Proceedings of the 45th IEEE Conference on Decision & Control
Manchester Grand Hyatt Hotel
San Diego, CA, USA, December 13-15, 2006

Adaptive Source Localization by Mobile Agents

Sandra H. Dandach, Barış Fidan, Soura Dasgupta and Brian D. O. Anderson

Abstract— **The problem of source localization has assumed increased importance in recent years. Applications include locating the source of a cellular phone signal, sensor localization or satellite localization. In this paper we formulate a continuous time adaptive localization algorithm that permits a mobile agent to estimate the location of a stationary source using only the measured distance from the source. The algorithm is shown to be exponentially asymptotically stable, under a persistent excitation condition that has an intuitively appealing interpretation. Exponential asymptotic stability indicates the ability of the algorithm to track modest movements of the source.**

I. INTRODUCTION

Over the last few years the problem of source localization has assumed increasing significance, [1]. Specifically source localization refers to an agent estimating the precise location of a source using information related to the relative position of the agent and the source. This information can be of different kinds, for example distance information, bearing information, power level information (which is indirectly related to distance) and even time difference of arrival (TDOA) information, where two agents are involved. *In this paper, we will focus on distance estimates only.* Examples where source localization is important are many. Thus for example a base station in a cellular communications network may have to estimate the location of a phone transmitting within its geographic region of coverage. Similarly, in sensor networks, groups of sensors may have to estimate the location of an object or a node to facilitate such tasks as routing, rescue, target tracking and to ensure proper network coverage.

At the outset, we note that such distances can be estimated through two possible means. The first, which we call active measurement, involves a source that actively emits a signal. In this case the signal intensity at the point of arrival at the agent location, together with characteristics of the propagation medium provides a distance estimate. Alternatively, in passive distance measurements an agent transmits signals of its own, and estimates the distance by measuring the time it takes for this signal reflected off the source to return.

There are two major strands characterizing research in this area. The first involves clusters of stationary agents that collaborate to localize a given source. In two dimensions this would generically require that at least three distinct non-collinearly situated agents use their distances from the source they seek to localize. To be precise, with just two agents, the source position can be determined to within a binary ambiguity. Occasionally, *a priori* information may be available which will resolve that ambiguity. Otherwise, a third agent needs to be involved. In three dimensions, one generically needs at least four agents that do not lie on the same two dimensional plane. A number of papers have proposed collaborative localization algorithms under a variety of assumptions concerning the manner in which distances are estimated, [2]-[6]. In the second strand a single mobile agent, exploits its motion to achieve localization, [7]-[9].

This paper belongs to the second category. It would seem relatively straightforward to achieve localization of a source with a mobile agent: one simply needs to take one distance measurement, move the agent, take another distance measurement, and then move it again to a position that is not collinear with the first two, and take a third measurement (and in three dimensions, a fourth measurment after another move). However, there are disadvantages to this, principally stemming from the fact that measurements are likely to be contaminated by noise, and from the fact that the source may move while the agent is moving to its new position. To address these concerns, we formulate a *continuous time algorithm* that adaptively localizes a source through known agent motion, in three dimensions. As noted, the first motivation for adaptation, as opposed to a stand alone batch algorithm stems from the need to achieve robust localization, given noisy measurements. A more sophsiticated argument will be used to show that we can localize not only stationary sources, but also those that undergo slow, but possibly persistent movements, albeit with some error.

In section II we formulate a continuous time algorithm that achieves exponentially fast localization of a stationary source under a *persistent excitation* (p.e.) condition on the path of the moving agent. To understand the significance of this algorithm we shall argue in section II that in continuous time, distance measurements together with their derivatives, should suffice under the right conditions to secure source localization. However, while distance measurements are directly available, their derivatives are not. Nor is it desirable to perform explicit differentiation, as poor noise

This work was supported by, US NSF grant ECS-0225530, an Australian Research Council Discovery Projects Grant and by National ICT Australia, which is funded by the Australian Government's Department of Communications, Information Tehcnology and the Arts and the Australian Research Council through the *Backing Australia's Ability* initiative and the ICT Centre of Excellence Program.

S. Dandach is with Dept. of Mechanical and Environmental Engineering, University of California, Santa Barbara, CA 93106, USA. sandra@engineering.ucsb.edu

B. Fidan and B. D. O. Anderson are with Australian National University, and National ICT Australia Ltd, Locked Bag 8001, Canberra ACT 2601 Australia. Baris.Fidan and Brian.Anderson@nicta.com.au

S. Dasgupta is with Dept. of Electrical and Computer Engineering, University of Iowa, Iowa City, IA 52242, USA. dasgupta@engineering.uiowa.edu

performance will ensue. The algorithm of this paper avoids such an explicit differentiation, and thus localizes without noise amplification. In section III we show that this algorithm can track a source experiencing slow, bounded but potentially persistent drift. In section IV we provide an intuitively appealing interpretation of the p.e. condition of section II. Section V provides simulation results. Section VI is the conclusion.

II. THE LOCALIZATION ALGORITHM

We deal with a source positioned at a location whose coordinates in three dimensions are in a 3×1 vector x. An agent whose coordinates at time t are in a 3×1 vector $y(t)$, must estimate x from the knowledge of its own position and its distance from the source

$$d(t) = \|y(t) - x\|.$$

In the sequel the following standing assumption will hold:

Assumption 2.1: The agent trajectory $y(t)$ is bounded and differentiable, as is its velocity $\dot{y}(t)$, and its acceleration $\ddot{y}(t)$. In particular for all t and positive M_i,

$$\|y(t)\| \leq M_1, \tag{II.1}$$

and

$$\|\dot{y}(t)\| \leq M_2. \tag{II.2}$$

This assumption ensures that the motion of the agent can be executed with finite force.

Now observe that with x a constant

$$\frac{d}{dt}\{d^2(t)\} = 2\dot{y}(t)^T(y(t) - x). \tag{II.3}$$

Thus, should the derivative of $d(t)$ and $\dot{y}(t)$ be available and $\dot{y}(t)$ span $\mathbb{R}^3$, the sensor location x can be estimated. Pursuing such an approach however would require explicit differentiation of measured signals with accompanying noise amplification. To avoid such explicit differentiation we invoke instead the device of state variable filtering, popularly employed in the adaptive systems literature, [10]. A novelty of our approach lies in the fact that while in the adaptive systems literature state variable filtering has generally involved signals that are linearly related, in this context the underlying relationships are nonlinear.

Indeed we consider the signals $\eta(t)$, $m(t)$ and $V(t)$, that are respectively the state variable filtered versions of $d^2(t)/2$, $\|y(t)\|^2/2$ and $y(t)$. These are given in (II.4 - II.9): For $\alpha > 0$ generate,

$$\dot{z}_1(t) = -\alpha z_1(t) + \frac{1}{2}d^2(t), \tag{II.4}$$

$$\eta(t) = -\alpha z_1(t) + \frac{1}{2}d^2(t), \tag{II.5}$$

$$\dot{z}_2(t) = -\alpha z_2(t) + \frac{1}{2}y^T(t)y(t), \tag{II.6}$$

$$m(t) = -\alpha z_2(t) + \frac{1}{2}y^T(t)y(t), \tag{II.7}$$

$$\dot{z}_3(t) = -\alpha z_3(t) + y(t) \tag{II.8}$$

and

$$V(t) = -\alpha z_3(t) + y(t). \tag{II.9}$$

Measurements $d(t)$ and the localizing agent's position $y(t)$, suffice to generate $\eta(t)$, $m(t)$ and $V(t)$ without any explicit differentiation. Assumption 2.1 ensures that all signals in (II.4)-(II.9) are bounded.

In the sequel we denote p as the derivative operator, i.e. $p \triangleq d/dt$. Further we say

$$a(t) \approx b(t),$$

if $a(t) - b(t)$ decays exponentially to zero. Then noting the stability of (II.4 - II.9) and the exponentially decaying nature of the relevant initial conditions we have in operator notation that,

$$\eta(t) \approx \frac{p}{p+\alpha}\left\{\frac{1}{2}d^2(t)\right\}, \tag{II.10}$$

$$m(t) \approx \frac{p}{p+\alpha}\left\{\frac{1}{2}y^T(t)y(t)\right\}, \tag{II.11}$$

and

$$V(t) \approx \frac{p}{p+\alpha}\{y(t)\}. \tag{II.12}$$

Then we establish the following key relationship between η, $m(t)$, $V(t)$ and x that assumes that x is a constant.

$$\begin{aligned} \eta(t) &\approx \frac{p}{p+\alpha}\left\{\frac{1}{2}d^2(t)\right\} \\ &\approx \frac{1}{p+\alpha}\left\{\dot{y}^T(t)(y(t)-x)\right\} \\ &\approx \frac{p}{p+\alpha}\left\{\frac{1}{2}y^T(t)y(t)\right\} - \left(\frac{p}{p+\alpha}\left\{y^T(t)\right\}\right)x \\ &\approx m(t) - V^T(t)x. \end{aligned} \tag{II.13}$$

Observe that (II.13) mirrors (II.3), but involves only signals whose generation requires no explicit differentiation. We now present the adaptive localization algorithm: With $\hat{x}(t)$ the estimate of x, choose for an adaptation gain $\gamma > 0$:

$$\dot{\hat{x}}(t) = -\gamma V(t)(\eta(t) - m(t) + V^T(t)\hat{x}(t)). \tag{II.14}$$

Define

$$\tilde{x}(t) = \hat{x}(t) - x. \tag{II.15}$$

Then because of (II.13), (II.14) becomes:

$$\dot{\tilde{x}}(t) \approx -\gamma V(t)V^T(t)\tilde{x}(t). \tag{II.16}$$

Localization will require that $\tilde{x}$ converge to zero. We then have the following theorem.

Theorem 2.1: Suppose assumption 2.1 holds and x is constant. Then with $\eta(t)$, $m(t)$ and $V(t)$ defined in (II.4) to (II.9), (II.16) is exponentially asymptotically stable if and only if there exist $\alpha_1, \alpha_2, T > 0$ such that for all $t \geq 0$

$$0 < \alpha_1 I \leq \int_t^{t+T} V(\tau)V^T(\tau)d\tau \leq \alpha_2 I. \tag{II.17}$$

Proof: It is well known, see e.g. [11], that the linear time varying system

$$\dot{z}(t) = -\gamma V(t)V^T(t)z(t) \qquad \text{(II.18)}$$

is exponentially asymptotically stable iff (II.17) holds. Hence the result follows.

■

The condition (II.17) is the celebrated p.e. condition. In section IV we interpret this condition further, and demonstrate that it is in accord with the physical intuition that underlies the localization problem. For the moment we pause to consider the broader implications of exponentially stability itself. It is well known in the adaptive systems literature [14] that such exponential convergence makes the algorithm robust to modest departures from idealizing assumptions. Certainly it will ameliorate the effects of noise perturbing the distance measurements. In the next section we show that in fact it also permits (II.14) to track slow, bounded though possibly large and persistent drift in the source location.

We may also use alternatives to (II.14) once (II.13) has been noted. For example, one could use for positive definite Γ, and nonnegative scalar γ_1,

$$\dot{\hat{x}}(t) = -\frac{\Gamma V(t)(\eta(t) - m(t) + V^T(t)\hat{x}(t))}{1+\gamma_1 V^T(t)V(t)}, \qquad \text{(II.19)}$$

which too exponentially localizes under (II.17), [12]. Thus, though the remaining sections will analyze (II.14), most of the analysis will directly translate to (II.19) as well.

III. Tracking drift

The analysis in the previous section assumed a stationary source specifically by assuming that x, the source location, is a constant vector. The fact that under the p.e. condition (II.17) the localization algorithm is exponentially asymptotically stable suggests the possibility of coping with departures from idealizing assumptions. One such departure of particular practical import is when the assumption of a stationary source is dropped. Rather, the source may experience slow but persistent drift that results in significant movement from its original position. One notes that should this drift eventually cease then the fact that under the conditions of theorem 2.1 convergence to this terminal position will occur, is almost immediate. Instead we treat here the case that the source potentially never ceases to move.

Specifically, we make the following assumption on source motion.

Assumption 3.1: The source trajectory $x(t)$ is bounded and differentiable, and its velocity $\dot{x}(t)$ is "small". In particular for all t and positive M_3 and ϵ,

$$\|x(t)\| \le M_3, \qquad \text{(III.1)}$$

and

$$\|\dot{x}(t)\| \le \epsilon. \qquad \text{(III.2)}$$

Observe that even though this assumption constrains the drift in the source to be bounded and slow, the net extent of the drift is permitted to be substantial. We remark that the bound (III.1) is also reasonable. Under assumption 3.1, $y(t)$ remains bounded for all time. If (III.1) failed, then the distance measurements from the agent to the source would be arbitrariliy large, and possibly along asymptotically parallel lines. In the limit, this is like having collinear measurement points that fundamentally impair the ability to localize without ambiguity.

Since the stationarity assumption on the source has now been relaxed, (II.13) must also be accordingly modified. Indeed we now have:

$$\begin{aligned}
\eta(t) &\approx \frac{p}{p+\alpha}\left\{\frac{1}{2}d^2(t)\right\} \\
&\approx \frac{1}{p+\alpha}\left\{(\dot{y}(t)-\dot{x}(t))^T(y(t)-x(t))\right\} \\
&\approx \frac{p}{p+\alpha}\left\{\frac{1}{2}y^T(t)y(t)\right\} - \left(\frac{1}{p+\alpha}\left\{\dot{y}^T(t)x(t)\right\}\right) \\
&\quad -f(t) \\
&\approx m(t) - \left(\frac{1}{p+\alpha}\left\{\dot{y}^T(t)x(t)\right\}\right) - f(t) \qquad \text{(III.3)}
\end{aligned}$$

where

$$f(t) = \left(\frac{1}{p+\alpha}\left\{\dot{x}^T(t)(y(t)-x(t))\right\}\right). \qquad \text{(III.4)}$$

Thus because of assumptions 2.1 and 3.1

$$|f(t)| \le F(t) \approx \frac{(M_1+M_3)\epsilon}{\alpha}. \qquad \text{(III.5)}$$

Focus now on the second term in (III.3). There holds:

$$\begin{aligned}
\frac{1}{p+\alpha}\left\{\dot{y}^T(t)x(t)\right\} &\approx e^{-\alpha t}\int_0^t e^{\alpha\tau}\dot{y}^T(\tau)x(\tau)d\tau \\
&= e^{-\alpha t}\left[\left(\int e^{\alpha\tau}\dot{y}(\tau)d\tau\right)^T x(\tau)\right]_0^t \\
&- e^{-\alpha t}\int_0^t\left(\int e^{\alpha\tau}\dot{y}(\tau)d\tau\right)^T\dot{x}(\tau)d\tau \\
&\approx \left(\int_0^t e^{-\alpha(t-\tau)}\dot{y}(\tau)d\tau\right)^T x(t) \\
&- e^{-\alpha t}\int_0^t\left(\int e^{\alpha\tau}\dot{y}^T(\tau)d\tau\right)\dot{x}(\tau)d\tau \\
&= \left(\int_0^t e^{-\alpha(t-\tau)}\dot{y}(\tau)d\tau\right)^T x(t) \\
&\quad -G(t) \\
&\approx V^T(t)x(t) - G(t),
\end{aligned}$$

where because of assumptions 2.1 and 3.1, $G(t)$ obeys

$$\begin{aligned}
|G(t)| &\le e^{-\alpha t}M_2\epsilon\int_0^t\int e^{\alpha\tau}d\tau d\tau \\
&\approx \frac{M_2\epsilon}{\alpha^2}.
\end{aligned}$$

Thus for a suitable K_1 depending only on M_1 to M_3 and α,

$$\begin{aligned}
|\eta(t) - m(t) + V^T(t)x(t)| &\le M(t) \\
&\approx K_1\epsilon. \qquad \text{(III.6)}
\end{aligned}$$

Then in view of Theorem 2.1 we have the following result.

Theorem 3.1: Suppose assumptions 2.1 and 3.1 hold, and there exist $\alpha_1, \alpha_2, T > 0$ such that for all $t \geq 0$ (II.17) holds. Then with $\eta(t)$, $m(t)$ and $V(t)$ defined in (II.4) to (II.9), $\hat{x}(t)$ given by (II.14) obeys for some K obtained from M_1 to M_3, α, T, α_1 and α_2,

$$\limsup_{t\to\infty} |\hat{x}(t) - x(t)| = K\epsilon.$$

We note that K itself depends on the convergence rate of (II.18), which according to [13] increases linearly with α_1 and declines quadratically with α_2 and linearly with T. Specifically, for all t,

$$\frac{\|x(t+T)\|^2 - \|x(t)\|^2}{\|x(t)\|^2} \leq \frac{4\alpha_1}{1+\alpha_2+\alpha_2^2}.$$

What theorem 3.1 shows is that under the p.e. condition in (II.17), one can track sustained but bounded drift in the source provided the drift is sufficiently slow, underscoring the robustness of the proposed localization algorithm. Section IV provides a physical interpretation of this p.e. condition.

IV. Persistent excitation

In this section we explore the meaning of the p.e. condition in (II.17). First observe that in three dimensions an agent cannot generically localize without ambiguity if its motion is confined to a plane, as there would be at least two points separated by this plane that may provide the same distance measurements. An exception arises when there is an additional level of nongenericity in that not only is the motion of the agent confined to a plane, but the source lies in the same plane. Similarly in two dimensions generically the motion cannot be exclusively collinear. Since the source location is unknown, for all practical purposes planar agent motion in three dimensions and collinear motion in two dimensions should be avoided. We now quantify the relationship between avoiding planar motion and satisfying (II.17).

To this end we first relate the p.e. condition on $V(t)$ to one on $\dot{y}(t)$, by exploiting techniques used to establish transfer of excitation conditions developed in [15] for adaptive identification and control problems. From (II.12), we have

$$\dot{V}(t) + \alpha V(t) = \dot{y}(t). \qquad \text{(IV.1)}$$

We are interested in showing that (II.17) holds iff a p.e. condition holds on $\dot{y}(t)$. The transfer of excitation results of [15] do not directly apply to this setting as they involve scalar inputs and furthermore require that the system relating the two signals be proper, which the V to $\dot{y}$ system is not.

To prove this result we invoke the following specialization of an inequality from [16], which has been used before in establishing transfer of excitation relations in [15].

Lemma 4.1: Suppose on a closed interval $\mathcal{I}$ of length Δ, a signal $w(t)$ is twice differentiable and for some ϵ and M'

$$|w(t)| \leq \epsilon \text{ and } |\ddot{w}(t)| \leq M' \quad \forall \quad t \in \mathcal{I}.$$

Then for some M independent of ϵ, $\mathcal{I}$ and M', and $M'' = \max(M', 2\epsilon\Delta^{-2})$ one has:

$$|\dot{w}(t)| \leq M(M''\epsilon)^{1/2} \quad \forall \quad t \in \mathcal{I}.$$

Then we have the following theorem.

Theorem 4.1: Suppose assumption 2.1 and (IV.1) hold. Then there exist $\alpha_1, \alpha_2, T > 0$ such that (II.17) holds for all $t \geq 0$ iff there exist $\beta_1, \beta_2, \bar{T} > 0$ such that for all $t \geq 0$

$$0 < \beta_1 I \leq \int_t^{t+\bar{T}} \dot{y}(\tau)\dot{y}(\tau)^T d\tau \leq \beta_2 I. \qquad \text{(IV.2)}$$

Proof: The upper bounds hold because of assumption 2.1 and the stability of the operator in (IV.1). We will thus focus on the two lower bounds by proving that the violation of one is equivalent to the violation of the other.

Suppose the lower bound in (IV.2) is violated. Then for all $\epsilon > 0$ and $T > 0$, there exists a t_0 and $\|\theta\| = 1$, such that

$$\int_{t_0}^{t_0+T} (\theta^T \dot{y}(\tau))^2 d\tau \leq \epsilon^2.$$

Thus from lemma 4.1 for some M, all $\epsilon > 0$, some $T_1(\epsilon)$, dependent only on the bound on $\ddot{y}(t)$ and ϵ, and all $T > T_1(\epsilon)$, there exists a t_0 and $\|\theta\| = 1$, for which

$$|\theta^T \dot{y}(t)| \leq M\epsilon^{1/2} \quad \forall t \in [t_0 + T_1(\epsilon), t_0 + T].$$

Thus because of (IV.1),

$$\left|\frac{d}{dt}\{\theta^T V(t)\} + \alpha\theta^T V(t)\right| \leq M\epsilon^{1/2} \quad \forall t \in [t_0+T_1(\epsilon), t_0+T].$$

As (IV.1) is exponentionally stable, for some K and T_2 dependent only on M and α, all $\epsilon > 0$, and all $T > T_1(\epsilon) + T_2$,

$$|\theta^T V(t)| \leq K\epsilon^{1/2}, \quad \forall t \in [t_0 + T_1(\epsilon) + T_2, t_0 + T].$$

This violates the lower bound of (II.17). Now suppose that the lower bound in (II.17) is violated. Then for all $\epsilon > 0$ and $T > 0$, there exists a t_0 and $\|\theta\| = 1$, such that

$$\int_{t_0}^{t_0+T} (\theta^T V(\tau))^2 d\tau \leq \epsilon^2.$$

Thus from lemma 4.1 for some M, all $\epsilon > 0$, some $T_1(\epsilon)$, dependent only on the bound on $\dot{V}(t)$ and ϵ, and all $T > T_1(\epsilon)$, there exists a t_0 and $\|\theta\| = 1$, for which

$$|\theta^T V(t)| \leq M\epsilon^{1/2}, \quad \forall t \in [t_0 + T_1(\epsilon), t_0 + T].$$

As $\ddot{y}(t)$ is bounded so is $\ddot{V}(t)$. Thus again from lemma 4.1 for some L, all $\epsilon > 0$, some $T_2(\epsilon)$, dependent only on the bounds on $\dot{V}(t)$, $\ddot{V}(t)$ and ϵ, and all $T > T_2(\epsilon)$, there exists a t_0 and $\|\theta\| = 1$, for which

$$|\theta^T \dot{V}(t)| \leq L\epsilon^{1/4}, \quad \forall t \in [t_0 + T_2(\epsilon), t_0 + T].$$

Consequently, because of (IV.1)

$$|\theta^T \dot{y}(t)| \leq \alpha M\epsilon^{1/2} + L\epsilon^{1/4}, \quad \forall t \in [t_0 + T_2(\epsilon), t_0 + T].$$

This violates the lower bound of (IV.2). ∎

Let us now tie (IV.2) to the avoidance of planar motion. Observe that should in particular the Gramian in (IV.2) be singular then because of assumption 2.1 for some unit θ, $\theta^T \dot{y}(t) = 0$ for all t. This in particular means that for this constant unit θ, there exists a constant C such that for all t, $\theta^T y(t) = C$ defining a planar motion. The p.e. condition in (IV.2) effectively requires that the localizing agent avoid in some sense planar motion. In particular we have the following tangible connection that among other things allows the verification of the p.e. condition from $y(t)$ alone. The proof is omitted due to space constraints.

Theorem 4.2: Under assumption 2.1, there exist $\alpha_1, \alpha_2, T > 0$ such that (II.17) holds for all $t \geq 0$ iff there exists a $\bar{T}$ and $\beta > 0$ such that for all t, there exist $\{t_1, \cdots, t_4\} \in [t, t+\bar{T}]$, $t_i \leq t_{i+1}$, such that

$$|\det([y(t_2) - y(t_1), y(t_3) - y(t_1), y(t_4) - y(t_1)])| > \beta. \tag{IV.3}$$

Note that

$$|\det([y(t_2) - y(t_1), y(t_3) - y(t_1), y(t_4) - y(t_1)])| = 0$$

implies that $y(t_i)$ are coplanar. The parameter β in (IV.3) measures how close they are to be situated on a plane. In effect this result shows that the p.e. condition can be verified by checking whether on each interval of a fixed length, there are at least four time points at which the agent positions are sufficiently removed from any single plane, specifically by testing (IV.3).

V. SIMULATION RESULTS

In this section, we provide simulation results to demonstrate the performance of the localization algorithm in Section II. In these examples, unless otherwise stated, the adaptation gain $\gamma = 1$.

Assume throughout that the mobile agent is moving along the trajectory $y(t) = [2 + 2\sin t,\ 2\cos 2t,\ 2\sin 0.5t]^T$ (m), and $\alpha = 1$. Consider the case where the source is moving around a nominal location at $x = [2, 3, 2]^T$ (m) and its trajectory is given by $x(t) = [2 + \sin 0.005t,\ 3 + \cos 0.005t,\ 2]^T$ (m). The net drift in the first two coordinates is 2, and thus substantial. The rate of drift (i.e ϵ in assumption 3.1) however is relatively small having an amplitude of 0.005. We expect from Theorem 3.1 that the agent will track the source movement with an error proportional to 0.005. This is verified in Figure 1. The estimate $\hat{x}(t)$ (est(x)), tracks the motion of $x(t)$ with barely discernible error.

Now consider the case when the distance measurements $d(t)$ are perturbed by a zero mean bandlimited white Gaussian noise, and $x = [2, 3, 2]^T$. Figures 2 to 4 consider noise variance of 0.001 (m^2), 0.005 (m^2) and 0.1 (m^2), respectively. While the first two are with unity adaptation gains, the last is with a much reduced adaptation gain of 0.01. We note that a noise variance of .001 (m^2) corresponds to an average noise amplitude of about 0.03 (m), which given the scale of the problem in terms of the actual distances involved, is quite reasonable. As expected in these two examples the tracking error scales with the noise magnitude.

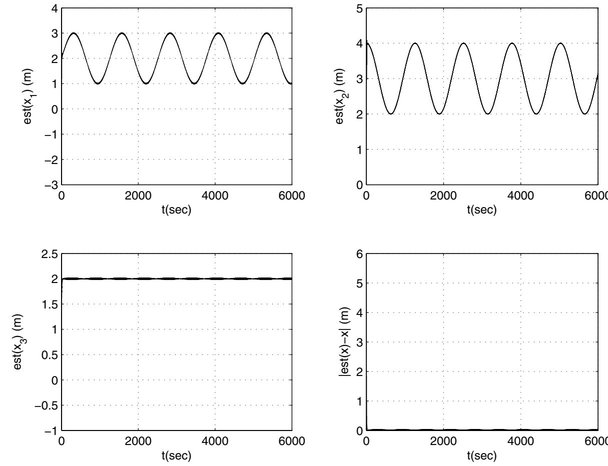

Fig. 1. Location estimation for $x(t) = [2 + \sin 0.005t,\ 3 + \cos 0.005t,\ 2]^T$ (m), $y(t) = [2 + 2\sin t,\ 2\cos 2t,\ 2\sin 0.5t]^T$ (m), $\alpha = 1$. The dashed curves correspond to the actual coordinates of the source and the solid curves show the estimate trajectories. Time scale 0-6000 seconds.

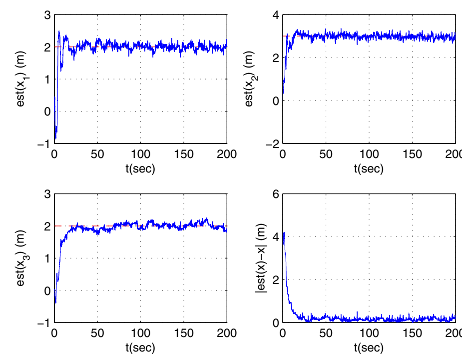

Fig. 2. Location estimation for $x = [2, 3, 2]^T$ (m), $y(t) = [2 + 2\sin t,\ 2\cos 2t,\ 2\sin 0.5t]^T$ (m), $\alpha = 1$. Noise in distance measurement with power .001 (m^2). The dashed lines correspond to the actual coordinates of the source and the solid curves show the estimate trajectories.

Figure 4 reveals the role of the adaptation gain. Thus in this example involving large noise, reduction of the adaptation gain to 0.01, reduces the effect of noise in the tracking error. The price is reduced convergence speed which impairs the ability to track source movements. This is to be expected, as a smooth noise performance requires a slow adaptation response, while the ability to track fast source movement requires fast adaptation. Nonetheless figure 5 shows that these competing requirements can be reconciled when the algorithm must track a slowly drifting source in the face of modest noise.

VI. CONCLUSION

In this paper we have presented a continuous time adaptive source localization algorithm that exploits an agent's mobility to localize a stationary source. Exponential asymptotic source localization has been shown to require the satisfaction of a persistent excitation condition, whose violation implies

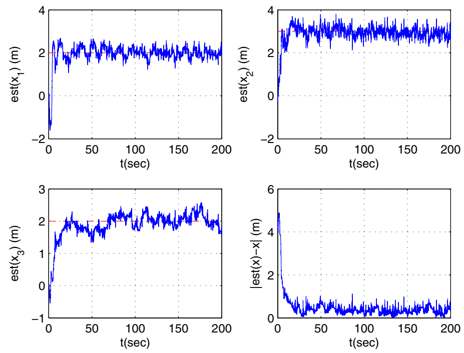

Fig. 3. Location estimation for $x = [2, 3, 2]^T$ (m), $y(t) = [2 + 2\sin t,\ 2\cos 2t,\ 2\sin 0.5t]^T$ (m), $\alpha = 1$. Noise in distance measurement with power .005 (m^2). The dashed lines correspond to the actual coordinates of the source and the solid curves show the estimate trajectories.

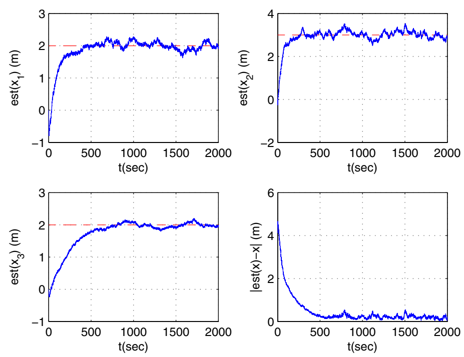

Fig. 4. Location estimation for $x = [2, 3, 2]^T$ (m), $y(t) = [2 + 2\sin t,\ 2\cos 2t,\ 2\sin 0.5t]^T$ (m), $\alpha = 1$. Noise in distance measurement with power 0.1 (m^2) and adaptation gain 0.01. The dashed lines correspond to the actual coordinates of the source and the solid curves show the estimate trajectories.

approximate planar agent motion. Under persistent excitation, the algorithm is shown to have the ability to track, slow, bounded but potentially persistent and nontrivial drift. Further this persistent excitation condition is independent of the source position and requires only that the agent avoid a planar motion sufficiently often and to a sufficient level.

Future direction of research includes tracking a mobile source that moves on a parameterized orbit, especially movement on the surface of a sphere, and localization on a spherical surface. Also of interest is when active distance measurements are undertaken with unknown source signal intensity and unknown characteristics of the broadcast medium.

It is also known that in two dimensions, the noise vector affecting three discrete measurements *cannot have independent entries*, [5], but must lie on a manifold. It maybe worth one's while to explore the implications of this fact in the face of continuous measurements and indeed also to three dimensions, in the hope that the resulting redundancy in the noise profile can be exploited to enhance noise amelioration.

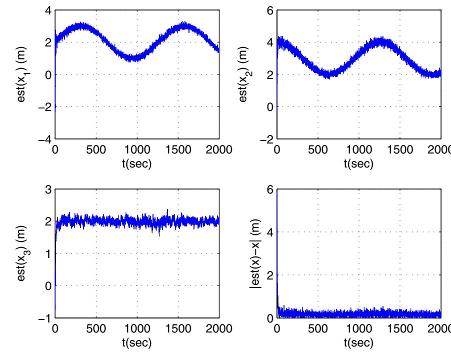

Fig. 5. Location estimation for $x(t) = [2 + \sin 0.005t,\ 3 + \cos 0.005t,\ 2]^T$ (m), (m), $y(t) = [2 + 2\sin t,\ 2\cos 2t,\ 2\sin 0.5t]^T$ (m), $\alpha = 1$. Noise in distance measurement with power 0.001 (m^2). The dashed lines correspond to the actual coordinates of the source and the solid curves show the estimate trajectories.

REFERENCES

[1] *IEEE Signal Processing Magazine*, July 2005.
[2] Patwari, N., Ash, J.N., Kyperountas, S., Hero, A.O., Moses, R.L., and Correal, N.S., "Locating the nodes: cooperative localization in wireless sensor networks", *IEEE Signal Processing Magazine*, pp. 54 - 69, July 2005.
[3] Ihler, A.T., Fisher, J.W., Moses, R.L., and Willsky, A.S., "Nonparametric belief propagation for self-localization of sensor networks", *IEEE Journal on Selected Areas in Communications,* pp. 809 - 819, April 2005.
[4] Roumeliotis, S.I. and Bekey, G.A., "Distributed multirobot localization" *IEEE Transactions on Robotics and Automation,* pp. 781 - 795, Oct. 2002.
[5] Cao, M., Anderson, B.D.O., and Morse, A.S., "Localization with imprecise distance measurements in sensor networks", in *Proceedings of 44th Conference on Decision and Controls*, December 2005, Seville.
[6] Hanbiao Wang, Len Yip, Kung Yao and Estrin, D., "Lower bounds of localization uncertainty in sensor networks", in *Proceedings of IEEE International Conference on Acoustics, Speech, and Signal Processing,* pp. 17-21 May 2004.
[7] Pathirana, P.N., Bulusu, N., Savkin, A.V., and Jha, S., "Node localization using mobile robots in delay-tolerant sensor networks", *IEEE Transactions on Mobile Computing,*, pp. 285 - 296, May-June 2005.
[8] Chandra, N. and Knapp, C., "Optimal hydrophone placements under random perturbations" *IEEE Transactions on Acoustics, Speech, and Signal Processing ,* pp., 860 - 864, May 1990.
[9] Fichtner, M., Grobmann, A., "A probabilistic visual sensor model for mobile robot localisation in structured environments", in *Proceedings of 2004 IEEE/RSJ International Conference on Intelligent Robots and Systems, 2004. (IROS 2004)*, pp. 1890 - 1895, 2004.
[10] Lion, P.M., "Rapid identification of linear and nonlinear systems", in *Prodeedings of Joint American Control Conference*, pp. 605-615, 1966.
[11] Anderson, B.D.O, "Exponential stability of linear equations arising in adaptive identification", *IEEE Transaction on Automatic Control*, pp. 530-538, 1977.
[12] P. A. Ioannou and J. Sun, *Robust Adaptive Control*. Upper Saddle River, NJ: Prentice-Hall, 1996.
[13] Sondhi, M. M. and Mitra, D., "New results on the performance of a well known class of adaptive filters", *Proceedings of IEEE*, pp. 1583-1597, 1976.
[14] Anderson, B. D. O., Bitmead, R. R., Johnson, C. R., Kokotovic, P. V., Kosut, R. L., Mareels, I. M. Y., Praly, L., and Riedle, B., *Stability of Adaptive Systems: Passivity and Averaging Analysis*, MIT Press, 1987.
[15] Dasgupta, S., Anderson, B.D.O and Tsoi, A.C., "Input conditions for continuous time adaptive systems problems," *IEEE Transactions on Automatic Control*, January 1990, vol. 35, pp. 78-82.
[16] Mitrinovic, D. S., *Analytic Inequalities*, Springer Verlag, 1970.