input-output equivalent.