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Collision Free Coordination of Autonomous Multiagent Systems

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***Abstract*—Inspired by natural swarming and flocking behaviors, in this paper we study a series of problems concerning motion control for autonomous multiagent formations moving in a two dimensional space and avoiding collision with each other as well as obstacles. We give a hierarchical task map and argue that if the formation is kept rigid via implementation of certain distance maintenance constraints, then no inter-agent collision can happen. We outline recently developed procedures based on graph theory for achieving this. We then take the study further by investigating and discussing two strategies for obstacle avoidance by the entire formation. We present some simulation results for the motion of an asymmetrically controlled formation with no global leader, in presence of multiple obstacles on the plane of interest.**

***Key words* — Multiagent System, Formation Control, Collision Avoidance, Hierarchical control**

I. INTRODUCTION

In recent years, the topic of cooperative control and control of multi-agent systems has gained much attention. The multi-agent systems can be considered as a group of combat robots, unmanned aerial vehicles, ground vehicles or underwater vehicles. Numerous research papers (for example, [1-9]) have already been published on formation control. These papers differ in factors such as the types of agent dynamics, control strategies, formation control tasks and the information flow between the agents.

Collision avoidance is an a priori requirement for coordination of autonomous multiagent formations. The observation of the schooling fish, flocking birds, swarming bees, inspires us to propose a hybrid two-level collision-free coordination of autonomous multiagent systems, in a way which may mimic the natural examples. The two-level collision-free coordination scheme consists of the lower level inter-agent collision avoidance control and the higher level formation-to-obstacle avoidance control. We will first comment on the inter-agent collision avoidance control and then focus more on obstacle avoidance control for a formation.

In brief, the lower level collision avoidance refers to avoiding collisions between two agents in the system during any given motion of the system. There are two general ways to tackle the problem: tackling it as a problem explicitly in its own right, or by arranging its solution to be a consequence of satisfying other system tasks. In the explicit case, if there are a large or growing number of agents in the system, too many sensed entities (targets, in this case other agents within the sensing range of an agent in the same system) will overload the agent, resulting in either failure to avoid collision or inability to perform other tasks.

Based on earlier studies of some of its authors, this paper is proposing use of *formation* as an implicit way of avoiding inter-agent collisions; formation control is achievable by maintaining the distances between enough nominated pairs of agents fixed. In formation control, we are specially interested in how to preserve the shape of the formation, so that the formation moves as a cohesive whole during required maneouvers. Note that this is a task considered under the constraints that individual agents are autonomous and they make decisions based on only their own observations and state.

Once the formation shape is maintained, i.e. inter-agent collision avoidance is guaranteed implicitly, then the higher level collision avoidance task is assigned cooperatively to the entire formation to avoid (external) obstacles. In this paper, we consider the *rigid (persistent)* [3] formation as a single entity and no splitting/rejoining task is contemplated, i.e. we propose strategies to move around obstacles while keeping the formation shape unchanged.

The contributions of this paper are a novel concept of viewing a formation so as to avoid inter-agent collision, and an exploratory study of obstacle avoidance given coordinated motion of a distance-based persistent formation. The main specific problem elaborated in the paper is coordinated collision-free motion control of a certain class of autonomous multiagent formations with asymmetric control structure, in the presence of multiple round obstacles. For simplicity, we present here only the case of two-dimensional multi-agent formations, and the number of agents is kept small (four agents) for better illustration.

In the recent literature, there exist a number of different approaches to the problem of obstacle avoidance for multi-agent formations. The two main classes of these approaches are: (i) avoiding the obstacles based on a priori knowledge of environment (planning based approaches) and (ii) dealing with the obstacles during the course of motion as they are detected (behavior based approaches). The reader may refer to [14-16] for further information about the first class, and [17-19] for the second. Our approach can be categorized in the second class. However, it is different from the existing approaches in this class, in the sense that it does not require a detailed map of environment. Moreover, it is easier to implement than these approaches.

The rest of the paper is organized as follows. Section II overviews the topic in a hierarchical framework summarizing various formation tasks. The main argument is that by keeping a formation rigid at all times, the inter-agent collision

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avoidance is guaranteed as a consequence. Section III provides the problem definition and a brief review on some recent graph rigidity theoretical results. In Section IV, we present the basic motion control laws proposed for the cohesive motion and obstacle avoidance problem described above using a "virtual vector field" approach. These control laws are then explicitly modified to realize obstacle avoidance for the entire formation, with simulation results presented in Section V. The paper ends with concluding remarks and suggestions for future work in Section VI.

II. A Hierarchical Framework

In control of multiple agents in a system, typically many control tasks are involved; in this subsection we study the tasks related to the motion decision and control of each agent and the formation. We propose to use a hierarchical task map shown in a pyramid as in Fig. 1. The tasks of interest that will be dealt in this paper are highlighted in Fig. 1.

There are many scenarios where one needs multiagent systems to perform a group of complicated tasks (for example, see [9].) The multiagent system can be easily formed into a formation by explicitly keeping the distances between each pair of agent at a desired value; however, this is not optimal. Developed from graph rigidity theory, there are ways of maintaining only a sufficient number (growing linearly with the number of agents) of inter-agent distances and then all other distances are maintained as a consequence. We shall explain in more detail in the next subsection; for the time being, let us assume the formation is formed and distance maintenance is a task of a subset of agents of the system. If the formation-keeping task is assigned as the top priority level task, then inter-agent collisions will not occur when any other task is given to the formation. This is equivalent to saying that the operational task is given such that the multiagent system is required to be in formation at all times, and the operational task then can be implemented by a set of operations on the entire formation, rather than the individual agents of the multiagent system. This avoids conflict between individual agent task assignment and group task assignment.

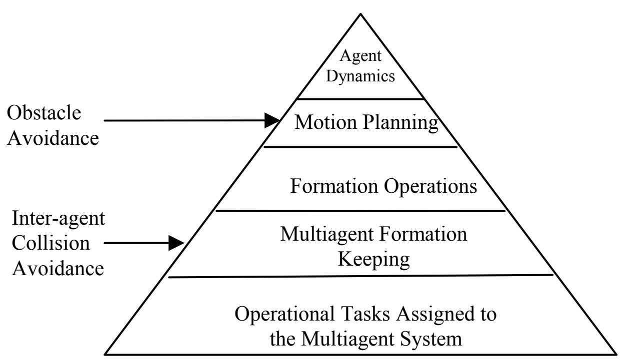

Fig. 1. The hierarchical task map for a multiagent system

There are several formation control tasks that can be contemplated, such as formation stabilization, flocking, merging, splitting, formation switching and cohesive motion. In formation stabilization, agents which are initially at arbitrary positions, converge to a particular geometrical configuration, thereby creating a formation. Formation stabilization of agents with nonholonomic agent dynamics is described in [2]. Flocking is a process by which speeds and directions of motion of initially arbitrarily positioned agents converge to a common value. Due to this convergence, the agents asymptotically move in a particular direction with a constant velocity [2]. In merging, two rigid sub-formations are combined together to form a single post-merged rigid formation [9]. This is done by introducing distance constraints between some of the agents in the two pre-merged formations. Splitting is the opposite process of merging, resulting in the formation of two post-split rigid formations from a single pre-split rigid formation [7].

Obstacle avoidance through formation operations can be done in various ways; we briefly discuss here two ways. Splitting/merging can be implemented as one way to get around obstacles [7, 9], as is suggested by observations of natural species: a formation can be split into two sub-formations prior to encountering some obstacles and then merged into a single one after passing around the obstacles. Another strategy is to simply avoid the obstacle by moving the entire formation to one side of the obstacle. From an energy point of view, this may not be efficient when the obstacles are significantly smaller than the geo-coverage of the entire formation; in this case, the displacement caused by moving all agents to one side of the small obstacle is simply a nuisance. It can be easily seen that in such cases, a splitting/merging strategy works better. However, in the case of multiple obstacles, before two post-split formations attempt to merge, some more obstacles may possibly lie in the planned path for merging or in the path of one of the sub-formations, and then the split formations may have to split again to avoid these obstacles. If this scenario keeps occurring then the formation may be split into components of trivial size, too trivial in fact to carry out any meaningful operational tasks previously assigned to the original collective group, or they may even fail to merge as a single one again. So in the presence of multiple obstacles, the strategy of obstacle avoidance by moving to one side is preferred in this paper.

Since the focus of the paper is to illustrate a hybrid scheme of collision avoidance, we will present two simple strategies for multiple obstacle avoidance with simulation results in Section V. We leave other possible approaches/strategies, including those based on splitting and merging, or even more complicated optimized solutions, as open problems.

III. Problem Definition

In this paper, we consider 2-dimensional point–agent formations whose shapes are required to be maintained during any collision free motion by explicitly maintaining the distances between certain agent pairs. The task of maintaining such a distance is assigned to one agent only of the pair. An associated directed graph can be defined, with a directed edge from vertex i to vertex j when agent i has to maintain its distance from agent j. A formation is called *rigid* if the distance between each pair of agents remains constant, i.e., the formation shape is maintained during any continuous motion provided that each agent satisfies all the distance constraints on it [6-8]. In a rigid formation, (explicit) maintenance of the distances between neighbor agents result in (implicit) maintenance of the remaining inter-agent distances and hence the formation shape as well. If each agent in a formation is able to satisfy all the constraints on it once all the other constraints within the formation are satisfied, then the formation is called *constraint-consistent*. A formation that is both *rigid* and *constraint-consistent* is called *persistent*. Rigorous definitions and analysis of *rigidity, constraint-consistence* and *persistence* can be found in [8]. Next we provide some fundamental characteristics of persistent formations relevant to our work, whose details can be found in [6, 8].

Noting that the out-degree of a vertex in a directed underlying graph is equal to the number of distance constraints on the corresponding agent, the *degree of freedom (DOF)* of an agent (for its motion in $\Re^2$) can be defined based on the out-degree of its representative vertex in the underlying graph. Given an agent A_i, if the out-degree of its representative vertex i is 0 then A_i has 2-DOF, i.e., it can move freely in $\Re^2$. If the out-degree is 1 then A_i can only rotate around the agent it follows in order to meet its distance constraint and hence has 1-DOF. If the out-degree is 2 or more then the motion of A_i completely depends on the agents it follows, i.e. it has 0-DOF.

For a persistent formation, it has been shown in [8] that the sum of DOFs of individual agents is at most 3 and for a minimally persistent formation, exactly 3, which is the same as the DOF of a free rigid (non-vertex) object in $\Re^2$ (2 for translation and 1 for rotation). Based on the distribution of these 3 DOFs, minimally persistent formations can be divided into two categories: Formations with the *leader-follower* structure where one agent has 2-DOF, another has 1-DOF and the rest have 0-DOF, and the *3-coleader* structure where three agents have 1-DOF and the rest have 0-DOF. In the *leader-follower* structure, the 2-DOF agent is called the *leader* and the 1-DOF is called the *first follower*. In the 3-*coleader* structure, the 1-DOF agents are called the *coleaders*. In both structures the 0-DOF agents are called the *(ordinary) followers.* Refer to Fig. 2 for examples of both structures.

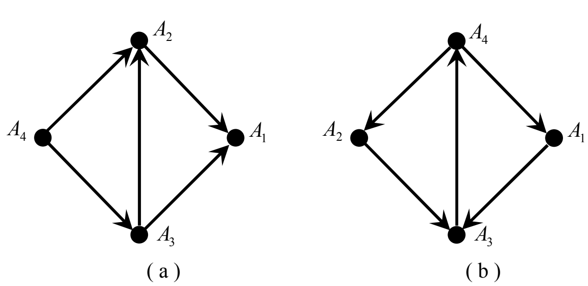

Fig. 2. The directed underlying graph of a persistent graph with (a) the leader-first follower structure; (b) the 3 coleader structure

From [8], it can be seen that the underlying graph of a formation with leader-follower structure may be acyclic (cycle-free) while any formation with the three-coleader structure has at least one cycle. Due to the presence of a cycle, the motions of the three coleaders are cyclically dependent on each other and hence the motion control for the formation requires a more implicit strategy. Some stability properties of leader-follower structures are partially investigated in [6]. However, to the best of our knowledge, there is no existing literature that deals with motion control of persistent (or rigid) formations with cycles.

In our work, we consider both of the categories above. One can verify via simple examples that there exist (minimally) persistent formations with the leader-follower structure where the first follower does not directly follow the leader but another (ordinary follower) agent, and there exist (minimally) persistent formations with the 3-coleader structure where the coleaders do not directly follow each other but some other agents. For simplicity, we assume that the first follower directly follows the leader in a leader-follower structured formation and the three coleaders follow each other in a 3-coleader structured formation.

Our control task is to move a given persistent formation F with say $m \geq 3$ agents $A_1, \ldots, A_m$ whose initial position and orientation are specified, to a new desired position and orientation in $\Re^2$ (the *xy*-plane) cohesively, i.e., without violating the persistence of F during the motion, using a decentralized strategy. The initial and final positions and orientations of F are defined by the initial positions p_{i0} and final positions p_{if} of the individual agents A_i for $i = 1, \ldots, m$, respectively. We assume that the final positions p_{if} are consistent with the desired inter-agent distances d_{ij} between neighbor agent pairs (A_i, A_j).

For each agent, we assume a velocity integrator kinematics, i.e., for each agent A_i we assume that

$$\dot{p}_i(t) = v_i(t) \qquad (1)$$

where $p_i(t) = (x_i(t), y_i(t)), v_i(t) = (v_{xi}(t), v_{yi}(t)) \in \Re^2$ denote the position and velocity of A_i at time t, respectively. The velocity $v_i(t)$ is considered as the control signal to be generated by the individual controller of agent A_i. It is required that $v_i(t)$ is continuous and satisfies $|v_i(t)| \leq \bar{v}$ for some constant maximum speed limit $\bar{v} > 0$ at any $t \geq 0$ for any $i \in \{1, \ldots, m\}$. We assume that each agent A_i knows its final desired position p_{if} and can sense its own position $p_i(t)$ and velocity $v_i(t)$ as well as the position $p_j(t)$ of each agent A_j it follows at any time $t \geq 0$. It is also assumed that the distance sensing range for an agent A_j is sufficiently larger than any desired distance it is explicitly tasked to maintain.

For each 1-DOF agent, the control laws below are chosen so that meeting the distance constraint has a higher priority than reaching the final desired position, i.e., a 1-DOF agent can undergo its DOF movement, only when its distance constraint is satisfied within a certain error bound.

Based on the agent kinematics (1) and the assumptions above, in the next section we develop control schemes to address the cohesive motion task defined above for both the leader-follower and the 3-coleader structures; and we modify the control laws for obstacle avoidance in Section V.

IV. Basic Control Laws

In this section, we briefly review the control laws for the cohesive motion of a persistent formation with the leader-follower structure and with the 3-coleader structure developed in [6], leaving the motivational and technical details to [6]. The designs are illustrated on a formation with one leader A_1, one first-follower A_2 and two (ordinary) followers A_3 and A_4 depicted in Fig. 2 (a). These control laws constitute the basics of the overall control scheme to be developed in this paper and their objective is solution of the problem in Section III, i.e. moving the formation cohesively to the target setting while avoiding inter-agent collisions, assuming that the region is free of obstacles. They will be modified in the later sections to cope with the cases with obstacles in the region.

A. Control Law for the 0-DOF Agents

Consider an (0-DOF, ordinary) follower agent A_i (for example A_4 in Fig. 2(a)) and the two agents A_j and A_k it follows (A_2 and A_3 for the example). Due to the distance constraints of keeping $|p_i(t)-p_j(t)|,|p_i(t)-p_k(t)|$ at the desired values of d_{ij}, d_{ik} respectively, at each time $t \geq 0$, the desired position $p_{id}(t)$ of A_i is the point whose distances to $p_j(t)$ and $p_k(t)$ are d_{ij}, d_{ik} respectively. Further, $p_{id}(t)$ must vary continuously. Assuming $|p_i(t)-p_{id}(t)|$ is sufficiently small, $p_{id}(t)$ can be explicitly determined as

$$p_{id}(t)=\bar{p}_{jk}(t,p_i(t))$$

where $\bar{p}_{jk}(t,p_0)$ for any $p_0 \in \Re^2$ denotes the intersection of the circles $C(p_j(t),d_{ij})$ and $C(p_k(t),d_{ik})$ that is closer to p_0, and in the notion $C(\cdot,\cdot)$ the first argument denotes the center and the second denotes the radius. Based on this observation, we use the following control law for the follower agents:

$$\begin{aligned} v_i(t) &= \bar{v}\beta_i(t)\,\delta_{id}(t)/|\delta_{id}(t)| \\ \delta_{id}(t) &= p_{id}(t)-p_i(t)=\bar{p}_{jk}(t,p_i(t))-p_i(t) \\ \beta_i(t) &= \begin{cases} 0, & |\delta_{id}(t)|<\varepsilon_k \\ \dfrac{|\delta_{id}(t)|-\varepsilon_k}{\varepsilon_k} & \varepsilon_k \leq |\delta_{id}(t)|<2\varepsilon_k \\ 1, & |\delta_{id}(t)| \geq 2\varepsilon_k \end{cases} \end{aligned} \tag{2}$$

where $\bar{v}>0$ is the constant maximum speed of the agents and $\varepsilon_k>0$ is a small design constant. In (2), the switching term $\beta_i(t)$ is introduced to avoid chattering due to small but acceptable errors in the desired inter-agent distances.

B. Control Law for the 1-DOF Agents

Consider a 1-DOF agent A_i which only maintains its distance to A_j. Note such an agent exists in both the leader-follower structure (for example A_2 in Fig.2(a), which is given a specific name of "first-follower") and the 3-coleader structure (for example, $A_1 A_2 A_3$ in Fig. 2(b) and they are all "coleaders"). Let us take the example of the leader-follower structure. Observe that once the 1-DOF agent (the first-follower) satisfies its distance constraint with the leader, it is free to rotate around the leader. For the example in Fig. 2(a), the first-follower can move on the circle with center A_1 and radius d_{21} provided that it does not need to use the whole of its velocity capacity to satisfy $|A_1A_2|=d_{21}$. Based on this observation and the assumption in Section III for 1-DOF agents, we propose the following control scheme for the first-follower agent A_i:

$$\begin{aligned} v_i(t) &= \beta_i(t)v_{i1}(t)+\sqrt{1-\beta_i^2(t)}\,v_{i2}(t) \\ \delta_{ji}(t) &= (\delta_{jix}(t),\delta_{jiy}(t))=p_j(t)-p_i(t) \\ \bar{\delta}_{ji}(t) &= |\delta_{ij}(t)|-d_{ij} \\ \beta_i(t) &= \begin{cases} 0, & |\bar{\delta}_{ji}(t)|<\varepsilon_k \\ \dfrac{|\bar{\delta}_{ji}(t)|-\varepsilon_k}{\varepsilon_k} & \varepsilon_k \leq |\bar{\delta}_{ji}(t)|<2\varepsilon_k \\ 1, & |\bar{\delta}_{ji}(t)| \geq 2\varepsilon_k \end{cases} \end{aligned} \tag{3}$$

where

$$v_{i1}(t)=\bar{v}\,\mathrm{sgn}\left(\bar{\delta}_{ji}(t)\right)\delta_{ji}(t)/|\delta_{ji}(t)| \tag{4}$$

$$\begin{aligned} v_{i2}(t) &= \bar{v}\bar{\beta}_i(t)\,\mathrm{sgn}\left(\delta_{if}^T(t)\bar{\delta}_{ji}^{\perp}(t)\right)\bar{\delta}_{ji}^{\perp}(t) \\ \delta_{if}(t) &= p_{if}(t)-p_i(t) \\ \bar{\delta}_{ji}^{\perp}(t) &= \left(-\delta_{jiy}(t),\delta_{jix}(t)\right)/|\delta_{ji}(t)| \\ \bar{\beta}_i(t) &= \begin{cases} 0, & |\delta_{if}(t)|<\varepsilon_f \\ \dfrac{|\delta_{if}(t)|-\varepsilon_f}{\varepsilon_f} & \varepsilon_f \leq |\delta_{if}(t)|<2\varepsilon_f \\ 1, & |\delta_{if}(t)| \geq 2\varepsilon_f \end{cases} \end{aligned} \tag{5}$$

$\varepsilon_k, \varepsilon_f>0$ are small design constants. In (3), via the switching term $\beta_i(t)$, the controller switches between a translational action (4) to satisfy $|A_iA_j| \cong d_{ij}$ and a rotational action (5) to move the agent A_i towards p_{if}, which can take place only when $|A_iA_j|$ is sufficiently close to d_{ij}.

In (5), $\bar{\delta}_{ji}^{\perp}(t)$ is the unit vector perpendicular to the distance vector $\delta_{ji}(t)=p_j(t)-p_i(t)$ with clockwise orientation with respect to the circle $C(p_j(t),d_{ij})$, and the term $\text{sgn}\left(\delta_{if}^T(t)\bar{\delta}_{ji}^{\perp}(t)\right)$ determines the orientation of motion that would move A_i towards A_{if}. The switching term $\overline{\beta}_i(t)$ is for avoiding chattering due to small but acceptable errors in the final position of A_i.

C. Control Law for the 2-DOF Agent

If a given agent A_i is the leader of the formation (e.g., A_1 in Fig. 2 (a)), since it does not have any constraint to satisfy, it can use its full velocity capacity only to move towards its desired final position p_{if}. Hence the velocity input at each time t can be simply designed as a vector with magnitude $\bar{v}$ in the direction of $p_{if}(t) - p_i(t)$:

$$v_i(t) = \bar{v}\overline{\beta}_i(t)\delta_{if}(t)\big/\left|\delta_{if}(t)\right| \tag{6}$$

$$\delta_{if}(t) = p_{if}(t) - p_i(t)$$

$$\overline{\beta}_i(t) = \begin{cases} 0, & \left|\delta_{if}(t)\right| < \varepsilon_f \\ \dfrac{\left|\delta_{if}(t)\right| - \varepsilon_f}{\varepsilon_f} & \varepsilon_f \le \left|\delta_{if}(t)\right| < 2\varepsilon_f \\ 1, & \left|\delta_{if}(t)\right| \ge 2\varepsilon_f \end{cases}$$

The switching term $\overline{\beta}_i(t)$ again prevents chattering due to small but acceptable errors in the final position of A_i.

The control laws (2)-(6) have been successfully tested via a number of simulations on various formations with leader-follower structure and the 3-coleader structure, for various initial and desired final settings. For more details refer to [6]. A formal proof of stability is still lacking.

V. OBSTACLE COLLISION AVOIDANCE

In this section, we improve the previous control laws by integrating the obstacle avoidance requirement. Similarly, we adopt in modified form the *virtual vector field* approach [10] for modeling of the obstacles, which is a variation of the potential function approach that is commonly used in articulated obstacle avoidance problems in robot and autonomous vehicle navigation [11-13]. The repulsive force increases when an agent gets closer to obstacles, and the force is "felt" by the agent and integrated in the calculation of its motion vector. However, it is worth mentioning that in almost all cases, the "force" (and the factor) keeping the formation rigid/persistent is much larger than the repulsive force.

As is common in any potential based artificial field approach, our method cannot straightforwardly deal with the problem of "local minima" created by multiple repulsive fields (the obstacles). We have investigated two possible approaches and tested these with many different obstacle layouts. Though we have not fully analyzed the convergence and robustness of both approaches, we present them in heuristic terms making comparisons between the two. We will begin by explaining our primitive approach to reducing the occurrence of local minima by a simple heuristic-based "line-of-sight" approach. Then we will move on to the "nearest-effective-obstacle" approach. In the sequel, each obstacle is assumed to be in the form of a circular disc, and hence can be represented as $O_i = C(c_{oi}, R_i)$ where c_{oi} is the center and R_i. is the radius. The radius affects the threshold of effectiveness of each obstacle. The larger R_i is, the farther O_i affects the coleaders in the formation. We have assumed the obstacles are comparable in size, if not all the same.

Due to space limitations, we only present the result for the formations with a 3-coleader structure. Both of the proposed methods turn out in simulations to cope with control of formations with a leader-follower structure as well.

A. Line-of-Sight Multiple Obstacle Avoidance

The intuition behind this strategy is simple, yet the result is convincing. Consider when a person's vision of a target (final destination) is blocked by obstacles; then the person may first seek to achieve a view of the target, rather than moving straight towards the obstacles, in the belief that somehow he will be able to get around the obstacle when closer to it. Thus, he may choose to move in the perpendicular direction in order to "see" the target, and then move to the target.

The coleader control laws presented in the previous section are modified for modified-virtual vector field based collision avoidance with an *obstacle avoidance unit vector*, which incorporates the "line-of-sight" idea. The only modification that has to be done is replacement of the vector $\delta_{if}(t)$ (instantaneous relative position of the target) with a virtual one $\delta_{if}^v(t)$ (instantaneous relative position of the virtual target), which has to be set according to the current coleader position, a set defined below termed the obstacle region set, and the final coleader position. An obstacle region is defined as that region surrounding and including an obstacle, such that if any coleader is in this region, one or more agents of the formation, if kept persistent, could lie in the space occupied by the obstacle, were the obstacle not there.

Note below that we only present the modification with reference to a coleader (1-DOF agent) control law, as mentioned previously: such modification can be easily generalized to control laws for the leader agent of a leader-follower formation.

Now we shall enumerate the cases showing how the presence of obstacles and obstacle regions affect the vector $\delta_{if}^v(t)$ corresponding to a coleader A_i . Let $O = \{O_1, \ldots, O_N\}$ be the set of all obstacles, and $P_i(t) \subseteq O$ be the set of obstacles of interest to A_i at time t. As before, let $p_i(t) = \left(x_i(t), y_i(t)\right)$ denote the current position of the coleader A_i, we have:

Case 1: If the line segment $[p_i(t)p_{if}]$ joining the current position of the coleader to its final position does not intersect with any of the N obstacle regions, then the virtual target is the same as the actual target, i.e. $\delta_{if}^v(t) = \delta_{if}(t)$.

Case 2: If the line segment $[p_i(t)p_{if}]$ joining the current position of the coleader to its final position intersects with any of the obstacle regions (as shown in Fig. 3, the coleader A's view to its destination A_f is blocked by the obstacle region of O_2), then the desired motion has to be in a direction perpendicular to $\delta_{if}(t)=(\delta_{ifx}(t),\delta_{ify}(t))=p_{if}(t)-p_i(t)$. Defining

$$\bar{\delta}_{if}^{\perp}(t)=(-\delta_{ify}(t),\delta_{ifx}(t))/|\delta_{if}(t)|$$

and using the same argument in Section IV.A (for determining the orientation of motion that would move A_i towards A_{if}), the instantaneous relative position $\delta_{if}^{v}(t)$ of the virtual target is set to

$$\delta_{if}^{v}(t)=\operatorname{sgn}(\xi_i^T(t)\bar{\delta}_{if}^{\perp}(t))\bar{\delta}_{if}^{\perp}(t) \quad (7)$$

where $\xi_i(t)$ is the *accumulated obstacle avoidance unit vector* affecting A_i at time t, which is defined below.

Case 3: If due to the "perpendicular motion" as required in Case 2, the coleader moves inside one or more obstacle regions at a particular time t, then based on the definition of the *obstacle avoidance unit vector*, $P_i(t)$ will be a nonempty set containing the obstacles, whose obstacle regions have the coleader inside them. For example in Fig. 3, suppose the coleader A, as a result of moving to its left-hand side, enters the intersection (shadowed) of the obstacle regions of O_1 and O_3, then $P_i(t)=\{O_1,O_3\}$. Now we can define $\xi_i(t)$ as follows:

Let $\rho_{ji}(t)$ be the distance vector between the center c_{oj} of the obstacle $O_j=C(c_{oj},R_j)$ and $p_i(t)$, defined as

$$\rho_{ji}(t)=p_i(t)-c_{oj}$$

Further define the normalized distance vector

$$\bar{\rho}_{ji}(t)=\rho_{ji}(t)/|\rho_{ji}(t)| \quad (8)$$

Then the obstacle avoidance vector $\xi_i(t)$ is defined as

$$\xi_i(t)=\frac{\sum_{O_j\in P_i(t)}\bar{\rho}_{ji}(t)}{\left|\sum_{O_j\in P_i(t)}\bar{\rho}_{ji}(t)\right|} \quad (9)$$

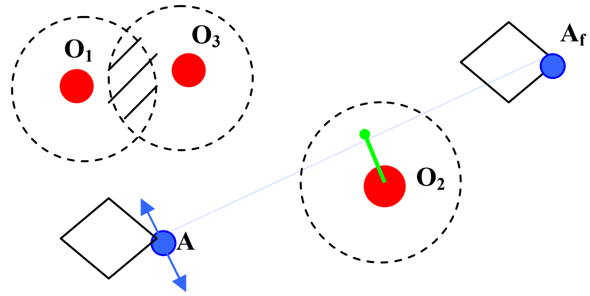

Fig.3. Illustration of the Line-of-Sight Obstacle Avoidance idea

B. Nearest Effective Obstacle Avoidance

Once again, the laws presented in Section III are modified in order to provide control actions to avoid obstacle collision. These modifications are in the form of introducing a new class of control vectors, local repulsive vectors, $\overline{\psi}_{ik}$, and a global repulsive vector, ψ_i. The local repulsive vectors are defined as

$$\overline{\psi}_{ik}=\begin{cases}0 & |p_i-c_{ok}|>\varepsilon_{\psi}\\ (p_i-c_{ok})/|p_i-c_{ok}| & |p_i-c_{ok}|\le\varepsilon_{\psi}\end{cases} \quad (10)$$

where p_i is the position of agent A_i, c_{ok} is the center position of the center of the nearest obstacle O_k to the agent A_i, and ε_{ψ} is a distance threshold. Unlike in the line-of-sight approach presented in the previous subsection, where the obstacle repulsive force only affects agents with positive DOFs, here, the local repulsive vector affects all agents.

As such, each obstacle is modeled to have a repulsive vector to keep the whole formation away, but only the repulsive vector of the *effective obstacle* is taken into consideration by the formation, the concept of the effective obstacle being defined below. Obstacle repulsive vectors are calculated by the following equation in which ψ_i is the repulsive vector of the effective obstacle, and R is the radius of the smallest circle surrounding the formation:

$$\psi_i=\begin{cases}0 & c_o\notin\Sigma(c_F,c_{Fd}) \text{ or } |c_o-c_F(t)|\ge 2R+R_o\\ \eta\dfrac{(c_F(t)-c_o)}{|c_F(t)-c_o|} & \textit{otherwise}\end{cases} \quad (11)$$

Here, c_o is the center position of the effective obstacle, and η is a design coefficient and takes a value between 0 and 1($\eta\approx 0.8$ in the simulations); further $c_F(t)$ and c_{Fd} denote the positions of the formation center of mass at time t and at the final desired position; $\Sigma(c_F,c_{Fd})$ is the rectangular region having sides parallel to the x and y axis of the global Cartesian coordinates and having c_F and c_{Fd} as two of its vertices. We also simulated using regions of other shapes.

The virtual relative target distance vector $\delta_{if}^{v}(t)$ (see Section V.A (7)) is redefined as

$$\delta_{if}^{v}(t) = \frac{\left(\bar{\delta}_{if}(t) + \psi_i(t) + \bar{\psi}_{ik}(t)\right)}{\left|\bar{\delta}_{if}(t) + \psi_i(t) + \bar{\psi}_{ik}(t)\right|} \qquad (12)$$

where $\bar{\delta}_{if}(t) = \delta_{if}(t) \big/ \left|\delta_{if}(t)\right|$.

To understand equation (12), let us consider the rectangular region $\Sigma(c_F, c_{Fd})$. Among the obstacles of interest, we call the one that is the closest to $c_F(t)$ and that is in $\Sigma(c_F, c_{Fd})$ as the *effective obstacle*. In Fig. 4, although R_3 is the smallest distance, O_3 is not considered as the effective obstacle because it is not in the rectangular region $\Sigma(c_F, c_{Fd})$.

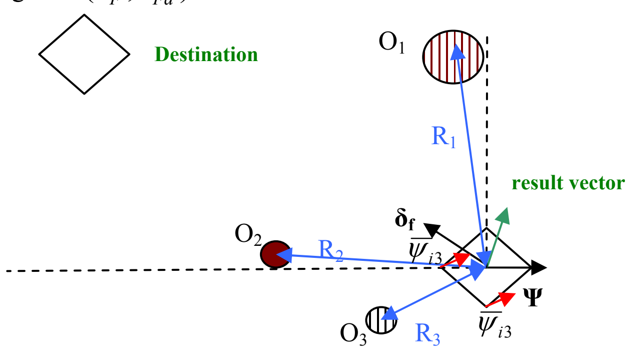

Fig.4. Illustration of the nearest effective obstacle avoidance approach

C. Simulation Results and Comparison

A simulation example is shown in Fig. 5, assuming point agents with velocity integrator dynamics and all the sensing and noiseless sensing and control. From the figure we can see that in the beginning of the motion, not all three coleaders have line-of-sight to their respective final destinations; thus they jointly move towards the right of the plane until time instant (t = 5sec). Then they all secure line-of-sight and so start moving towards the final destinations; prior to (t = 10 sec) they are pushed away because of the repulsive factor generated by a nearby obstacle and then they move straight again until reaching the final destination at (t = 18 sec). Note in this approach, we have implicitly assumed that the agents have the capability to sense the presence of all obstacles (shown as circular objects) in this example. These obstacles, shown as circular objects in the figure, indeed have to lie within the sensing radius of these agents in the real application.

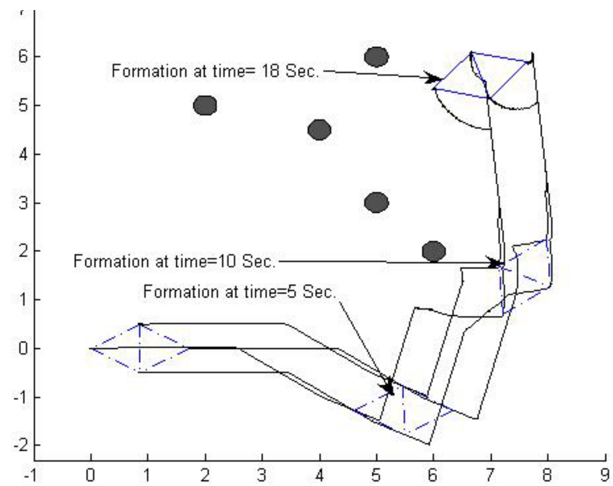

Fig.5. Simulation result for a formation with 3-coleader structure that is controlled using the line-of-sight approach

The same initial and final position scenario is tested with the nearest effective obstacle approach. The result is depicted in Fig. 6. From the figure (and in comparison to Fig. 5) we can see that this approach halves the time taken for the formation to reach its final destination. On the other hand, the shorter path and time is achieved at the cost of some level of formation deformation (shape change) during the motion. For instance, at (t = 5 sec) when the formation approaches two nearby obstacles, the agents in the same formation receive repulsive forces from different effective obstacles (one of the two), which results in deformation of the formation. Note that this phenomenon does not result in any inter-agent collision, the severity (measured by the changes in the formation shape) being determined by the layout of the obstacles. This is not desirable in applications where the formation rigidity is essential (i.e. inter-agent distances are required be constant over time).

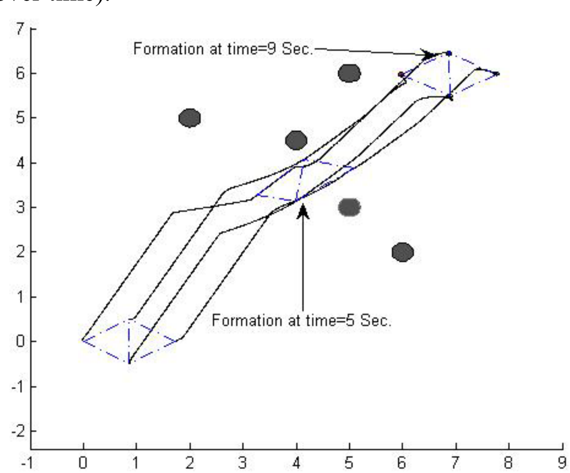

Fig.6. Simulation result for a formation with 3-coleader structure that is controlled using the nearest obstacle avoidance approach

VI. Conclusions

This paper has introduced a hierarchical control scheme for collision avoidance by a multiagent system in conjunction with moving the formation as a rigid entity from one part of the plane to another part, while avoiding multiple obstacles that may lie on a direct shortest path. The proposed approach utilizes the property that if there are a sufficient number of distances maintained between pairs of agents of a formation, so that the entire formation is kept as a rigid/persistent whole during any cohesive motion, then the possibility of inter-agent collision is avoided or at least diminished. Then the problem is reduced to motion control of the entire formation with obstacle avoidance, which is tackled using a modified virtual vector field approach.

Two simple strategies for avoiding the local minima problem in potential-function-based multiple obstacles avoidance problems have been presented. A simple simulation model assuming point agents with velocity integrator dynamics and static circular obstacles is investigated to demonstrate the effectiveness of the proposed schemes.

The control and sensing in the simulations are assumed to be noiseless. In real world applications, the scheme clearly has to be implemented to be robust to noise, and scalability and complexity are among other aspects to be considered. Future investigations and comparisons between the two approaches are required, to perhaps conclude if one is always superior to the other for a given performance metric or class of scenarios. This will allow the selection of an appropriate control scheme in advance and hence we obtain a scenario/pattern-based formation control paradigm.

Moreover, the provision of a formal stability analysis and proof of avoidance of a local minimum as a robust property are among immediate future tasks. Lastly, obstacle avoidance by means of splitting and merging of formations is yet another future problem.

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