A BELEVITCH SYNTHESIS VIA STATE-SPACE TECHNIQUES *

by

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In this paper a state-variable scattering passive synthesis based on the concept of resistance extraction as used in the classical Belevitch Synthesis [1] is discussed.

In the Belevitch synthesis, an $n \times n$ bounded-real (BR) scattering matrix $S(s)$ is synthesised as a lossless coupling network N_c terminated in r unit resistors (see figure) where $r \geq \rho$, with ρ being the normal rank of the resistivity matrix $[I_n - S'(-s)S(s)]$. (Superscript $'$ denotes transpose). The network N_c is specified in terms of a scattering matrix $S_c(s)$ obtained by bordering $S(s)$ into a para-unitary matrix. Thus if $S_c(s)$ is the scattering matrix of N_c, partitioned according to the ports as:

$$S_c = \begin{bmatrix} S_{11}(s) & S_{12}(s) \\ S_{21}(s) & S_{22}(s) \end{bmatrix}$$

then it is necessary that $S_{11}(s) = S(s)$ and the para-unitary constraint requires that $S_c'(-s)S_c(s) = S_c(s)S_c'(-s) = I_{n+r}$. As a result, the matrices $S_{12}(s)$, $S_{21}(s)$ and $S_{22}(s)$ may be found using the relatively simple Gauss factorization or the more elaborate and complicate polynomial factorization, corresponding to the Belevitch synthesis or the Oono and Yasuura synthesis [1] respectively. The method of Oono and Yasuura can easily predict the number of reactive elements needed in synthesising $S_c(s)$ (i.e. the degree of $S_c(s)$). Both methods, however, require an excess number of reactive elements in general.

Although from the theory of equivalent realizations Oono and Yasuura

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are able to obtain a minimal reactive synthesis only after a rather extensive manipulation [1], it is possible to present one such synthesis using state-space ideas in a very simple fashion. The synthesis rests on the following result:

<u>Lemma</u> [2] Let $S(s)$ be an $n \times n$ rational matrix with $S(\infty)$ finite and let $\{F_0, G_0, H_0, J\}$ be a minimal realization of $S(s)$. Suppose that all poles of $S(s)$ lie in the strict left half plane and $I_n - S(s)$ is nonsingular almost everywhere including at ∞. Then $S(s)$ is bounded-real, if and only if, there exist a symmetric positive-definite P, and real matrices L_0 and W_0 such that

$$PF_0 + F_0'P = -H_0H_0' - L_0L_0' \tag{1a}$$

$$-PG_0 = H_0J + L_0W_0 \tag{1b}$$

$$I_n - J'J = W_0'W_0 \tag{1c}$$

Observe that if T is any nonsingular matrix such that $T'T = P$, then on setting $F = TF_0T^{-1}$, $G = TG_0$, $H' = H_0'T^{-1}$ and $L' = L_0'T^{-1}$, (1) reduces to

$$F + F' = -HH' - LL' \tag{2a}$$

$$-G = HJ + LW_0 \tag{2b}$$

$$I_n - J'J = W_0'W_0 \tag{2c}$$

The real matrices L and W_0' can be chosen with dimensions $k \times \rho$ and $n \times \rho$ respectively, where k is the degree of $S(s)$, i.e. they are chosen with the minimum number of columns ρ.

Now compute an orthogonal $(n + \rho) \times (n + \rho)$ matrix of

$$J_c = \begin{bmatrix} J & J_{12} \\ W_0 & J_{22} \end{bmatrix} \tag{3a}$$

with

$$J_{12}J_{12}' = I_n - JJ' \tag{3b}$$

and

$$J_{22} = -(W_0 W_0')^{-1} W_0 J' J_{12} \qquad (3c)$$

Define matrices F_c, G_c and H_c by

$$F_c = F \; ; \; H_c = [H \; \vdots \; L] \quad \text{and} \quad G_c = [G \; \vdots \; G_2] \qquad (4a)$$

where

$$G_2 = -HJ_{12} - LJ_{22} \qquad (4b)$$

then it is simple to verify that the $(n + \rho)x(n + \rho)$ matrix

$$S_c(s) = J_c + H_c'(sI - F_c)^{-1}G_c \qquad (5)$$

is a para-unitary matrix; moreover the upper left (nxn) block is

$$J + H'(sI - F)^{-1}G = S(s).$$

Hence

> <u>Theorem</u> Let $\{F,G,H,J\}$ be a minimal realization for an nxn bounded-real $S(s)$ with $I_n - S(s)$ being nonsingular almost everywhere including at ∞. Then $S_c(s)$ of (5) with F_c, G_c, H_c and J_c defined by (3) and (4) is the scattering matrix of a lossless coupling network such that terminating the network of $S_c(s)$ at the last ρ ports in unit resistors yields a passive synthesis of $S(s)$. Moreover the number of reactive elements used is a minimum.

The last part of the theorem follows from the fact that the degree of $S_c(s)$ is the same as the dimension k of a minimal F. Note further that the conditions hat $I_n - S(s)$ has a normal rank n and has full rank at ∞ simply mean that a $Z(s)$ description exists for $S(s)$ and that $Z(\infty)$ is finite. Unfortunately the construction procedure of $S_c(s)$ above does not yield a symmetric matrix even if $S(s)$ is, and hence will not give a reciprocal synthesis in general. The computation of a symmetric $S_c(s)$ is not as simple as for the nonsymmetric case above. The starting point is to obtain a minimal realization $\{F_1, G_1, H_1, J\}$ for $S(s)$ (see [2]) such that

$$F_1 + F_1' = -H_1H_1' - L_1L_1' \quad (6a)$$
$$-G_1 = H_1J + L_1W_1 \quad (6b)$$
$$I_n - J^2 = W_1'W_1 \quad (6c)$$
$$\Sigma F_1 = F_1'\Sigma \quad (6d)$$
$$\Sigma G_1 = -H_1' \quad (6e)$$

and

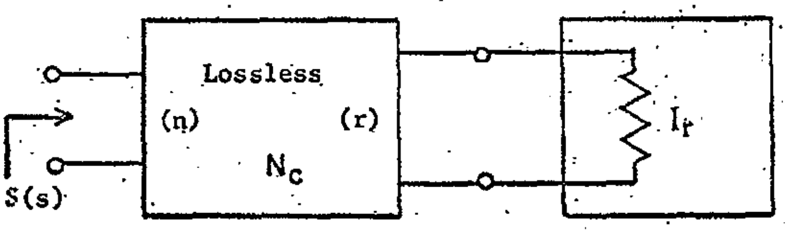
$$\Sigma = [I_{k_1} \dotplus (-1)I_{k_2}] \quad (6f)$$

where $k_1 + k_2 = k$ and $\dotplus$ denotes direct sum. Here the number of columns r of matrices L_1 and W_1' is generally greater than ρ. This has a physical interpretation, recall that r is the number of resistors required for the synthesis of $S(s)$. Thus if a minimum number of reactive elements is to be used and no gyrators allowed, more than the minimum number of resistors is required [1].

The basis of computing a realization $\{F_c, G_c, H_c, J_c\}$ of a symmetric para-unitary matrix $S_c(s)$ in (5) rests primarily on computing an orthogonal matrix $J_c = \begin{bmatrix} J & W_1' \\ W_1 & K_{22} \end{bmatrix}$, or in other words on computing K_{22}, such that $\Sigma L_1 = H_1W_1' + L_1K_{22}$. Then with matrices F_c, G_c, and H_c defined in the similar fashion as (4), a required $S_c(s)$ is obtained.

[1] R.W. Newcomb, "Linear Multiport Synthesis", McGraw Hill, New York, 1966.

[2] S. Vongpanitlerd and B.D.O. Anderson, "Scattering Matrix Synthesis Via Reactance Extraction", Technical Report EE-6903, Department of E.E., University of Newcastle, Australia, June 1969.



Resistor Extraction