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Switching Control for Robust Autonomous Robot and Vehicle Platoon Formation Maintenance

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Abstract— **In this paper, we analyze the problem of acquiring and maintaining desired inter-agent distances for non-hierarchical autonomous multi-agent platoons (1-dimensional formations), where the agents can be autonomous robots or vehicles, using only agent-wise relative position measurements. In our analysis, we consider the existence of measurement noises, which may cause the failure of controllers ignoring these noises in keeping the positions of the agents bounded. We design a switching control scheme that addresses the above problem, i.e., that acquires and maintains the desired inter-agent distances while guaranteeing that the agent positions are kept bounded.**

I. INTRODUCTION

In various modern technical application areas, there exist networked systems which are in the form of an array or a lattice of similar dynamic sub-systems (or agents). Some examples of such networked systems are automated highway systems [1], [2], formations of autonomous ground/aerial/underwater vehicles and robots [3], [4], sensor networks [5], [6], micro-electro-mechanical arrays [7], etc.

In parallel with the wide range of application areas of such networked systems, there has been significant amount of research interest and studies in stability and control theory of spatially interconnected systems. Some main topics being investigated in this framework are the relations among individual sub-system dynamics, dynamic interactions within local sub-system neighborhoods and the stability of the whole interconnected system [8]; distributed control design [9], [10]; and individual and cumulative affects of various disturbance sources to the behavior of the interconnected system as a whole [2], [11].

In this paper, we consider a decentralized control problem related to all of the three topics mentioned above and propose a decentralized control scheme to solve this problem robustly to certain measurement noises. The particular practical framework in which we investigate this problem will be formation maintenance and inter-agent distance keeping for autonomous robot and vehicle platoons, noting that there are similar problems (in which the proposed control scheme has potential applications) in other practical frameworks.

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The particular problem we consider is keeping the inter-agent distances between consecutive agents[1] in a platoon of vehicles or mobile robots at certain desired values, in the existence of distance measurement noises. For simplicity and clarity, we assume that all the agents move on a fixed linear motion axis (x-axis) and each agent A_i of the platoon has single-integrator kinematics

$$\dot{x}_i(t) = v_i(t), \ \forall t \geq 0 \tag{1}$$

where $x_i, v_i \in \Re$ denote, respectively, the position (on the x-axis) and the velocity of A_i. It is also assumed that the output of the individual controller of each agent A_i is v_i.

The main focus and contribution of the paper is solving the above distance keeping problem using a decentralized non-hierarchical control scheme that is robust to measurement noises. Here the term "non-hierarchical" indicates that all of the individual controllers of the agents are identical, e.g. there is no leader or guide agent(s).

The properties and limitations of such decentralized non-hierarchical control schemes in autonomous formation control have been discussed in several places (see, e.g. [12], [13]) assuming that the individual controllers have certain classical forms, e.g. they are simple proportional (P) or proportional-integral (PI) controllers. The results derived in these works were quite pessimistic. For such simple and classical controller forms, one can observe various scenarios with certain instability results due to some initial condition or noise effects. An example for the particular distance-keeping problem above is given in Section IV, where the measurement noises cause the platoon to move unboundedly. In order to overcome the shortcomings of simple classical controllers, a particular non-linear switching technique is employed in our control design. Note that such switching techniques are widely used in nonlinear and adaptive control under the names dead-zone, sigma switching, projection, hysteresis-switching, etc. [14], [15].

As a final remark of this section, it is worth mentioning that although the 1-dimensional setting in this paper complies with or can be easily adapted to a set of application areas such as longitudinal (single-lane) control in automated-highways [1], [2], formation control of autonomous robot/vehicle strings [16], etc., the settings of many practical autonomous formation control problems are 2 or 3-dimensional [3], [4]. For such problems, this paper should be considered as a potential framework to facilitate

[1] In this paper, an ordered 1-dimensional vehicle (or mobile robot) array is called a *platoon*, and each vehicle (robot) in the array is called an *agent*.

establishing the main structure of the decentralized control scheme.

II. TWO-AGENT PLATOON FORMATIONS

In this section, as a simple setting to have better insight to the general problem of formation maintenance of multi-agent vehicle platoons, we consider the problem of maintaining a 2-agent platoon formation that is described below and depicted in Figure 1. The focus of the design will be on maintaining the desired inter-agent distance and guaranteeing that the agents remain within a bounded region all the time given the existence of distance measurement noises. We consider a point-agent model with single-integrator dynamics and assume that each agent can measure its position relative to the other (with some noise) but it cannot measure the absolute position of itself (over the 1-dimensional axis, i.e. the lane, of interest). The problem of maintaining the platoon formation can be formulated as follows:

Problem 1: Consider two agents A_1 and A_2 each of which can move in both directions on a (1-dimensional) axis with kinematics (1). Denote the positions of A_1 and A_2 on the axis at each time $t \geq 0$ as $x_1(t)$ and $x_2(t)$, respectively, where $x_1(0) < x_2(0)$. Agent A_1 has a sensor to measure its position relative to A_2 at each $t \geq 0$ with some noise as

$$d_1(t) = x_1(t) - x_2(t) + n_1(t)$$

where n_1 denotes the corresponding sensor noise of A_1. Similarly, A_2 measures its position relative to A_1 as

$$d_2(t) = x_2(t) - x_1(t) + n_2(t)$$

where n_2 denotes its sensor noise. n_1 and n_2 are bounded from above by some constant $\bar{n} > 0, i.e., |n_1(t)| \leq \bar{n}$ and $|n_2(t)| \leq \bar{n}, \forall t$. Each agent A_i has a control law of the form

$$v_i(t) = f_i(d_i(t), D_i)$$

where $D_1 = -D, D_2 = D$ and $D > 0$ is the desired value for the distance $x_2 - x_1$.[2] The task is to design the control laws, i.e., select the functions f_i such that (i) x_1 and x_2 are kept bounded and (ii) the distance $x_2 - x_1$ converges to the ball $B(D, \gamma\bar{n})$, where $\gamma > 0$ denotes a predefined constant and $B(\alpha_1, \alpha_2)$ denotes the interval $[\alpha_1 - \alpha_2, \alpha_1 + \alpha_2]$ for any $\alpha_1 \in \Re, \alpha_2 > 0$.

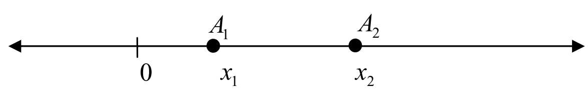

Fig. 1. Problem 1: It is desired to keep the agents A_1 and A_2 at a desired distance D in the existence of measurement noises.

For this initial problem, we consider two cases based on the controller constraints, and design a control scheme for each case.

[2]The opposite signs of $d_1(t) - n_1(t)$ and $d_2(t) - n_2(t)$ (and D_1 and D_2) indicate that each agent knows if it is at the left or right of the other.

A. Hierarchical Formation Control

As the first case, we consider the structures of f_1 and f_2 being allowed to be different. This case is a trivial one with low practical value. Nevertheless, it is analyzed here for completeness. We can simply select

$$\begin{array}{l} f_1(d_1, D_1) = 0, \forall d_1, D_1 \\ f_2(d_2, D_2) = -k(d_2 - D_2) \end{array}$$

where the gain term k is chosen as $k = k_c$ for some constant $k_c > 0$. Then, $x_1(t) = x_1(0)$ is obviously bounded for all $t \geq 0$. To analyze the boundedness and convergence of x_2, let us consider $e = x_2 - x_1 - D$. We have

$$\dot{e} = -k(d_2 - D_2) = -ke - kn_2$$

and hence

$$e(t) = e(0)\mathrm{e}^{-kt} - k\int_0^t \mathrm{e}^{-k(t-\tau)} n_2(\tau) d\tau$$

which can be easily shown to be bounded and converge to $B(0, \bar{n})$ as $t \to \infty$. Since $e = x_2 - x_1 - D$, x_2 is bounded as well and converges to the ball $B\left(x_1(0) + D, \bar{n}\right)$ as $t \to \infty$.

B. Non-Hierarchical Formation Control

As the second and main case we focus on in this paper, we consider non-hierarchical platoons where the individual agent controllers are required to have the same structure, i.e. $f_1 \equiv f_2$. We will present the analysis only for the sub-case where the noise upper-bound $\bar{n}$ is known since it resembles the corresponding situation in practice fairly well and yet its analysis is not as complicated as the contrary case where the noise upper-bound $\bar{n}$ (exists but) is unknown[3]. Focusing back on Problem 1, we propose the control scheme

$$\begin{array}{l} f_i(d_i, D_i) = -k\left(|\delta_i|\right)\delta_i, \\ \delta_i = d_i - D_i \end{array} \tag{2}$$

where the gain term $k : [0, \infty) \to [0, \bar{k}]$ (depicted in Figure 2 is a function of the error term δ_i defined for some design constants $\bar{k}, \bar{\delta} > 0$ as follows:

$$k(\delta) = \begin{cases} 0 & if \quad \delta \leq \bar{n} \\ \bar{k}\frac{\delta - \bar{n}}{\bar{\delta}} & if \quad \bar{n} < \delta \leq \bar{n} + \bar{\delta} \\ \bar{k} & if \quad \delta > \bar{n} + \bar{\delta} \end{cases} \tag{3}$$

Using the control law (2), for $e = x_2 - x_1 - D$, we have

$$\begin{array}{l} \dot{e} = \dot{x}_2 - \dot{x}_1 \\ = -k\left(|\delta_2|\right)\delta_2 + k\left(|\delta_1|\right)\delta_1 \\ = -k\left(|e + n_2|\right)(e + n_2) - k\left(|e - n_1|\right)(e - n_1) \end{array}$$

Defining $V(e) = e^2/2$, we have

$$\begin{array}{l} \dot{V} = -k\left(|e + n_2|\right)\left(e^2 + en_2\right) - k\left(|e - n_1|\right)\left(e^2 - en_1\right) \\ = -2\left(k_1 + k_2\right)V + \left(k_1 n_1 - k_2 n_2\right)e \\ \leq -2k_{12}V + k_{12}\bar{n}\,|e| = -k_{12}V - k_{12}\left(|e|^2\big/2 - \bar{n}\,|e|\right) \\ \leq -k_{12}V + k_{12}\bar{n}^2/2 \end{array}$$

[3]The complete analysis and design of a control scheme meeting the task of Problem 1 for this contrary case (under the assumption that $\bar{n} << D$) are available from the authors in pre-print form. The mentioned analysis and control design demonstrate that the condition of knowing an upper bound for the measurement noise amplitudes can be relaxed by slightly modifying the control laws presented in this paper.

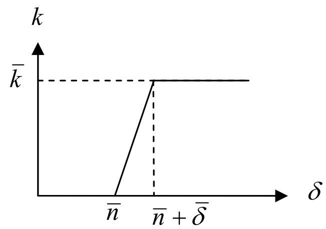

Fig. 2. The switching parameter k of the control scheme (2).

where $k_1(t) = k\left(|e(t) - n_1(t)|\right)$, $k_2(t) = k\left(|e(t) + n_2(t)|\right)$, $k_{12}(t) = k_1(t) + k_2(t)$, $\forall t \geq 0$.

We want to show that V is bounded and converges to $[0, 2\bar{n}^2]$ as $t \to \infty$, i.e., for any given $\varepsilon > 0$ there exists some time instant $t_V \geq 0$ such that $V(t) \leq 2\bar{n}^2 + \varepsilon$, $\forall t \geq t_V$. Assume that at a given time instant t_0, $V(t_0) > 2\bar{n}^2 + \varepsilon$ (If there is no such instant, obviously, $t_V = 0$). For any time instant $t \geq t_0$ for which $V(t) \geq 2\bar{n}^2 + \varepsilon$, we have $|e(t)| \geq 2\sqrt{\bar{n}^2 + \varepsilon/2} = 2\bar{n} + \bar{\varepsilon}$ where $\bar{\varepsilon} = 2\sqrt{\bar{n}^2 + \varepsilon/2} - 2\bar{n} > 0$, which implies that $|e(t) - n_1(t)| \geq |e(t)| - \bar{n} \geq \bar{n} + \bar{\varepsilon}$, $|e(t) + n_2(t)| \geq |e(t)| - \bar{n} \geq \bar{n} + \bar{\varepsilon}$ and hence $k_1(t), k_2(t) \geq \bar{k}\min\{\bar{\varepsilon}/\bar{\delta}, 1\}$. Therefore,

$$\begin{aligned} \dot{V} &\leq -k_{12}V + k_{12}\bar{n}^2/2 \\ &\leq -2\bar{k}\left(V - \bar{n}^2/2\right)\min\{\bar{\varepsilon}/\bar{\delta}, 1\} \\ &< -3\bar{k}\bar{n}^2\min\{\bar{\varepsilon}/\bar{\delta}, 1\} \end{aligned}$$

for any time instant $t \geq t_0$ for which $V(t) \geq 2\bar{n}^2 + \varepsilon$, which implies that there exists some $\bar{t}_V > t_0$ such that $V(\bar{t}_V) = 2\bar{n}^2 + \varepsilon$. For any $t \geq \bar{t}_V$, $V(t)$ satisfies $V(t) \leq \bar{n}^2/2$ or $\dot{V}(t) \leq 0$. Therefore, $V(t) \leq 2\bar{n}^2 + \varepsilon$, $\forall t \geq t_V = \bar{t}_V$. Since V is bounded and converges to $[0, 2\bar{n}^2]$ as $t \to \infty$, we obtain the following result.

Proposition 1: Consider Problem 1. The control scheme (2),(3) guarantees that the error $e(t) = x_2(t) - x_1(t) - D$ is bounded and converges to $B(0, 2\bar{n})$ as $t \to \infty$.

Remark 1: Since for any $t \geq \bar{t}_V, V(t) \leq n^2/2$ or $\dot{V}(t) \leq 0$, if V enters in an interval $[0, R_V]$ at some time $\bar{t}_0 \geq \bar{t}_V$, where $\bar{n}^2/2 \leq R_V \leq 2\bar{n}^2$, then it will remain in $[0, R_V]$ for all $t \geq \bar{t}_0$. In other words, for any $\bar{n}^2/2 \leq R_V \leq 2\bar{n}^2$ and $\bar{t}_0 \geq \bar{t}_V \geq 0$, if $V(\bar{t}_0) \leq R_V$ then $V(t) \leq R_V, \forall t \geq \bar{t}_0$. Consequently, for any $\bar{n} \leq R_e \leq 2\bar{n}$ and $\bar{t}_0 \geq \bar{t}_V \geq 0$, if $|e(\bar{t}_0)| \leq R_e$ then $|e(t)| \leq R_e, \forall t \geq \bar{t}_0$.

Next we analyze the behavior of

$$\begin{aligned} \dot{x}_1(t) &= k\left(|e(t) - n_1(t)|\right)\left(e(t) - n_1(t)\right) \\ \dot{x}_2(t) &= -k\left(|e(t) + n_2(t)|\right)\left(e(t) + n_2(t)\right) \end{aligned} \tag{4}$$

Since, at any time instant t, we have

$$\begin{aligned} e(t) < 0 \quad &\Rightarrow \quad e(t) - n_1(t) < \bar{n},\ e(t) + n_2(t) < \bar{n} \\ e(t) > 0 \quad &\Rightarrow \quad e(t) - n_1(t) > -\bar{n},\ e(t) + n_2(t) > -\bar{n} \\ e(t) = 0 \quad &\Rightarrow \quad |e(t) - n_1(t)| \leq \bar{n},\ |e(t) + n_2(t)| \leq \bar{n} \end{aligned}$$

and using continuity of x_1, x_2 we obtain the following proposition and its immediate corollary:

Proposition 2: Consider Problem 1 and the control scheme (2),(3). Given the error $e(t_0) = x_2(t_0) - x_1(t_0) - D$ at a specific time $t_0 \geq 0$:

1) If $e(t_0) < 0, \dot{x}_1(t) \leq 0, \dot{x}_2(t) \geq 0, \forall t \geq t_0$.
2) If $e(t_0) > 0, \dot{x}_1(t) \geq 0, \dot{x}_2(t) \leq 0, \forall t \geq t_0$.
3) If $e(t_0) = 0, \dot{x}_1(t) = \dot{x}_2(t) = 0, \forall t \geq t_0$.

Corollary 1: Consider Problem 1 and the control scheme (2),(3). For any given initial error $e(0) = x_2(0) - x_1(0) - D > -D$, the following hold:

1) If $e(0) \leq 0, \dot{x}_1(t) \leq 0, \dot{x}_2(t) \geq 0$ and $x_2(0) - D \leq x_1(t) \leq x_2(t) \leq x_1(0) + D, \forall t \geq 0$.
2) If $e(0) \geq 0, \dot{x}_1(t) \geq 0, \dot{x}_2(t) \leq 0$ and $x_1(0) \leq x_1(t) \leq x_2(t) \leq x_2(0), \forall t \geq 0$.
3) $|e(t)|$ is a non-increasing function of time t.
4) $x_1(t)$ and $x_2(t)$ remain bounded for all $t \geq 0$.

Combining Proposition 1 and Corollary 1, we obtain the following theorem summarizing this section's main results:

Theorem 1: Consider Problem 1 and the control scheme (2),(3). For any given initial positions $x_1(0), x_2(0)$ satisfying $x_1(0) < x_2(0)$, the following hold:

1) $x_1(t)$ and $x_2(t)$ remain bounded for all $t \geq 0$.
2) The distance $x_2(t) - x_1(t)$ converges to the ball $B(D, 2\bar{n})$ as $t \to \infty$.

Remark 2: In (4), $\dot{x}_1, \dot{x}_2$ are non-zero if and only if $|e(t) - n_1(t)| > \bar{n}, |e(t) + n_2(t)| > \bar{n}$, respectively. Using this fact, it can be shown that for a certain class of noise functions $n_1(t), n_2(t)$ including sinusoidal signals with amplitude $\bar{n}$, the error signal $e(t) \to 0$ as $t \to \infty$. Similar phenomena apply for the many-agent platoons considered in Section III.

In the rest of the paper, where we analyze more complex platoon formations, we only focus on the non-hierarchical case with similar assumptions to those of this subsection, i.e., we assume that the control structures are required to be identical for all the agents and an upper bound for the magnitudes of the distance measurement noises is known.

III. PLATOONS WITH THREE OR MORE AGENTS

In this subsection, we extend our analysis in Section II-B to platoon formations with an arbitrary number of agents. Note here that this extension is not straightforward, especially when the number of agents is greater than 3, because of the interactions between the consecutive distance error terms (e_i, which will be defined in the sequel) and dependence of the dynamics of each agent other than the leftmost and rightmost ones on two such error terms rather than one, which makes the analysis method used to obtain Proposition 2 impractical in the many-agent case. Next, we reformulate Problem 1 for an arbitrary number of agents. Note that, as we assume identical controller structures for the agents and known noise upper bounds as in Section II-B from now on, we directly impose these assumptions in the problem definition.

Problem 2: Consider a platoon of m ordered agents $A_1, A_2, \ldots, A_m$ each of which can move in both directions on a (1-dimensional) axis (see Figure 3) with kinematics (1), where $m \geq 3$. For each $i \in \{1, \ldots, m\}$, let us denote the position of A_i on the axis at each time $t \geq 0$ by $x_i(t)$ where $x_1(0) < x_2(0) < \cdots < x_m(0)$. Each agent A_i has a sensor to measure its position relative to each of its immediate neighbors A_j, i.e. each A_j such that $j \in N_i = \{j \,||i-j| = 1, j \in \{1, \ldots, m\}\}$, with some noise as

$$d_{ij}(t) = x_i(t) - x_j(t) + n_{ij}(t)$$

where n_{ij} denotes the corresponding sensor noise of A_i whose magnitude is bounded from above by some known constant $\bar{n} > 0, i.e., |n_{ij}(t)| \leq \bar{n}, \forall t$. Each agent A_i can move with a control law of the form

$$v_i = f_i\left(\{d_{ij}, D_{ij} \,| j \in N_i\}\right)$$

where $D_{ij} = -D_{ji}$ is the desired value of the position difference $x_i - x_j$[4]. The task is to design the control laws, i.e., select the functions f_i such that (i) x_i is kept bounded for any $i \in \{1, \ldots, m\}$, (ii) the distance $x_i - x_j$ converges to the ball $B(D_{ij}, \gamma \bar{n})$ for any $i \in \{1, \ldots, m\}$ and any $j \in N_i$, where $\gamma > 0$ denotes a predefined constant and (iii) f_i has a common structure valid for any agent $A_i, i.e., f_i \equiv f_j, \forall i, j \in \{1, \ldots, m\}$.

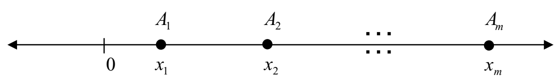

Fig. 3. Problem 2: It is desired to keep each agent A_i at a desired distance $|D_{ij}|$ from each of its immediate neighbors A_j ($j \in N_i$) in the existence of measurement noises.

As a generalization of the control law (2), we propose the following control scheme for Problem 2:

$$\begin{aligned} f_i\left(\{d_{ij}, D_{ij} \,| j \in N_i\}\right) &= -\sum_{j \in N_i} k\left(|\delta_{ij}|\right)\delta_{ij} \\ \delta_{ij} &= d_{ij} - D_{ij} \end{aligned} \tag{5}$$

where the switching term $k : [0, \infty) \to [0, \bar{k}]$ is defined by (3) as in Section II-B.

In analyzing the behavior of the formation in Problem 2 with control scheme (5), we follow the same steps as in Section II-B. First, let us define the error signals $e_i(t) = x_{i+1}(t) - x_i(t) - D_{(i+1)i} = -\left(x_i(t) - x_{i+1}(t) - D_{i(i+1)}\right)$ for $i = 1, \ldots, m-1$, noting that by definition $e_i(t) = \delta_{(i+1)i}(t) - n_{(i+1)i}(t) = -\left(\delta_{i(i+1)}(t) - n_{i(i+1)}(t)\right)$. Note also that the requirement (ii) of Problem 2 is equivalent to driving all the e_i signals to $B(0, \gamma\bar{n})$ for some predefined constant $\gamma > 0$. For each $i \in \{2, \ldots, m-2\}$, differentiating e_i we have

$$\begin{aligned} \dot{e}_i &= \dot{x}_{(i+1)} - \dot{x}_i \\ &= -\sum_{j \in N_{i+1}} k\left(\left|\delta_{(i+1)j}\right|\right)\delta_{(i+1)j} + \sum_{l \in N_i} k\left(|\delta_{il}|\right)\delta_{il} \\ &= -k_i e_i + k_{(i-1)2} e_{i-1} + k_{(i+1)1} e_{i+1} + (k_{i1} n_{i(i+1)} \\ &\quad -k_{i2} n_{(i+1)i} - k_{(i+1)1} n_{(i+1)(i+2)} + k_{(i-1)2} n_{i(i-1)}) \end{aligned}$$

[4]The opposite signs of $d_{ij}(t) - n_{ij}(t)$and $d_{ji}(t) - n_{ji}(t)$, as well as those of D_{ij} and D_{ji}, indicate the assumption that each agent knows if it is at the left or right of each of its neighbors.

where

$$\begin{aligned} k_{j1}(t) &= k\left(\left|e_j(t) - n_{j(j+1)}(t)\right|\right) \\ k_{j2}(t) &= k\left(\left|e_j(t) + n_{(j+1)j}(t)\right|\right) \\ k_j(t) &= k_{j1}(t) + k_{j2}(t) \end{aligned}$$

for $t \geq 0, j \in \{1, \ldots, m-1\}$. Similarly, we have

$$\begin{aligned} \dot{e}_1 &= -k_1 e_1 + k_{21} e_2 + (k_{11} n_{12} - k_{12} n_{21} - k_{21} n_{23}) \\ \dot{e}_{m-1} &= -k_{m-1} e_{m-1} + k_{(m-2)2} e_{m-2} + k_{(m-1)1} n_{(m-1)m} \\ &\quad -k_{(m-1)2} n_{m(m-1)} + k_{(m-2)2} n_{(m-1)(m-2)} \end{aligned}$$

Defining $V(e) = \frac{e^T e}{2} = \frac{1}{2}\sum_{i=1}^{m-1} e_i^2$, we have

$$\begin{aligned} \dot{V} &= e^T \dot{e} \\ &= -\sum_{i=1}^{m-1} k_i e_i^2 + \sum_{i=1}^{m-2}\left(k_{i2} + k_{(i+1)1}\right) e_i e_{i+1} \\ &\quad + \sum_{i=1}^{m-2}\left(k_{(i+1)1} n_{(i+1)(i+2)} + k_{i2} n_{(i+1)i}\right)\left(e_{i+1} - e_i\right) \\ &\quad + k_{11} n_{12} e_1 - k_{(m-1)2} n_{m(m-1)} e_{m-1} \\ &= -k_{11}\left(e_1^2 - n_{12} e_1\right) - k_{(m-1)2}\left(e_{m-1}^2 + n_{m(m-1)} e_{m-1}\right) \\ &\quad - \sum_{i=2}^{m-1}\left(\left(e_{i-1} - e_i\right)\left(k_{(i-1)2}\left(e_{i-1} + n_{i(i-1)}\right)\right.\right. \\ &\quad \left.\left. -k_{i1}\left(e_i - n_{i(i+1)}\right)\right)\right) \end{aligned} \tag{6}$$

A. Noiseless Measurement Case

To analyze the case when there is no measurement noise, i.e. $n_{ij}(t) = 0, \forall t \geq 0, \forall i, j$, we first state a lemma about some characteristics of the switching function (3):

Lemma 1: The switching function $k : [0, \infty) \to [0, \bar{k}]$ given by (3) satisfies the following:

(i) $(a - b)\left(k(|a|)a - k(|b|)b\right) \geq 0, \forall a, b \in \Re$.

(ii) $|k\left(|a|\right)a - k\left(|b|\right)b| \leq 2\bar{k}\,|a - b|, \forall a, b \in \Re$.

(iii) For any given $\forall a, n \in \Re$ with $|n| \leq \bar{n}$, $k(|a+n|)(a+n)a \geq \frac{1}{2}\bar{k}\left(a^2 - 4\bar{n}^2\right)$. Furthermore, if $|a| \geq 2\bar{n} + \bar{\delta}$, $k(|a+n|)(a+n)a \geq \frac{1}{2}\bar{k}\left(a^2 - \bar{n}^2\right)$.

(iv) For any given $\forall a, b, n_1, n_2 \in \Re$ with $|n_1|, |n_2| \leq \bar{n}$, $\left(k(|a+n_1|)(a+n_1) - k(|b+n_2|)(b+n_2)\right)(a - b) \geq \frac{1}{2}\bar{k}\left((a-b)^2 - 16\bar{n}^2\right)$. Furthermore, if $|a| \geq 2\bar{n} + \bar{\delta}$ or $|b| \geq 2\bar{n} + \bar{\delta}$, $\left(k(|a+n_1|)(a+n_1) - k(|b+n_2|)(b+n_2)\right)(a-b) \geq \frac{1}{2}\bar{k}\left((a-b)^2 - 4\bar{n}^2\right)$.

Proof: The results follow using simple analysis. The details are omitted here due space considerations. ∎

For the noiseless case, from (6) and Lemma 1(i) we have

$$\begin{aligned} \dot{V} &= -k_{11} e_1^2 - k_{(m-1)2} e_{m-1}^2 \\ &\quad - \sum_{i=2}^{m-1}\left(\left(e_{i-1} - e_i\right)\left(k_{(i-1)2} e_{i-1} - k_{i1} e_i\right)\right) \leq 0 \end{aligned} \tag{7}$$

noting that $k_{(i-1)2}(t) = k\left(|e_{i-1}(t)|\right), k_{i1}(t) = k\left(|e_i(t)|\right)$. Since $V(t) \geq 0, \dot{V}(t) \leq 0, \forall t \geq 0$, we have $0 \leq V(t) \leq V_0, \forall t \geq 0$, where $V_0 = V(0) \geq 0$. Therefore $e_i, \sum_{j=1}^{i} e_i$ are bounded for any $i \in \{1, \ldots, m-1\}$.

Using (3) and (7), it can be seen that for any given $\varepsilon > 0$, $\dot{V} \leq -\bar{k}\min\{\frac{\varepsilon}{\delta}, 1\}\bar{n}^2$ whenever $\max\{|e_1|, |e_{m-1}|\} \geq$

$\bar{n}+\varepsilon$. Furthermore, again for any given positive $\varepsilon < \bar{n}$, $(e_{i-1}-e_i)\left(k_{(i-1)2}e_i - k_{i1}e_i\right) \geq \bar{k}\min\{\frac{\varepsilon}{\delta},1\}\varepsilon^2$ whenever $|e_{i-1}-e_i| \geq \varepsilon$ and $\max\{|e_{i-1}|,|e_i|\} \geq \bar{n}+\varepsilon$. Hence it follows that for any given positive $\varepsilon < \bar{n}$, $\dot{V} \leq -\bar{k}\min\{\frac{\varepsilon}{\delta},1\}\varepsilon^2$ when $\max\{|e_1|,\cdots,|e_{m-1}|\} \geq \bar{n}+\frac{m}{2}\varepsilon$. Hence we have established that $e_i(t) \to B(0,\bar{n})$ as $t \to \infty$ for any i.

Next we analyze the behavior of x_i for $i \in \{1,\ldots,m-1\}$. Using (7) and Lemma 1 (i), we obtain

$$\dot{V}(t) \leq -k_{11}(t)e_1^2(t) \leq 0, \forall t \geq 0 \quad (8)$$

Integrating (8), we obtain

$$V(t) - V_0 \leq -\int_0^t k_{11}(\tau)e_1^2(\tau)d\tau, \forall t \geq 0$$

Hence using $V(t) \geq 0, \dot{V}(t) \leq 0, \forall t \geq 0$, we have

$$\int_0^t k_{11}(\tau)e_1^2(\tau)d\tau \leq V_0, \forall t \geq 0 \quad (9)$$

Since $k_{11}(t) \geq 0, \forall t \geq 0$ and $k_{11}(t) = k(|e_1(t)|) = 0$ whenever $|e_1(t)| \leq \bar{n}$, (9) implies $\forall t \geq 0$ that

$$\left|\int_0^t k_{11}(\tau)e_1(\tau)d\tau\right| \leq \frac{1}{\bar{n}}\int_0^t k_{11}(\tau)e_1^2(\tau)d\tau \leq \frac{V_0}{\bar{n}}$$

Hence, noting that $\dot{x}_1(t) = k\left(|e_1(t)|\right)e_1(t) = k_{11}(t)e_1(t)$, we have

$$|x_1(t)| \leq |x_1(0)| + \frac{V_0}{\bar{n}}, \forall t \geq 0$$

i.e. x_1 is bounded. Since we have already established $e_i, \sum_{j=1}^{i} e_i$ are bounded for any $i \in \{1,\ldots,m-1\}$, this further implies that x_i is bounded for any i. This completes our analysis for the noiseless measurement case, the results of which are summarized in the following proposition:

Proposition 3: Consider Problem 2 and the control scheme (5),(3). Assume that the distance measurements are noiseless, i.e. $n_{ij} = 0$ for any $i \in \{1,\ldots,m\}, j \in N_i$. For any given initial positions $x_1(0) < x_2(0) < \cdots < x_m(0)$, the following hold:

1) $x_i(t)$ remains bounded for all $t \geq 0$ for any $i \in \{1,\ldots,m\}$.
2) The distance $x_i(t) - x_j(t)$ converges to the ball $B(D_{ij},\bar{n})$ as $t \to \infty$ for any $i \in \{1,\ldots,m\}, j \in N_i$.

B. Noisy Measurement Case

Applying Lemma 1 (iii),(iv) to (6), we obtain

$$\begin{aligned}\dot{V} \quad &\leq \quad -\frac{1}{2}\bar{k}\left(e_1^2 - 4\bar{n}^2\right) - \frac{1}{2}\bar{k}\left(e_{m-1}^2 - 4\bar{n}^2\right) \\ &\quad -\frac{1}{2}\bar{k}\sum_{i=2}^{m-1}\left((e_{i-1}-e_i)^2 - 16\bar{n}^2\right) \quad (10) \\ &= \quad -\frac{1}{2}\bar{k}\left(e_1^2 + e_{m-1}^2 + \sum_{i=2}^{m-1}(e_{i-1}-e_i)^2 - (16m-24)\bar{n}^2\right)\end{aligned}$$

Noting that the right hand side of (10) is negative for $(e_1,\ldots,e_{m-1})$ out of a certain ellipsoid around the origin of $\Re^{m-1}$, we have that V is decreasing outside of this ellipsoid. Hence we have that each e_i converges to a ball $B(0,\gamma\bar{n})$ for some γ whose value is determined by the ellipsoid mentioned above. Hence we have the following result:

Proposition 4: Consider Problem 2 and the control scheme (5),(3). For any given initial positions $x_1(0) < x_2(0) < \cdots < x_m(0)$, $x_i(t) - x_j(t)$ converges to the ball $B(D_{ij},\gamma\bar{n})$ as $t \to \infty$ for some predefined constant $\gamma > 0$ and any $i \in \{1,\ldots,m\}, j \in N_i$.

Using Proposition 4 and Lemma 1, the analysis may be completed to establish convergence of x_i and convergence sets of e_i. At the time of submission this analysis was incomplete. However, the following conjecture has been derived intuitively, and verified for $m = 3,4$ analytically and for $m \geq 5$ with numerous simulations:

Conjecture 1: Consider Problem 2 and the control scheme (5),(3). For any given initial positions $x_1(0) < x_2(0) < \cdots < x_m(0)$, the following hold:

1) $x_i(t)$ remains bounded $\forall t \geq 0$, $\forall i \in \{1,\ldots,m\}$.
2) $x_{i+1}(t) - x_i(t)$ converges to $B(D_{(i+1)i}, 2\min\{i, m-i\}\bar{n})$ as $t \to \infty$ for any $i \in \{1,\ldots,m-1\}$.

IV. A SIMULATION EXAMPLE

Consider a platoon of six agents A_1,A_2,A_3,A_4,A_5,A_6, whose initial positions (in meters) are $x_1(0) = 0$, $x_2(0) = 3$, $x_3(0) = 6$, $x_4(0) = 9$, $x_5(0) = 12$, $x_6(0) = 15$, i.e. there is 3 m between each neighbor agent pair initially. Let the desired distances between the neighbor agent pairs be given by $D_{21} = D_{43} = D_{65} = 2$ m and $D_{32} = D_{54} = 1$ m. Assume that the noise upper bound is $\bar{n} = 0.1$ m.

In order to demonstrate the shortcomings of simple classical controllers mentioned in Section I, we have first used the proportional control law (without switching) $f_i\left(\{d_{ij}, D_{ij} \,|j \in N_i\}\right) = -\sum_{j \in N_i}\bar{k}\delta_{ij}$ for each of the agents. For various $\bar{k}$ and noise settings, (e.g. $\bar{k} = 3$, $n_{12}(t) = -\bar{n}/2$, $\forall\ t \geq 0$, and $n_{ij}(t) = 0$, $\forall\ t \geq 0$ for any other neighbor agent pair A_i, A_j) we have observed that the platoon escapes to infinity in time although the distance errors are bounded and converge to a vicinity of zero.

Next, we have applied the decentralized control scheme of Section III, i.e. (3),(5), with the control parameters $\bar{\delta} = 0.02$ and $\bar{k} = 3$, for various settings including the following cases:

Case 1. No measurement noise: $n_{ij}(t) = 0$, $\forall\ t \geq 0$ for any neighbor agent pair A_i, A_j.

Case 2. Constant measurement noises[5]: $n_{12}(t) = n_{21}(t) = n_{23}(t) = n_{32}(t) = n_{34}(t) = \bar{n}$, $n_{43}(t) = n_{45}(t) = n_{54}(t) = n_{56}(t) = n_{65}(t) = -\bar{n}$, $\forall\ t$.

Case 3. Periodic (sinusoidal) measurement noises: $n_{i(i+1)}(t) = \bar{n}\sin t$ and $n_{(i+1)i}(t) = \bar{n}\cos t$ for $i \in \{1,2,3,4,5\}$.

Case 4. Random measurement noises: $n_{ij}(t)$ is a time-varying random number with a uniform distribution within the interval $[-\bar{n},\bar{n}]$ for each $t \geq 0$, for any neighbor agent pair A_i, A_j.

[5]Magnitudes of the noises in this case are specially chosen to demonstrate a worst case scenario.

Simulation results (in terms of time-histories of the inter-agent distance error terms $e_1, \ldots, e_5$ and the location x_1 of agent A_1) for Case 2 and Case 3 (only, due to space limitations) are shown in Figures 4 and 5. As can be seen in these figures, all the error terms e_i and x_1 remain bounded and asymptotically converge to certain constant values, which complies with Conjecture 1. The convergence rate is sufficiently fast for all the cases. In Case 1, it is observed that each $|e_i|$ converges to $\bar{n} = 0.1$ m, confirming the statement of Proposition 3. In Case 2, $|e_1|, |e_5| \to 2\bar{n}$, $|e_2|, |e_4| \to 4\bar{n}$, $|e_3| \to 6\bar{n}$, which demonstrates that the upper bound for the size of the error convergence set given in Conjecture 1 can be reached in some cases. In Cases 3 and 4, each $|e_i|$ converges to zero, performing better than Case 1 and demonstrating the phenomena mentioned in Remark 2.

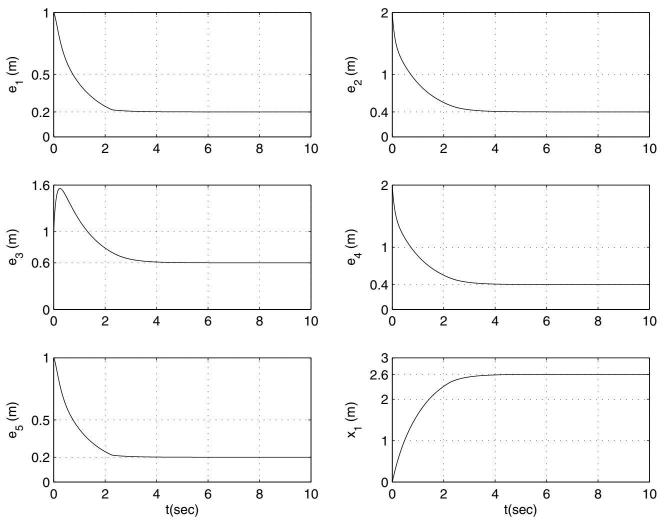

Fig. 4. Simulation results for Case 2.

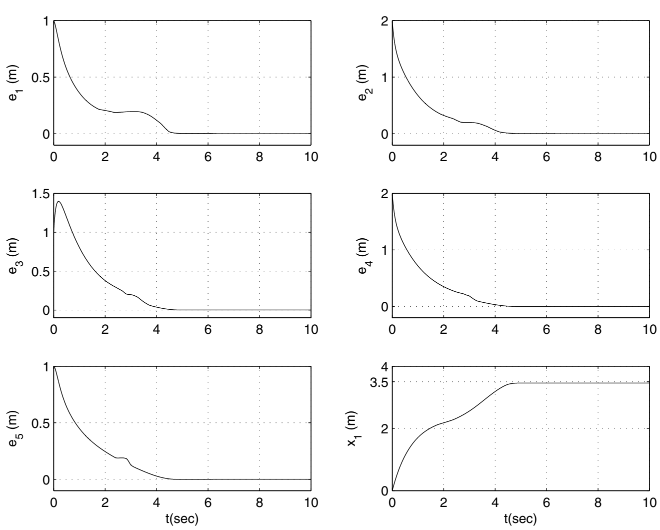

Fig. 5. Simulation results for Case 3.

V. CONCLUSION

We have analyzed the problem of acquiring and maintaining desired inter-agent distances for non-hierarchical autonomous multi-agent platoons of robots or vehicles, using only agent-wise relative position measurements. We have designed a robust switching control scheme addressing this problem in the existence of measurement noises, which might cause instability problems using classical control designs ignoring such noises. The stability and robustness properties of the proposed control scheme are established analytically and demonstrated via simulations.

A particular future direction is adaptation and application of the proposed control scheme to other similar networked system problems mentioned in Section I. As a closely related problem, velocity equalization of vehicles in vehicle platoons together with keeping the distances between consequent vehicle pairs constant using a similar control scheme is being investigated for an advanced version of this paper.

In another follow-up research direction, the authors are currently studying the extensions of the proposed control scheme for acquisition and maintenance of 2 and 3-dimensional autonomous formations.

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