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Checking if controllers are stabilizing using closed-loop data

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***Abstract*— Suppose an unknown plant is stabilized by a known controller. Suppose also that some knowledge of the closed-loop system is available and on the basis of that knowledge, the use of a new controller appears attractive, as may arise in iterative control and identification algorithms, and multiple-model adaptive control. The paper presents tests using a limited amount of experimental data obtained with the existing known controller for verifying that introduction of the new controller will stabilize the plant.**

I. INTRODUCTION

Let $[P, C_0]$ be a feedback control interconnection. The symbols P and C_0 denote respectively the plant and the controller. The Multiple Input Multiple Output (MIMO) case is considered here. The transfer function $P(s)$ is not known while the transfer function $C_0(s)$ is known. The closed-loop interconnection $[P, C_0]$ is known to be internally stable and is available for experiments. Let C_1 denote a new controller which has been designed to replace C_0 in the loop. In this paper, we develop tests to check whether C_1 (instead of C_0) stabilizes the feedback loop. These tests are based on the knowledge of $C_0(s)$ and $C_1(s)$ and on data obtained from experiments on the closed-loop system $[P, C_0]$, but not directly on P. The tests are based on gross properties of the behaviour of the closed-loop, and so should exhibit significant tolerance of noise.

It should be noticed that many iterative control design methods have been developed to use closed-loop data obtained from an existing closed loop system in order to update the current controller with a controller with better performance [7], [8], [10]. Iterative data based control methods are mainly focused on the objective of performance improvement which is typically an objective competing with the robust stability of the designed closed loop [4], [11]. Therefore, alongside data based iterative control design methods a number of stability tests have been developed to ascertain stability of the new controller before implementing the controller in the loop. Existing tests are based either on the identification of a parametric 'full order' model of the current closed-loop transfer function or on the estimation of frequency bounds on the magnitude of the current closed-loop transfer function [3], [6], [9], [17].

One may argue that a mismatch exists between the nature of these tests and their usual application. Iterative methods as [7], [8], [10], [17] are based on limited closed loop experiments which are intended to obtain information for the design of small controller changes, see also [1], [2], [5], [12]. The existing validation tests are based on the identification of the full dynamics of the current closed-loop system. Hence the amount of experimental effort required for validation purposes, can apparently be much larger that the amount of experimental effort required for the design of the controller update. In contrast to this fact we will show in Section IV that our validation test requires gathering of information only on a limited known frequency region whose size depends on the size of the controller change. Hence the experimental effort is linked to the size of the controller update.

In this paper we put forward the use of phase information. Our validation tests rely on estimating the phase of the current closed-loop transfer functions. The use of the phase information to ascertain closed-loop stability derives from the Nyquist stability criterion and leads to validation tests which assess necessary and sufficient stability conditions. This is in contrast with methods based on magnitude bounds from which only sufficient conditions can be derived. We will show that our validation experiments have can reflect the limitation on the size of the controller update imposed by the closed loop experimental setting. In particular it will be shown that if the controller change has limited size then it is sufficient to obtain an estimate of the phase of the current closed loop system only up to a certain known finite frequency. This fact makes the validation tests practical from the experimental point of view.

The paper is organized as follows. In Section II we recall coprime factors representations and stability results in this framework. In this work we adopt coprime factors representations because they allow us to obtain very neat statements and simple derivations. In Section III we present the result which defines the experimental setting for a stability test based on phase information. Some stability falsification and validation tests are derived in Section IV. Numerical illustrations and conclusions complete the paper.

II. COPRIME FACTOR REPRESENTATIONS AND STABILITY

We shall denote by $\mathscr{H}_\infty$ the space of functions bounded and analytic in the open right-half complex plane, and the same function spaces with prefix $\mathscr{R}$ their real-rational proper subspaces. The plant is assumed to be a MIMO linear time-invariant system with m inputs and p outputs. The transfer

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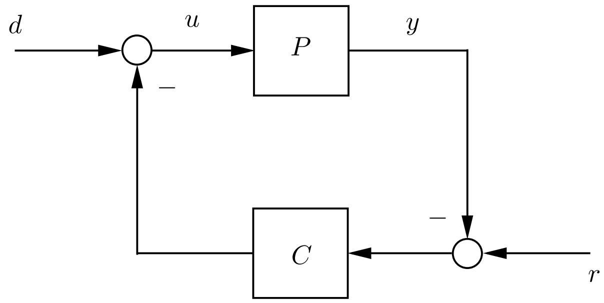


Fig. 1. Standard Feedback Configuration

function of the plant belongs to $\mathscr{R}^{m\times p}$,the set of real rational transfer functions, and is denoted by P. The transfer function of the controller is denoted by C. In this work we will use coprime factor representations of P and C, and without further comment we adopt as a standing assumption that the plant and all controller transfer functions are always proper. Hence, in this section we collect definitions and stability results related to this representation.

Definition 1: The interconnection $[P,\, C]$ (Fig. 1) is "well-posed" if the transfer function matrix mapping $\left[\begin{smallmatrix} r\\ d\end{smallmatrix}\right]$ to $\left[\begin{smallmatrix} y\\ u\end{smallmatrix}\right]$ exists. Put another way, $[P,\, C]$ is well-posed if $(I-CP)^{-1} \in \mathscr{R}$. In this case, these four transfer functions can be written as

$$\begin{bmatrix} y\\ u\end{bmatrix} = \begin{bmatrix} P\\ I\end{bmatrix}(I-CP)^{-1}\begin{bmatrix} -C & I\end{bmatrix}\begin{bmatrix} r\\ d\end{bmatrix} = H(P,C)\begin{bmatrix} r\\ d\end{bmatrix}.$$

Definition 2: The interconnection $[P,\, C]$ is said to be "internally stable" if it is well-posed and $H(P,C)\in\mathscr{RH}_\infty$; i.e., each of the four transfer functions in $\left[\begin{smallmatrix} r\\ d\end{smallmatrix}\right] \mapsto \left[\begin{smallmatrix} y\\ u\end{smallmatrix}\right]$ belongs to $\mathscr{RH}_\infty$.

Definition 3: The ordered pair $\{N, M\}$, with $M, N \in \mathscr{RH}_\infty$, is a right-coprime factorization (*rcf*) of $P \in \mathscr{R}$ if M is invertible in $\mathscr{R}$, $P = NM^{-1}$, and N and M are right-coprime over $\mathscr{RH}_\infty$. Furthermore, the ordered pair $\{N, M\}$ is a normalized *rcf* of P if $\{N, M\}$ is a *rcf* of P and $M^*M + N^*N = I$. □

Definition 4: The ordered pair $\{\tilde U, \tilde V\}$, with $\tilde U, \tilde V \in \mathscr{RH}_\infty$, is a left-coprime factorization (*lcf*) of $C\in\mathscr{R}$ if $\tilde V$ is invertible in $\mathscr{R}$, $C = \tilde V^{-1}\tilde U$, and $\tilde U$ and $\tilde V$ are left-coprime over $\mathscr{RH}_\infty$. Furthermore, the ordered pair $\{\tilde U, \tilde V\}$ is a normalized *lcf* of C if $\{\tilde U, \tilde V\}$ is a *lcf* and $\tilde V\tilde V^* + \tilde U\tilde U^* = I$. □

Then, we define

$$G := \begin{bmatrix} N\\ M\end{bmatrix}, \tag{1}$$

$$\tilde K := \begin{bmatrix} -\tilde U & \tilde V\end{bmatrix}, \tag{2}$$

where G and will be referred to as the graph symbols of P, and $\tilde K$ will be referred to as the inverse graph symbol of C. Then the following results hold.

Theorem 5: [16, Proposition 1.9] Let G and $\tilde K$ be defined as in (1) and (2). Then the following are equivalent:

a) $[P,\, C]$ is internally stable;
b) $(\tilde KG)^{-1}\in\mathscr{RH}_\infty$;
c) $\det(\tilde KG)(j\omega)\neq 0\ \forall\omega$ and $\operatorname{wno}\det(\tilde KG) = 0$. □

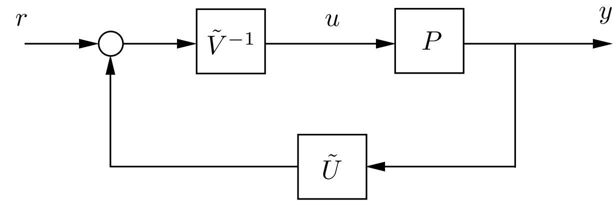


Fig. 2. Controller Implementation with $C = \tilde V^{-1}\tilde U$

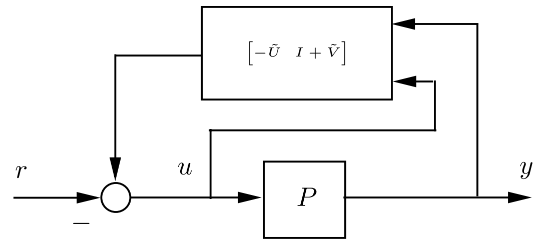


Fig. 3. Alternative Implementation of $C = \tilde V^{-1}\tilde U$

In this work we will also refer to the "Observer-form implementation" of the controller, see [16, Chapter 5]. In this form the factor $\tilde V^{-1}$ of C is implemented in the feed-forward path and the factor $\tilde U$ of C is implemented in the feedback path as depicted in Fig. 2 (in Figures 2, 3 and 4 we omit the signal d because it is not relevant to the discussion).

This is typically done in order for the poles and zeros of the controller not to impose restrictions on the response from r to y. Simple manipulations show that the controller equation can also be rewritten as:

$$u = \begin{bmatrix} -\tilde U & I+\tilde V\end{bmatrix}\begin{bmatrix} y\\ u\end{bmatrix} - r$$

which is depicted in Fig. 3. This figure shows why this configuration is referred to as the observer-form.

III. Experimental setting for the stability tests

The following theorem defines the experimental setting for the stability tests proposed in this paper.

In the theorem we will refer to the *unwrapped* phase of a transfer function which is the phase of the frequency response when it is in the form of a continuous function of the frequency [14].

Theorem 6: Let $[P, C_0]$ be internally stable. Let $C_0 = \tilde V_0^{-1}\tilde U_0$ and $C_1 = \tilde V_1^{-1}\tilde U_1$ be left coprime factorizations over $\mathscr{RH}_\infty$. Consider the configuration in Fig. 4 and define T to be

$$T = [-\tilde U_1 \quad \tilde V_1]\begin{bmatrix} P(I-C_0P)^{-1}\\ (I-C_0P)^{-1}\end{bmatrix}\tilde V_0^{-1}$$

i.e. the mapping $T : r \to z$ in Fig. 4.

Let arg denote the unwrapped phase. Then the following are equivalent:

a) $[P, C_1]$ is internally stable;
b) $T^{-1}\in\mathscr{RH}_\infty$;
c) $\det T(j\omega)\neq 0\ \forall\omega$ and $\operatorname{wno}\det T = 0$;
d) $\det T(j\omega)\neq 0\ \forall\omega$ and $\arg\det T(j\infty) = \arg\det T(j0)$.

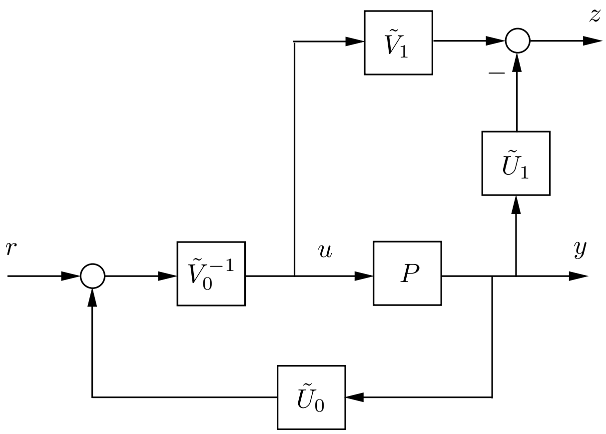


Fig. 4. Experimental setting

Proof We have that $T = (\tilde{K}_1 G)(\tilde{K}_0 G)^{-1}$. The proof is completed by noticing that: (b) and (Theorem 5 part b) are equivalent since $(\tilde{K}_0 G)$, $(\tilde{K}_0 G)^{-1} \in \mathscr{RH}_\infty$; (c) and (Theorem 5 part c) are equivalent since $\{[P, C_0]$ is internally stable $\Leftrightarrow$ $\{\det(\tilde{K}_0 G)(j\omega) \neq 0\ \forall\omega$ and $\operatorname{wno}\det(\tilde{K}_0 G) = 0\}$ and $\operatorname{wno}\det(T) = \operatorname{wno}\det(\tilde{K}_1 G) - \operatorname{wno}\det(\tilde{K}_0 G)$; (d) and (c) are equivalent because $T \in \mathscr{RH}_\infty$ and is bi-proper and therefore $\operatorname{wno}\det(T) = \mathscr{Z}(T) = \frac{1}{\pi}[\arg\det T(j\infty) - \arg\det T(j0)]$ where $\mathscr{Z}(T)$ denotes the number of open RHP zeros of T. $\square$

If the plant P is unknown, one cannot explicitly construct the transfer function T in closed-form. However, the stable mapping from r to z (resulting from $T : r \to z$) can be studied in a safe experiment, i.e. one where no instability can occur, as shown in Fig. 4. Even though we do not have an explicit characterization of T when P is unknown, the reference signal r and the computed output signal z (computed as a filtered version of the measured signals $\left[\begin{smallmatrix} y \\ u \end{smallmatrix}\right]$ via $\tilde{K}_1$) can be used to infer the required properties of T.

In this work we adopt the observer form implementation of the controller depicted in Fig. 2. If one is concerned in having to split up the physical controller in two coprime factors before injecting the reference signal, then the following implementation will circumvent the concerns.

Let $\left[\begin{smallmatrix} X \\ Y \end{smallmatrix}\right]$ be a right inverse of $\begin{bmatrix} -\tilde{U}_0 & \tilde{V}_0 \end{bmatrix}$ (i.e., in other words, let $P_0 = XY^{-1}$ be some plant that stabilizes $C_0 = \tilde{V}_0^{-1}\tilde{U}_0$ and satisfies the corresponding Bezout identity. Note that P_0 does not have to be an estimate of P). Then, it is easy to see in Fig. 5 that

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix} r \quad \text{and} \quad \begin{bmatrix} y \\ u \end{bmatrix} = H(P, C_0) \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}.$$

Since $H(P, C_0) = G(\tilde{K}_0 G)^{-1}\tilde{K}_0$, it easily follows that

$$\begin{bmatrix} y \\ u \end{bmatrix} = G(\tilde{K}_0 G)^{-1} r$$

as is the mapping from r to $\left[\begin{smallmatrix} y \\ u \end{smallmatrix}\right]$ in Fig. 2 and Fig. 3. Note that the requirement $\begin{bmatrix} -\tilde{U}_0 & \tilde{V}_0 \end{bmatrix} \left[\begin{smallmatrix} X \\ Y \end{smallmatrix}\right] = I$ can be relaxed to $\begin{bmatrix} -\tilde{U}_0 & \tilde{V}_0 \end{bmatrix} \left[\begin{smallmatrix} X \\ Y \end{smallmatrix}\right] = Z$ where Z is a unit in $\mathscr{RH}_\infty$ since the

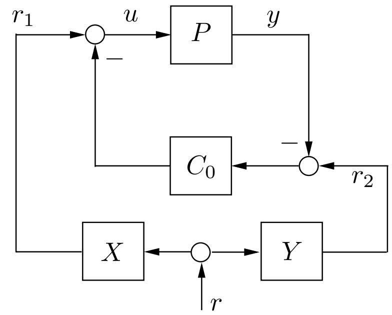


Fig. 5. Alternative experimental setting

transfer function from r to $\left[\begin{smallmatrix} y \\ u \end{smallmatrix}\right]$ then becomes

$$\begin{bmatrix} y \\ u \end{bmatrix} = G(\tilde{K}_0 G)^{-1} Z r = G(\hat{\tilde{K}}_0 G)^{-1} r$$

with $\hat{\tilde{K}} = Z^{-1}\tilde{K}_0$, i.e. only changing the particular coprime factor representation of the controller.

An interesting observation is that there are several plants P_0 that stabilize C_0 and furthermore there are several coprime factorizations of $P_0 = Y^{-1}X$. This choice can be used in the synthesis of X and Y to determine the frequency and bandwidth characteristics of the physical reference signals r_1 and r_2. This facilitates the experiment by allowing the engineer to control the excitation characteristics of the feedback interconnection via alteration of the frequency characteristics of the reference signals r_1 and r_2.

IV. Data-based stability tests

In this section we develop data-based stability tests based on the experimental setting defined in Section III. The tests aim at verifying condition (d) in Theorem 6. We introduce the following assumptions.

Assumption 7: The factors $\tilde{V}_0$ and $\tilde{V}_1$ are such that $\tilde{V}_0(j\infty) = \tilde{V}_1(j\infty) = I$. $\square$

Assumption 8: The transfer functions PC_0 and PC_1 are strictly proper. $\square$

Assumption 7 is without loss of generality and assumption 8 captures a typical situation. Notice that the transfer function T can be written as

$$T = \tilde{V}_1(I - C_1 P)(I - C_0 P)^{-1}\tilde{V}_0^{-1}. \tag{3}$$

Hence under Assumptions 7 and 8 we have that

$$\det T(j\infty) = \frac{\det \tilde{V}_1(j\infty)}{\det \tilde{V}_0(j\infty)} \frac{\det(I - C_1 P)(j\infty)}{\det(I - C_0 P)(j\infty)} = 1\,.$$

Therefore $\det T(j\infty)$ is strictly positive and known and will be used as a datum for the verification of condition (d) in Theorem 6.

To start with we have the following *falsification* test based on step responses.

Theorem 9: Let the suppositions of Theorem 6 and Assumptions 7 and 8 hold. Let e_i denote a reference signal where a step is applied at the $i-$th input while the other inputs are kept at 0. Perform m experiments with reference signal $r(t) = e_i(t), i = 1, \ldots, m$ up to steady state conditions. Let $\bar{z}_i$ be the steady state output of T recorded in each experiment and define $\bar{Z} = [\bar{z}_1, \ldots, \bar{z}_m]$. Then

$$[P, C_1] \text{ is internally stable } \Rightarrow \det \bar{Z} > 0\,.$$

Therefore if $\det \bar{Z} \leq 0$, stability of $[P, C_1]$ is falsified.

Proof A necessary condition for condition (d) in Theorem 6 to hold true is that $\det T(j0)$ and $\det T(j\infty)$ have the same sign. By the final value Theorem we have

$$\bar{Z} = [\bar{z}_1 \ \bar{z}_2, \ \cdots \ \bar{z}_m] = \lim_{s\to 0} s\left[T(s)\frac{1}{s}\right] = T(j0)\,.$$

Hence $\det T(j0) = \det \bar{Z}$. The proof is completed by noticing that if condition (d) in Theorem 6 holds true, then $\det T(j0)$ must have the same sign of $\det T(j\infty)$, which was set to be 1 without loss of generality. □

The experimental test devised above is quite simple to carry out; it simply consists in recording the steady states of m step responses. However such an experiment can only be used to check a necessary stability condition.

Condition (d) in Theorem 6 can be verified in both its necessary and sufficient parts by using more sophisticated identification techniques. In principle, one could inject a white noise signal r or a full sine sweep, measure the corresponding output z and compute the *full* frequency response for T. However, this is not practical and hence one needs to determine an alternative, smarter, experiment. The key point that has to be noticed in designing the experiment is that there is no need to estimate the full frequency response of T but what is instead needed is to measure its frequency response up to a certain finite frequency ω_0. The measurement can tolerate significant error, as its purpose is simply to allow computation of a certain phase change. A way to estimate ω_0 can be worked out from the structure of the transfer function T. We have the following result.

Lemma 10: Let the suppositions of Theorem 6 hold. Then the transfer function T has the following expression.

$$T = I + T' \tag{4}$$

$$T' = [-(\tilde{U}_1 - \tilde{U}_0) \quad (\tilde{V}_1 - \tilde{V}_0)] \begin{bmatrix} P(I - C_0P)^{-1} \\ (I - C_0P)^{-1} \end{bmatrix} \tilde{V}_0^{-1}. \tag{5}$$

Proof The expression for T' is derived as follows

$$\begin{aligned} T' &= \tilde{K}_1 G(\tilde{K}_0 G)^{-1} - I \\ &= (\tilde{K}_1 - \tilde{K}_0)G(\tilde{K}_0 G)^{-1} \\ &= \left[-(\tilde{U}_1 - \tilde{U}_0) \quad (\tilde{V}_1 - \tilde{V}_0)\right] \begin{bmatrix} P \\ I \end{bmatrix} (I - C_0P)^{-1}\tilde{V}_0^{-1} \end{aligned}$$

The last expression coincides with (5). □

The expression for the transfer function T presented in the lemma shows that T is the sum of a known term (i.e. I) and a term which, under Assumptions 7 and 8, is strictly proper. Hence it can be expected that measuring the frequency response of T up to a frequency where the response of T' has vanished is enough to characterize the full frequency response of T. This fact is illustrated in the following theorem specialized for the SISO case.

Theorem 11: Let the suppositions of Theorem 6 and Assumption 7 and 8 hold. Let P be a SISO transfer function. Let $\omega_0 \in [0, \infty)$ be a frequency such that $|T'(j\omega)| \leq 1 \ \forall \omega \geq \omega_0$ i.e.

$$\left|\tilde{V}^{-1}(j\omega)\right| \left|\frac{-(\tilde{U}_1 - \tilde{U}_0)P + (\tilde{V}_1 - \tilde{V}_0)}{1 - C_0P}(j\omega)\right| \leq 1 \ \forall \omega \geq \omega_0. \tag{6}$$

Then the condition

$$T(j\omega) \neq 0 \ \forall \omega \text{ and } \pi\left[\frac{\arg T(j\omega_0)}{\pi}\right] = \arg T(j0) \tag{7}$$

where $[\cdot]$ denotes the closest integer, is equivalent to condition (d) in Theorem 6.

Proof The proof consists in showing that

$$\arg T(j\infty) = \pi\left[\frac{\arg T(j\omega_0)}{\pi}\right]. \tag{8}$$

Lemma 10 shows that $T(j\infty) = I$ under Assumption 7 and 8. Hence, in the SISO case, the inequality

$$|T(j\omega) - 1| \leq 1 \quad \forall\, \omega \geq \omega_0 \tag{9}$$

certainly implies (8). Inequality (9) is equivalent to (6). □

Note that since $T(j\infty) = I$, then $\arg T(j\infty)$ is an integer multiple of 2π and hence both $\pi[\arg T(j\omega_0)/\pi]$ and $\arg T(j0)$ needs to be an integer multiple of 2π for condition (7) to hold. The two theorems presented in this section outline experimental tests to assess stability of $[P,\, C_1]$ before inserting controller C_1 in the loop. Theorem 9 holds for the MIMO case and implies a very simple experiment which consists in recording the steady state value of m step responses. The outcome of the test can only be used to falsify stability of $[P,\, C_1]$. Theorem 11 holds for the SISO case and implies the estimation of the frequency response of the current closed loop system up to a certain frequency ω_0. The Theorem states a necessary and sufficient condition for the stability of $[P,\, C_1]$. For the application of Theorem 11 it is important to note that under Assumption 7 and 8 the left hand side of inequality (6) tends to zero as ω tends to infinity. In practice, it is reasonable to assume that one has a rough estimate of the bandwidth of the current closed-loop $[P,\, C_0]$ which can then be used be used to obtain a possibly conservative estimate of ω_0 by assuming that the left hand side of inequality (6) remains below one over some known high-frequency region. Notice that the left-hand side of inequality (6) depends on the size of the controller change. A small controller change certainly implies a smaller frequency ω_0 and hence reduced experimental effort. The estimate of the frequency response of T up to frequency ω_0 can be obtained using either parametric or non parametric estimation methods [13][15]. The unwrapped phase can be obtained with phase unwrapping techniques [14]. It seems that Theorem 11 extends to the MIMO case quite readily and the remaining question is how to easily device an experiment and compute and interpret the corresponding MIMO results.

V. Simulation Examples

In this section, we consider a MIMO system and a SISO system to illustrate the advantages and effectiveness of the stability tests proposed in Theorem 6 and Theorem 11. Although the theorems do not assume that the plant is known, for the sake of simulation the underlying unknown plants are given.

A. Example 1: A MIMO System

Let the unknown plant, $P \in \mathscr{R}^{2\times 2}$, be given by

$$P = \frac{1}{s^2+2s+4}\begin{bmatrix} -(s-2) & 2(s+0.5) \\ -3 & -(s-2) \end{bmatrix}$$

and let C_0 be a stabilizing controller, $[P,\, C_0] \in \mathscr{RH}_\infty$, given by

$$C_0 = \frac{2(s+2)(s^2+2s+4)}{s(s+1)(s^2+2s+7)}\begin{bmatrix} (s-2) & 2(s+0.5) \\ -3 & (s-2) \end{bmatrix}$$

with a left coprime factorization, $C_0 = \tilde{V}_0^{-1}\tilde{U}_0$,

$$\tilde{V}_0 = \frac{(s+1)}{(s^2+3.89s+3.8)(s^2+1.94s+2.58)(s^2+2.03s+4.07)}\begin{bmatrix} \tilde{V}_0^{11} & \tilde{V}_0^{12} \\ \tilde{V}_0^{21} & \tilde{V}_0^{22} \end{bmatrix}$$

$$\tilde{U}_0 = \frac{(s+2)(s^2+2s+4)}{(s^2+3.89s+3.8)(s^2+1.94s+2.58)(s^2+2.03s+4.07)}\begin{bmatrix} \tilde{U}_0^{11} & \tilde{U}_0^{12} \\ \tilde{U}_0^{21} & \tilde{U}_0^{22} \end{bmatrix}$$

$$\begin{aligned}
\tilde{V}_0^{11} &= -0.22s(s^2+4.72s+6.01)(s^2+2.24s+4.51) \\
\tilde{V}_0^{12} &= 0.71s(s+2.03)(s^2+1.98s+3.8) \\
\tilde{V}_0^{21} &= 0.27s(s-3.12)(s+2.04)(s^2+2s+3.9) \\
\tilde{V}_0^{22} &= -0.71s(s+1.93)(s+0.2)(s^2+2.02s+4.14) \\
\tilde{U}_0^{11} &= -0.437(s+1.65)(s^2+1.31s+1.81) \\
\tilde{U}_0^{12} &= -0.872(s+1.88)(s^2+1.96s+2.94) \\
\tilde{U}_0^{21} &= 0.545(s+2.36)(s^2+2.44s+3.74) \\
\tilde{U}_0^{22} &= -0.341(s+1.62)(s^2+0.78s+2.35)\ .
\end{aligned}$$

Theorem 6 puts forward a solution to the problem of checking in advance using collected closed-loop data if the controller C_1 given here by

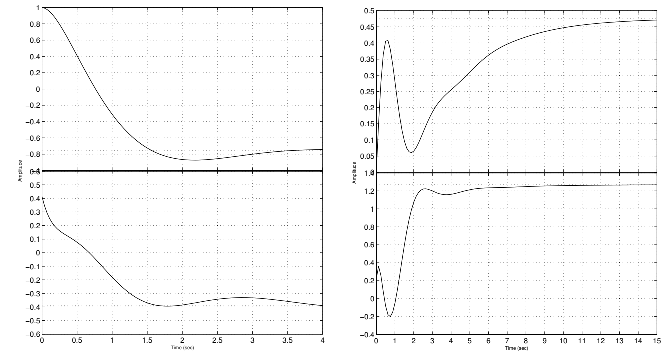
$$C_1 = \frac{2(s^2+2s+4)(s-2)}{(s^2+s+1)(s^2+2s+7)}\begin{bmatrix} (s-2) & 2(s+0.5) \\ -3(s-0.33) & (s-2) \end{bmatrix}$$

with a left coprime factorization, $C_1 = \tilde{V}_1^{-1}\tilde{U}_1$,

$$\tilde{V}_1 = \frac{(s^2+s+1)(s^2+2s+7)}{(s+3.15)(s+2.04)(s+1.85)(s+0.34)(s^2+2.04s+4.09)}\begin{bmatrix} \tilde{V}_1^{11} & \tilde{V}_1^{12} \\ \tilde{V}_1^{21} & \tilde{V}_1^{22} \end{bmatrix}$$

$$\tilde{U}_1 = \frac{(s-2)(s^2+2s+4)}{(s+3.15)(s+2.04)(s+1.85)(s+0.34)(s^2+2.04s+4.09)}\begin{bmatrix} \tilde{U}_1^{11} & \tilde{U}_1^{12} \\ \tilde{U}_1^{21} & \tilde{U}_1^{22} \end{bmatrix}$$

$$\begin{aligned}
\tilde{V}_1^{11} &= -0.22(s+2.95)(s+2.06) \\
\tilde{V}_1^{12} &= -0.13(s+1.93) \\
\tilde{V}_1^{21} &= -0.03(s+7.43)(s+1.97) \\
\tilde{V}_1^{22} &= -0.16(s+1.83)(s+0.64) \\
\tilde{U}_1^{11} &= -0.43(s-1.015)(s+1.8)(s+0.42) \\
\tilde{U}_1^{12} &= -0.87(s+2.01)(s+3.54)(s+0.27) \\
\tilde{U}_1^{21} &= 0.88(s-3.6)(s+1.85)(s+0.33) \\
\tilde{U}_1^{22} &= -0.43(s+0.13)(s^2+2.92s+2.17)\ .
\end{aligned}$$



(a) $r(t) = step(t)\cdot e_1$ (b) $r(t) = step(t)\cdot e_2$

Fig. 6. Step Responses

We setup the experimental configuration of Fig. 4 and perform two experiments with reference signals $r(t) = step(t)\cdot e_1$ and $r(t) = step(t)\cdot e_2$. The step responses are shown in Fig. 6 and the steady state of $T: r \to z$ are given in

$$\bar{Z} = \begin{bmatrix} -0.75 & 0.476 \\ -0.391 & 1.27 \end{bmatrix}$$

with $\det(\bar{Z}) = -0.7664 < 0$ and hence the stability of $[P,\, C_1]$ is falsified. Indeed, computing $H(P,\, C_1)$ shows that it has three RHP poles which conforms with the results.

B. Example 2: A SISO system

This example demonstrates the effectiveness of the stability tests proposed in Theorem 11 when the results of Theorem 6 stops shorts of unfalsifying the proposed controller C_1.

Let the unknown SISO plant be given by

$$P = \frac{-186.66(s-5)(s+4.5)}{(s+10)^2(s+7)(s+6)}$$

and let C_0 be a stabilizing controller, $[P,\, C_0] \in \mathscr{RH}_\infty$, given by

$$C_0 = \frac{0.021(s+10.92)(s+8.87)(s+7.31)(s+5.93)}{(s^2+8.6s+19.84)(s^2-0.603s+5.34)}$$

with a left coprime factorization, $C_0 = \tilde{V}_0^{-1}\tilde{U}_0$,

$$\tilde{V}_0 = \frac{(s^2+8.603s+19.84)(s^2-0.602s+5.34)}{(s^2+8.64s+19.97)(s^2+1.83s+6.96)}$$

satisfying Assumption 7, $\tilde{V}_0(j\infty) = 1$, and

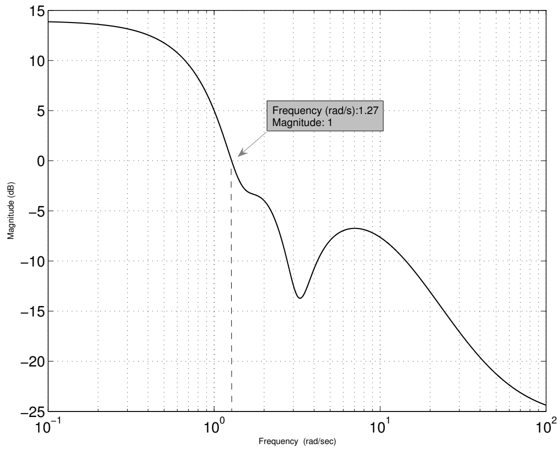
$$\tilde{U}_0 = \frac{0.021(s+10.92)(s+8.87)(s+7.31)(s+5.93)}{(s^2+8.64s+19.97)(s^2+1.83s+6.96)}\ .$$

Suppose that the data collected from the closed-loop suggests the use of a new controller C_1 given by

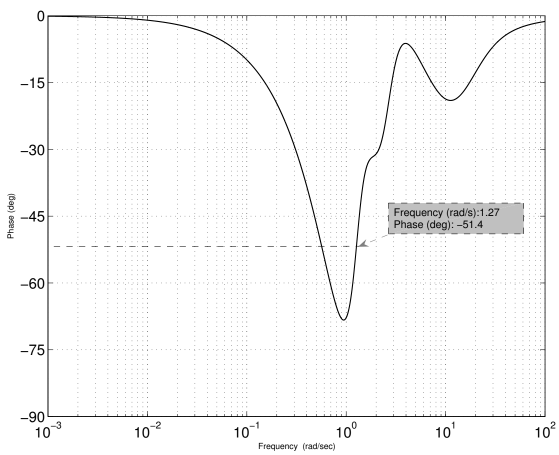
$$C_1 = \frac{0.33(s+0.586)(s+2.99)(s+3.416)}{(s+2)(s^2+2.26s+3.52)}$$

with a left coprime factorization, $C_1 = \tilde{V}_1^{-1}\tilde{U}_1$,

$$\tilde{V}_1 = \frac{(s+2)(s^2+2.26s+3.52)}{(s+1.87)(s^2+2.81s+3.712)}$$



(a) Magnitude Response of $T' = T - 1$



(b) Phase Response of T

Fig. 7. Example 2: Magnitude and Phase responses

satisfying Assumption 7, $\tilde{V}_1(j\infty) = 1$, and

$$\tilde{U}_1 = \frac{0.33(s+0.586)(s+2.99)(s+3.416)}{(s+1.87)(s^2+2.81s+3.712)} .$$

Setting up the experimental configuration of Fig. 4 for simulation and utilizing Theorem 6 to check if C_1 is stabilizing, we perform experiments with reference signal $r(t) = step(t)$ and the step response is measured at the output z. The steady state of $T : r \to z$ is $\bar{z} = 4.74 > 0$ which does not falsify the stability of $[P, C_1]$. Thus, we shall use the results of Theorem 11 to check if C_1 is stabilizing. As shown in Fig. 7a, the simulation reveals that $|T - 1| \leq 1 \ \forall \omega \geq 1.27 \ rad/s$. Given that $\arg T(j0) = 0$ and $\arg T(j\omega_0) = -0.285\pi$ as shown in Fig. 7b, then the condition in Theorem 11 holds and hence C_1 is stabilizing. Indeed, computing $H(P, C_1)$ shows that C_1 is stabilizing.

VI. Conclusions

We have proposed tests for MIMO and SISO systems to validate for stability the closed-loop system formed by a controller and a plant, whose exact transfer function is not known, a priori of the actual physical connection of the controller to the plant. The tests assume that the plant is connected to a stabilizing controller and that the resulting closed loop system is available for experiments. The general framework for our validation tests has been established in Theorem 6. The result of Theorem 11 shows, for the SISO case, that our validation tests require to gather information on the frequency response of the current control system over a limited bandwidth. Current research effort is set on the extension of this result to the MIMO case. We will also investigate which system identification methods are more appropriate to gather the information required.

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