Consequently, to avoid this outcome it is desirable to design an approach which can estimate the location of those UAVs which temporarily lose their GPS connection.

In this paper, we propose using range measurements in conjunction with an extended Kalman filter (EKF) to estimate the location of those UAVs temporarily losing their GPS connection. Specifically, we consider a group of UAVs with possibly different functionalities cooperating with each other to complete a mission, e.g., reconnaissance, video imaging, etc. These UAVs are able to measure distance between each other. During the flight, some UAVs may temporarily lose their GPS connection. However at any time, there are at least two UAVs, which are able to maintain their GPS connection. A UAV without GPS connection uses the proposed EKF, whose observations are the distance measurements to UAV with GPS navigation, to estimate its own location.

The rest of the paper is organized as follows. In Section 2 we discuss the general principle of localizing UAVs with range measurements. Section 3 introduces the design of the EKF. Section 4 introduces the UAV trajectories obtained from a recent trial conducted by DSTO in Australia with three UAVs. These data are used in the later simulation. Section 5 presents the simulation results. Finally, conclusions are given in Section 6.

2. LOCALIZING WITH RANGE MEASUREMENTS

Generally speaking, in 2D scenarios range measurements to three non-collinear nodes with known location (i.e., UAVs with known location) are required to uniquely determine the location of a node. When range measurements to only two nodes with known location are available, a situation referred to as flip ambiguity may occur. This is illustrated in Fig. 1.

For the specific problem considered in this paper, i.e., estimating the location of a UAV temporarily losing its GPS connection, the location of the UAV before it loses its GPS connection may be usable to resolve the flip ambiguity shown in Fig. 1. That is, the UAV location before GPS outage may help to determine on which side of the two known nodes (i.e., two UAVs with known location) the UAV is located. Therefore range measurements to two UAVs with known location may be sufficient to uniquely determine the location of a UAV temporarily losing its GPS connection. However it should
be noted that using range measurements to only two UAVs with known location, the flip ambiguity problem may still occur in some special cases. This is shown in Fig. 2. When this happens, additional information such as the maximum angular velocity of the UAV has to be considered to resolve the ambiguity.

In summary, depending on the movement pattern of the UAV and the geometric relationship between the UAV temporarily losing its GPS connection and the UAVs with GPS connection, in 2D range measurements to two UAVs with GPS connection may be sufficient to estimate the location of the UAV temporarily losing its GPS connection.

3. DESIGN OF THE EXTENDED KALMAN FILTER

It is assumed that the movement of the UAV in the vertical plane and in the horizontal plane can be decoupled [2]. This paper considers the movement of UAV in the horizontal plane only, i.e., it considers a 2D localization problem. This assumption is widely used to simplify the modeling of aircraft dynamics [2]. It is further justified by the fact that for the considered applications, the UAVs stay at approximately the same altitude.

The dynamic model of a UAV is given by the following continuous-time equations:

$$\dot{x}(t) = v(t) \cos \eta(t)$$  \hspace{1cm} (1)
$$\dot{y}(t) = v(t) \sin \eta(t)$$  \hspace{1cm} (2)
$$\dot{\eta}(t) = \omega(t)$$  \hspace{1cm} (3)
$$\dot{\omega}(t) = \varepsilon_\omega(t)$$  \hspace{1cm} (4)
$$\dot{v}(t) = \epsilon_v(t)$$  \hspace{1cm} (5)

where at time $t$ the state variables $\{x(t), y(t)\}$ are the coordinates of the UAV in the horizontal plane, $\eta(t)$ is the heading of the UAV, $\omega(t)$ is the angular speed of the UAV (i.e., the rate of change of the UAV heading measured in radians per second) and $v(t)$ is the ground speed of the UAV, i.e., a scalar. $\varepsilon_\omega(\cdot)$ and $\epsilon_v(\cdot)$ are assumed to be stationary, Gaussian, zero mean white noise processes, mutually independent, with known covariances $E[\varepsilon_\omega(t)\varepsilon_\omega(s)]$ and $E[\epsilon_v(t)\epsilon_v(s)]$ given by $Q_\omega \delta(t - s)$ and $Q_v \delta(t - s)$ respectively, $\delta$ is the Kronecker delta function. They are used to model the acceleration of the UAV speed caused by wind and control maneuver, etc. The values of $Q_\omega$ and $Q_v$ are derived from the maximum angular speed of the UAV and an empirical observation of the typical variations in the linear speed of the UAV respectively.

A discrete-time equation set corresponding to Eq. 1 through Eq. 5 will be an approximation because of the nonlinearities in the continuous-time model, and because of the treatment of noise terms. The easiest approximation is that based on an Euler approximation of the equations. This is a well known procedure for deterministic equations, but is also the basis for approximating stochastic equations, and is grounded in Itô calculus [4]. The discrete-time UAV dynamic model is shown in the following.

$$x_{k+1} = x_k + v_k \Delta t_k \cos \eta_k$$  \hspace{1cm} (6)
$$y_{k+1} = y_k + v_k \Delta t_k \sin \eta_k$$  \hspace{1cm} (7)
$$\eta_{k+1} = \eta_k + \omega_k \Delta t_k$$  \hspace{1cm} (8)
$$\omega_{k+1} = \omega_k + \varepsilon_\omega,k$$  \hspace{1cm} (9)
$$v_{k+1} = v_k + \epsilon_v,k$$  \hspace{1cm} (10)

The discrete-time state vector is $\theta_k = [x_k, y_k, \eta_k, \omega_k, v_k]^T$. It should approximate the value of the continuous time state vector at the $k^{th}$ distance measurement time, call it $t_k$. $\Delta t_k$ is the time interval between the $k + 1^{th}$ and $k^{th}$ distance measurement updates. The sequences $\{\varepsilon_\omega,k\}$ and $\{\epsilon_v,k\}$ are stationary zero mean white Gaussian sequences of random variables, mutually independent. The covariance of the first of these discrete-time sequence is [6]:

$$E[\varepsilon_\omega,k\varepsilon_\omega,j] = Q_\omega \Delta t_k \delta_{kj}.$$  \hspace{1cm} (11)

Similarly the covariance for the $\{\epsilon_v,k\}$ process is:

$$E[\epsilon_v,k\epsilon_v,j] = Q_v \Delta t_k \delta_{kj}.$$  \hspace{1cm} (12)

This means that one can replace Eq. 9 and Eq. 10 by:

$$\omega_{k+1} = \omega_k + \gamma_{\omega,k} \sqrt{\Delta t_k}$$  \hspace{1cm} (13)
$$v_{k+1} = v_k + \gamma_{v,k} \sqrt{\Delta t_k}.$$  \hspace{1cm} (14)
The sequences \( \{ \gamma_w,k \} \) and \( \{ \gamma_v,k \} \) are mutually independent, stationary zero mean white Gaussian sequences of random variables, with covariances \( Q_w \) and \( Q_v \) respectively. The advantage of this form is that it displays dependence of discrete-time noise on the interval between successive updates.

In summary, the discrete-time dynamic model of the UAV is taken as in Eq. 15.

\[
\theta_{k+1} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix} \theta_k + \begin{pmatrix}
\Delta t_k \cos \eta_k \\
\Delta t_k \sin \eta_k \\
0 \\
0 \\
\end{pmatrix}
\]

The measurement equation is:

\[
d_k = \sqrt{\left( x_k - x_0 \right)^2 + \left( y_k - y_0 \right)^2} + \varepsilon_d,k
\]

where \( \{ x_0, y_0 \} \) are the coordinates of the UAV from which the \( k^{th} \) distance measurement is received and \( \{ \varepsilon_d,k \} \) is a stationary zero mean white Gaussian sequence of random variables, which is used to model the distance measurement error. This sequence is assumed to be independent of \( \{ \varepsilon_w,k \} \) and \( \{ \varepsilon_v,k \} \). Note that in real-world application, it is possible that more than one distance measurements is received from multiple UAVs at approximately the same time. In that case the measurement equation can be expanded to include multiple equations like Eq. 16. In this paper, we only consider the more generic situation that at any time, only one distance measurement is received.

Given the non-linear dynamic equation shown in Eq. 15 and the non-linear measurement equation shown in Eq. 16, an estimate of the UAV location \( \{ x_k, y_k \} \) as \( k \) varies can be obtained using the extended Kalman filter [5]. Both the dynamic equation and the measurement equation are linearized by keeping the first order term in the Taylor series expansion and ignoring the higher order terms. The procedure of the EKF is shown in the following for completeness.

- **State prediction:**
  \[
  \hat{\theta}_{k+1|k} = f[\hat{\theta}_{k|k}],
  \]
  where the function \( f \) is defined in Eq. 15.
- **Computes the state prediction covariance matrix:**
  \[
  P_{k+1|k} = f_{x}[\hat{\theta}_{k|k}]P_{k|k}f_{x}^\prime[\hat{\theta}_{k|k}] + U_k Q U_k^\prime
  \]
  where \( U_k \) is defined in Eq. 15 and \( f_{x}[\hat{\theta}_{k|k}] \) is the Jacobian of \( f \) evaluated at \( \hat{\theta}_{k|k} \).
- **Computes the predicted measurement:**
  \[
  \hat{d}_{k+1|k} = h[\hat{\theta}_{k+1|k}],
  \]
  where the function \( h \) is defined in Eq. 16.
- **Computes the measurement prediction covariance:**
  \[
  S_{k+1} = h_{x}[\hat{\theta}_{k+1|k}]P_{k+1|k}h_{x}^\prime[\hat{\theta}_{k+1|k}] + R,
  \]
  where \( h_{x}[\hat{\theta}_{k+1|k}] \) is the Jacobian of \( h \) evaluated at \( \hat{\theta}_{k+1|k} \).
- **Computes the filter gain:**
  \[
  W_{k+1} = P_{k+1|k}h_{x}[\hat{\theta}_{k+1|k}]\gamma_{k+1|k}
  \]
  where \( \gamma_{k+1|k} \) is called the innovation and defined as:
  \[
  \gamma_{k+1|k} = d_{k+1} - \hat{d}_{k+1|k}
  \]
- **Updates the state estimate:**
  \[
  \hat{\theta}_{k+1|k+1} = \hat{\theta}_{k+1|k} + W_{k+1}\gamma_{k+1|k}
  \]
  where \( \lambda_{k+1} \) is called the innovation and defined as:
  \[
  \lambda_{k+1} = d_{k+1} - \hat{d}_{k+1|k}
  \]
- **Updates the state covariance matrix:**
  \[
  P_{k+1|k+1} = P_{k+1|k} - W_{k+1}S_{k+1}W_{k+1}^\prime
  \]
  The initial values \( \hat{\theta}_{0|0} \), \( P_{0|0} \) can be chosen empirically or can be computed from the state of the UAV before GPS outage. The impact of the initial values on the state estimate normally vanishes exponentially fast [5].

### 4. The UAV Trajectories

The proposed approach is validated by simulation using real UAV trajectories. In this section, we give an introduction to the experimental data used in the simulation.

Three UAVs are considered, which are named UAV 1, UAV 2 and UAV 3 respectively. During the flight, the following GPS data were collected: time, geocentric latitude, geocentric longitude, altitude, time of arrival, pulse width, signal frequency and amplitude. The longitude, latitude and height (LLH) coordinates recorded by the UAV are not well suited for navigation and tracking problems because linear motion becomes non-linear in these coordinates. In comparison, a local coordinate system whose X and Y axes are in the local horizon and Z axis points to the local Zenith is much better suited and is the industry standard. Therefore, the geocentric latitude and longitude location information is first converted into a local coordinate system, which is shown in Fig. 3. \( XYZ \) is the local coordinate system. The origin of the local coordinate system is randomly chosen to be the starting location of UAV 1. In the figure, \( \phi \) and \( \gamma \) are the geocentric latitude and longitude respectively, \( \phi' \) is the geodetic latitude [7]. Refer to [8] for a detailed description on how to convert the LLH coordinates into local coordinates.

Figures 4 - 6 show the UAV paths in 2D during a flight of 1 hour and 50 minutes. The location of the UAV is estimated from GPS, hereby referred to as GPS location. In
the local coordinate system, the $z$ coordinates of all three UAVs vary within a range of $[-4m, 6m]$ during the flight. This small variation in the $z$ coordinate may be attributable to the Earth’s curvature and is ignored. It is also noticed that the GPS location of UAV 1 varies in a very small range. In the experiment, UAV 1 is simply a receiver installed on a stationary post. Therefore this variation in the GPS location of UAV 1 is only an artifact reflecting the GPS measurement error.

It is noted that for UAV 2, it may lose its GPS connection for a maximum interval of 54s. Fig. 7 shows the time interval between adjacent GPS measurements for UAV 2. This figure shows that the scenario, which motivates the research in this paper, may indeed occur in real application. Even in the benign environment of the experiment, hardware failure and satellite obscuration due to the UAV wings when it is banking can cause GPS outage.

5. SIMULATION

In this section, we demonstrate simulation results, which validate the proposed approach. As we shall now explain, these simulation results use real world data from a recent trial conducted by DSTO in Australia with three UAVs, for both simulation purposes, and the data is also used for validation.

At the present time, the UAVs are not equipped to determine inter-UAV distances; hence, as indicated in the introduction, loss of a GPS connection is likely currently to be fatal. For this reason, the real world data we use is entirely data obtained when all UAVs do actually have a GPS connection (except UAV 2 may temporarily lose its GPS connection as indicated in Fig. 7). From this data, we are able to simulate loss of a GPS connection and acquisition of inter-UAV distance measurements in the following way. A certain time series of intervals of synthetic GPS outage is postulated (as discussed in more detail below). During these intervals, inter-UAV distances are synthesized at discrete instants of time. This is done by taking the actual GPS measurements, determining the corresponding inter-UAV distance, and then adding on to the resulting value a Gaussian random variable with zero mean and standard deviation of $10m$. This is delivered to the algorithm as a (synthesized) inter-UAV distance, and the Kalman filter is run with this data.

Validation occurs by comparing the estimated UAV tracks delivered by the Kalman filter with those in the original real
The choice of \( P \) and \( Q \) is critical for the filter performance and should be chosen carefully based on an empirical estimate as 
\[
|P| = 1000, \quad |Q| = 1.
\]

However, a very large deviation of \( P \) has little impact on the filter performance; \( P \) is chosen to be 
\[
|P| = 1000, \quad |Q| = 1.
\]

Fig. 7: Time interval between adjacent GPS measurements for UAV 2.

In real applications, the value of \( R \) can be obtained via a priori calibration of the distance measurement equipment. The distance measurement can be obtained by a simple round trip timing mechanism.

Fig. 8 illustrates the performance of the proposed EKF. A total of 9,718 of distance measurements to UAV 1 and UAV 2 are made during the 6,618s interval.

The flip ambiguity problem shown in Fig. 2 did not occur in the simulation. It is considered that in addition to distance measurements, knowledge of the UAV dynamic model is also employed in the EFK. This knowledge of UAV dynamics may help to alleviate the effect of the flip ambiguity problem.

The UAV location obtained from GPS is used as the “true location” of the UAV. The path of UAV 3 starts from the rectangular on the right side of the figure. Apparently, the estimated location has larger error on this part of the figure. As time evolves, the estimated location gradually converges to the true location, which is evidenced by much less deviation from the true location on the left side of the figure. Fig. 9 and 10 shows the variation of error in \( \hat{x} \) (i.e., \( \hat{x} - x \)) and the variation of error in \( \hat{y} \) (i.e., \( \hat{y} - y \)) respectively.

Fig. 8: An illustration of the performance of the proposed EKF.

In the simulation, the first two state variables of the initial state vector are chosen to be the initial GPS location of UAV 3 and other state variables are chosen randomly. The initial value of \( P \) is chosen based on an empirical estimate as 
\[
|P| = diag(1000, 1000, 0.3, 0.01, 1).
\]

It is found that generally the choice of \( P \) has little impact on the filter performance; however a very large deviation of \( P \) from its true value does cause the divergence of the filter. The value of \( Q \) is chosen based on an empirical estimate as 
\[
|Q| = diag(0.0003, 10).
\]

The choice of \( Q \) is critical for the filter performance and \( Q \) should be chosen carefully based on an in-depth understanding of the UAV dynamics. The value of \( R \) is chosen to be 100.

Fig. 9: Variation of error in \( \hat{x} \) with Time.

The performance of the proposed EKF is measured by six metrics, i.e., the mean error of the estimated \( x \), \( E(\hat{x} - x) \), and
Table 1: Performance of the Extended Kalman Filter. Unit: m.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>$E(\hat{x} - x)$</th>
<th>$\sigma(\hat{x} - x)$</th>
<th>$E(\hat{y} - y)$</th>
<th>$\sigma(\hat{y} - y)$</th>
<th>$E(\hat{d})$</th>
<th>$\sigma(\hat{d})$</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>10.5836</td>
<td>46.3876</td>
<td>-10.9167</td>
<td>38.1720</td>
<td>38.8275</td>
<td>48.2123</td>
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<tr>
<td>2</td>
<td>8.7641</td>
<td>45.6735</td>
<td>-11.7851</td>
<td>37.2149</td>
<td>37.2149</td>
<td>47.6161</td>
</tr>
<tr>
<td>3</td>
<td>10.9220</td>
<td>47.6161</td>
<td>-11.8615</td>
<td>38.5942</td>
<td>39.2297</td>
<td>49.7761</td>
</tr>
<tr>
<td>4</td>
<td>10.1834</td>
<td>62.1998</td>
<td>-12.3481</td>
<td>40.1211</td>
<td>39.0115</td>
<td>64.9048</td>
</tr>
<tr>
<td>5</td>
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<td>39.2946</td>
<td>39.0629</td>
<td>52.1936</td>
</tr>
<tr>
<td>6</td>
<td>9.2793</td>
<td>45.4631</td>
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<td>38.9082</td>
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</tr>
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<tr>
<td>Average</td>
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<td>49.8001</td>
<td>-11.8909</td>
<td>38.4917</td>
<td>39.0629</td>
<td>51.6034</td>
</tr>
</tbody>
</table>

Fig. 10: Variation of error in $\hat{y}$ with Time.

the corresponding standard deviation of this error, $\sigma(\hat{x} - x)$, the mean error of the estimated $y$, $E(\hat{y} - y)$, and the corresponding standard deviation of the error, $\sigma(\hat{y} - y)$, the mean value of the distance between the estimated location and the true location, $E(\hat{d})$, where

$$\hat{d} = \sqrt{(\hat{x} - x)^2 + (\hat{y} - y)^2},$$

and the corresponding standard deviation $\sigma(\hat{d})$.

Table 1 shows the results from ten simulations repeated with different random seed. The last row shows the average result of the ten simulations. 9,718 location estimates are obtained in each simulation and the first 2,000 location estimates have been removed when calculating the performance metrics.

As shown in the Table, both the estimate of $x$ and the estimate of $y$ have a bias. As a reference, the value of $x$ varies within the range of $[-5,000, 1,000]$ and the value of $y$ varies within the range of $[-6,000, 3,000]$. These have been shown in Fig. 6. Therefore, the value of the bias is comparatively small. However, both biases have a fairly consistent trend in all ten simulations. $E(\hat{x} - x)$ is always around 9m and $E(\hat{y} - y)$ is always around $-11m$. Further study may be required to investigate the cause of the biases and their implications on the design of the EKF. All three values $\sigma(\hat{x} - x)$, $\sigma(\hat{y} - y)$ and $\sigma(\hat{d})$ are around $40 - 50m$. There are four possible sources that may contribute to the error:

- Distance measurement error. The standard deviation of the distance measurement error is 10m.
- Location error of GPS. This error has been shown in Fig. 4, which is around 10m.
- The effect of an inaccurate dynamic model.
- Inaccurate knowledge of the error covariance matrix $Q$ in Eq. 18.

6. Conclusion

In this paper, we proposed an EKF to estimate the location of the UAV using inter-UAV distance measurements. The proposed method may solve the practical problem that during a flight, a UAV may temporarily lose its GPS connection for a rather long time period.

We have shown that if a simple round trip timing mechanism is added to the UAVs, so that they can measure the range to other UAVs, they will be able to navigate in a GPS-denied environment as long as they are within the communication range of at least two other UAVs which have reliable GPS coordinates. The accuracy of the location estimate of the GPS-denied UAV depends on the relative position of the other UAVs but we have found that in a typical scenario in which the UAVs are separated approximately $5km$ apart and the standard deviation of the range measurement error is $10m$, the GPS-denied UAV can be localized to within $40m$ of its true location.

In our future work, we shall consider evaluating the optimality and robustness of the proposed EKF using extensive experimental data.

References