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Sequential Localization of Networks*

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The sensor network localization problem with distance information is to determine the positions of all sensors in a network given the positions of some sensors and the distances between some pairs of sensors. The sensors whose positions are given are said to be *anchors*. Knowing the positions of sensors is essential in many network algorithms such as geographic routing and coverage. The sensor network localization problem is solvable if and only if the network is "localizable". A network in $\mathbb{R}^d$ is said to be *localizable* if there exists exactly one position in $\mathbb{R}^d$ corresponding to each non-anchor sensor such that the given inter-sensor distances are satisfied. The authors of [1] use rigidity theory to give the necessary and sufficient conditions for a network to be localizable. However, the process of localizing a network has been shown to be NP-hard even when the network is known to be localizable [2]. This leaves us to consider the more refined question of what kinds of localizable networks can we efficiently localize? This has been investigated in [3] and we extend the results of that paper. While some ingenious heuristics-based schemes have been proposed [4], [5], [6], [7], we are interested in provably correct localization algorithms and the kinds of networks that can be efficiently localized by them. In the following we will give a localization algorithm that consists of a finite number of steps to be carried out sequentially. We will also give some classes of networks that can be efficiently localized by the algorithm. In the process, we will raise a number of graph theoretic questions related to rigidity theory. The resolution of these questions can in turn help us shed light on which networks can be efficiently and provably correctly localized. This is another indication that the sensor network localization problem and rigidity theory are intricately related.

We begin by giving some terms and definitions to be used in the exposition which follows. A *configuration* $p = \{p_1, \ldots, p_n\}$ in d-dimensional space is a set of n points in $\mathbb{R}^d$ labelled $p_1, \ldots, p_n$. A configuration p is said to be *generic* if the coordinates of points in p are algebraically independent over the rationals. Two configurations $p = \{p_1, \ldots, p_n\}$ and $q = \{q_1, \ldots, q_n\}$ of n points are *congruent* if for all $i, j \in \{1, \ldots, n\}$, the distance between p_i and p_j is equal to the distance between q_i and q_j. A *point formation* of n points at a configuration $p = \{p_1, \ldots, p_n\}$ consists of p and a simple undirected graph $\mathbb{G}$ with vertex set $\mathcal{V} = \{1, \ldots, n\}$, and is denoted by $(\mathbb{G}, p)$. If (i, j) is an edge in $\mathbb{G}$, then we say the *length* of edge (i, j) in the point formation $(\mathbb{G}, p)$ is the distance between p_i and p_j. A point formation $(\mathbb{G}, p)$ is *globally rigid* in $\mathbb{R}^d$ if p and q are congruent configurations in $\mathbb{R}^d$ whenever $(\mathbb{G}, p)$ and $(\mathbb{G}, q)$ have the same edge lengths. A graph $\mathbb{G}$ is said to be *generically globally rigid* in $\mathbb{R}^d$ if $(\mathbb{G}, p)$ is globally rigid in $\mathbb{R}^d$ whenever p in $\mathbb{R}^d$ is generic. Since *almost all* configurations are generic, we have that $(\mathbb{G}, p)$ is globally rigid in $\mathbb{R}^2$ for *almost all* configurations p in $\mathbb{R}^2$ if $\mathbb{G}$ is generically globally rigid in $\mathbb{R}^2$. A graph that is generically globally rigid in $\mathbb{R}^2$ is said to be *minimally generically globally rigid* in $\mathbb{R}^2$ if the removal of any edge causes the graph to not be generically globally rigid in $\mathbb{R}^2$.

A sensor network with n sensors is modelled by a point formation $(\mathbb{G}, p)$, where each sensor corresponds to exactly one vertex of $\mathbb{G}$, and vice versa, with (i, j) being an edge of $\mathbb{G}$ if i and j are distinct and the distance between the corresponding sensors is known, and $p = \{p_1, \ldots, p_n\}$ where p_i is the position of the sensor corresponding to vertex i. We say that $\mathbb{G}$ is the graph of the network, and p is the configuration of the network. Vertex v of $\mathbb{G}$ is called an *anchor vertex* if the sensor corresponding to v is an anchor, and a *sensor vertex* otherwise. Since *almost all* configurations are generic, we have that the configurations of networks are *almost always* generic. Henceforth, we will only consider networks with generic configurations. It is known that if the configuration of a network in $\mathbb{R}^2$ is generic, then the network is localizable if and only if it has at least 3 non-collinear anchors and the graph of the network is generically globally rigid in $\mathbb{R}^2$.

A graph is said to be a *trilateration graph with trilateration ordering* $v_1, \ldots, v_n$ if its vertices can be relabelled as $v_1, \ldots, v_n$ so that the subgraph induced by v_1, v_2, v_3 is complete and each v_i with $i > 3$ is adjacent to at least three distinct vertices v_j which "precede" it in the ordering, where by *precede* we mean $j < i$ [1]. It is shown in [1] that trilateration graphs are generically globally rigid in $\mathbb{R}^2$. A graph is a *bilateration graph with bilateration ordering*

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$v_1, \ldots, v_n$ if its vertices can be relabelled as $v_1, \ldots, v_n$ so that the subgraph induced by v_1 and v_2 is complete, and each v_i, $i > 2$, is adjacent to at least two distinct vertices v_j which precede it in the ordering. It is easy to see that bilateration graphs are 2-connected but not necessarily 3-connected. We define the *maximal anchor-free subgraph* of the graph $\mathbb{G}$ of a network to be the maximal subgraph of $\mathbb{G}$ containing only sensor vertices. We say that $\mathbb{G}$ is *partially acyclic* if its maximal anchor-free subgraph is acyclic. A graph is *edge 2-connected* if there exists two paths with no edge in common between any two vertices. The *second power* of $\mathbb{G} = (\mathcal{V}, \mathcal{E})$, written $\mathbb{G}^2$, is the graph with vertex set $\mathcal{V}$ and edge set $\mathcal{E} \cup \mathcal{E}^2$, where $(i, j) \in \mathcal{E}^2$ just in case $i, j \in \mathcal{V}$ and there exists $k \in \mathcal{V}$ such that $(i, k), (k, j) \in \mathcal{E}$. It is known that the second power of an edge 2-connected graph is generically globally rigid in $\mathbb{R}^2$ [3]. We show that the second power of an edge 2-connected graph is also a bilateration graph.

In the following, we will describe a localization algorithm for networks in $\mathbb{R}^2$ that consists of a finite number of steps to be carried out sequentially. We will then use trilateration, bilateration and edge 2-connected graphs to identify and construct some networks that can be efficiently localized by this algorithm.

Let $\mathbb{N}$ be a network of n sensors labelled 1 through n where sensor i is positioned at $\pi(i)$, and $\pi(1), \pi(2), \ldots, \pi(n)$ are distinct points in $\mathbb{R}^2$. Suppose that $1, 2, \ldots, m$ are the labels of $\mathbb{N}$'s anchors and that $m \geq 3$. Let $\mathbb{G} = (\mathcal{V}, \mathcal{E})$ be the graph of $\mathbb{N}$. Without loss of generality, suppose that for each $i \in \{1, 2, \ldots, n\}$, vertex i of $\mathbb{G}$ corresponds to sensor i and vice versa. For each $v \in \mathcal{V}$, let $\mathcal{N}(v)$ denote the set consisting of all vertices u where $(u, v) \in \mathcal{E}$, and for each $u \in \mathcal{N}(v)$ write d_{uv} for the distance between sensors u and v. By an *assignment* for $\mathbb{N}$ is meant any function $\alpha : \{1, 2, \ldots, n\} \to \mathbb{R}^2$. An assignment for $\mathbb{N}$ is *consistent* if for all $v \in \{1, 2, \ldots, n\}$, $||\alpha(u) - \alpha(v)|| = d_{uv}$ for all $u \in \mathcal{N}(v)$, and $\alpha(v) = \pi(v)$ whenever sensor v is an anchor, i.e. $v \in \{1, 2, \ldots, m\}$. Hence, $\mathbb{N}$ is localizable if and only if there is exactly one consistent assignment for $\mathbb{N}$.

Let $2^{\mathbb{R}^2}$ be the power set of $\mathbb{R}^2$ and write $\mathbb{R}_+$ for the set of positive real numbers. Let $f : 2^{\mathbb{R}^2} \times \mathbb{R}_+ \to 2^{\mathbb{R}^2}$ denote the function $(\mathcal{S}, d) \longmapsto \mathcal{S}'$ where $\mathcal{S}'$ is the set of $p \in \mathbb{R}^2$ such that $||p - q|| = d$ for some $q \in \mathcal{S}$. If $\mathcal{S}$ is not empty, then geometrically $f(\mathcal{S}, d)$ is the union of all points in the plane which lie on circles with the same radius d centered at the points in $\mathcal{S}$. Of course if $\mathcal{S}$ is empty then so is $f(\mathcal{S}, d)$ and conversely. We will be especially interested in the case when $\mathcal{S}$ is a non-empty "finite set" and $d > 0$, where by *finite set* we mean a set with a finite number of points in $\mathbb{R}^2$. In this case $f(\mathcal{S}, d)$ is simply the union of a finite number of circles in the plane which all have radius d. An easily verified property of f is that if $u \in \mathcal{N}(v)$, and $\mathcal{S}(u)$ is a set for which $\pi(u) \in \mathcal{S}(u)$, then $\pi(v) \in f(\mathcal{S}(u), d_{uv})$. We call this the *position keeping* property of f.

Let $\mathbb{S}$ denote the set of all non-empty subsets of $\mathbb{R}^2$ with finitely many elements. Let q be a positive integer no smaller than 2 and write $\mathbb{S}^q$ for the q-fold Cartesian product of $\mathbb{S}$ with itself. Similarly, let $(\mathbb{R}_+)^q$ denote the q-fold Cartesian product of $\mathbb{R}_+$ with itself. Our aim is to define a function $g_q : \mathbb{S}^q \times (\mathbb{R}_+)^q \to 2^{\mathbb{R}^2}$ in such a way so that for each $\{\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_q\} \in \mathbb{S}^q$ and $\{d_1, d_2, \ldots, d_q\} \in (\mathbb{R}_+)^q$, $g_q(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_q, d_1, d_2, \ldots, d_q)$ is at most a finite set. Furthermore, we shall require the definition of g_q to be such that whenever there are distinct points $u_i \in \mathcal{S}_i$, $i \in \{1, 2, \ldots, q\}$, if $v \in \mathbb{R}^2$ satisfies $\|v - u_i\| = d_i$, $i \in \{1, 2, \ldots, q\}$, then v must be a point in $g_q(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_q, d_1, d_2, \ldots, d_q)$. Defining $g_q(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_q, d_1, d_2, \ldots, d_q)$ in the most obvious way as the intersection of the sets $f(\mathcal{S}_i, d_i)$, $i \in \{1, 2, \ldots, q\}$, will not be adequate for it may be the case that the resulting intersection is a continuous circle of points in the plane rather than a finite set.

Let $\mathcal{I} = \bigcap_{j=1}^{q} \mathcal{S}_j$, and let $p_1, p_2, \ldots, p_k$ denote the elements of $\mathcal{I}$. For any set $\mathcal{S} \in \mathbb{S}$, and any subset $\mathcal{T} \subset \mathcal{S}$, let $\mathcal{S} \backslash \mathcal{T}$ denote the complement of $\mathcal{T}$ in $\mathcal{S}$. The following definition of g_q satisfies both of the requirements listed above:

$$g_q(\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_q, d_1, d_2, \ldots, d_q) = \Big(f(\mathcal{S}_1 \backslash \mathcal{I}, d_1) \cap f(\mathcal{S}_2, d_2) \cap \ldots \cap f(\mathcal{S}_q, d_q) \Big) \bigcup \Big(\bigcup_{i=1}^{k} \; f(\{p_i\}, d_1) \cap f(\mathcal{S}_2 \backslash \{p_i\}, d_2) \cap \ldots\ldots \cap f(\mathcal{S}_q \backslash \{p_i\}, d_q) \Big) \tag{1}$$

An ordering $v_1, v_2, \ldots, v_n$ of the vertices in $\mathcal{V}$ for which

$$v_i = i, \; i \in \{1, 2, \ldots, m\}$$

and at least one of the sets

$$\mathcal{M}(v_i) = \mathcal{N}(v_i) \cap \{v_1, v_2, \ldots, v_{i-1}\}, \quad i \in \{m+1, m+2, \ldots, n\}$$

is non-empty is called a *sweep* of $\mathbb{N}$ and is denoted by $[v]$. Hence, if an ordering $v_1, v_2, \ldots, v_n$ is a sweep, then there is at least one vertex v_i where $i > m$ and v_i is adjacent to at least one vertex preceding it. We shall require the networks we consider to have at least one "finite position generating" sweep $v_1, v_2, \ldots, v_n$, where by *finite position generating* we mean that each vertex v_i, $i > m$, is adjacent to at least two distinct vertices preceding it. If $v_1, v_2, \ldots, v_n$ is a finite position generating sweep, then each of the $\mathcal{M}(v_i)$, $i > m$, is a set of at least two elements.

In the following, we will describe a recursive procedure for computing a sequence of finite sets for each vertex $v \in \mathcal{V}$, i.e. $\mathcal{S}(v,1),\ \mathcal{S}(v,2),\cdots\mathcal{S}(v,i),\cdots$, such that each $\mathcal{S}(v,i)$ is a finite set, $\pi(v) \in \mathcal{S}(v,i)$, and if $i < j$, then $\mathcal{S}(v,i) \supset \mathcal{S}(v,j)$. We begin by choosing a finite position generating sweep $[v]$ of $\mathbb{N}$. For $i \in \{m+1, m+2, \ldots, n\}$, let

$$\mathcal{M}(v_i) = \mathcal{N}(v_i) \cap \{v_1, v_2, \ldots, v_{i-1}\}$$

We denote the cardinality of $\mathcal{M}(v_i)$ by q_i and the elements of $\mathcal{M}(v_i)$ by $u_{i1}, u_{i2}, \ldots, u_{iq_i}$. We define the sets $\mathcal{S}(v_i,1),\ i \in \{1,2,\ldots,n\}$ as follows. For $i \in \{1,2,\ldots,m\}$, we define

$$\mathcal{S}(v_i,1) = \{\pi(v_i)\} \tag{2}$$

and for $i \in \{m+1, m+2, \ldots, n\}$, we define

$$\mathcal{S}(v_i,1) = g_{q_i}(\mathcal{S}(u_{i1},1), \mathcal{S}(u_{i2},1), \ldots, \mathcal{S}(u_{iq_i},1), d_{u_{i1}v_i}, d_{u_{i2}v_i}, \ldots, d_{u_{iq_i}v_i}) \tag{3}$$

Note that since $[v]$ is assumed to be a finite position generating sweep, each $\mathcal{M}(v_i)$ has at least 2 elements and so $q_i \geq 2$. Hence, for $i \in \{m+1, m+2, \ldots, n\}$, g_{q_i} is defined and $\mathcal{S}(v_i,1)$ is a finite set because the image of g_{q_i} consists only of finite sets. Since $\mathcal{S}(v_i,1)$, $i \in \{1,2,\ldots,m\}$, are also finite sets because of (2), we have that $\mathcal{S}(v,1)$ is a finite set for each $v \in \mathcal{V}$. Note also that $\pi(v_i) \in \mathcal{S}(v_i,1),\ v_i \in \mathcal{V}$. This is clearly true for $i \in \{1,2,\ldots,m\}$ because of (2). The assumption that $\pi(v)$, $v \in \mathcal{V}$, are distinct together with the definition of g_{q_i} and the position keeping property of f imply that $\pi(v_i) \in \mathcal{S}(v_i,1)$ for $i \in \{m+1, m+2, \ldots, n\}$. We call the computation of $\mathcal{S}(v,1),\ v \in \mathcal{V}$, the *computation of the initial sweep* of $\mathbb{N}$.

Now suppose that the initial sweep of $\mathbb{N}$ has been computed. The sets $\mathcal{S}(v,2),\ v \in \mathcal{V}$, are computed as follows. Let $[u] = u_1, u_2, \ldots, u_n$ be a sweep, and let $\mathcal{M}(u_i) = \mathcal{N}(u_i) \cap \{u_1, u_2, \ldots, u_{i-1}\}$ for $i \in \{m+1, m+2, \ldots, n\}$. Note that $[u]$ need not be a finite position generating sweep. For $i \in \{1,2,\ldots,m\}$ we define

$$\mathcal{S}(u_i,2) = \{\pi(u_i)\} \tag{4}$$

and for $i \in \{m+1, m+2, \ldots, n\}$ we define

$$\mathcal{S}(u_i,2) = \mathcal{S}(u_i,1) \bigcap_{w \in \mathcal{M}(u_i)} f(\mathcal{S}(w,2), d_{wu_i})\ if\ \mathcal{M}(u_i) \neq \emptyset \tag{5}$$

$$\mathcal{S}(u_i,2) = \mathcal{S}(u_i,1)\ if\ \mathcal{M}(u_i) = \emptyset \tag{6}$$

For each u_i, $i \in \{m+1, m+2, \ldots, n\}$, (5) and (6) implies that $\mathcal{S}(u_i,2)$ must be a finite set since $\mathcal{S}(u_i,1)$ is a finite set. Moreover, since $\mathcal{S}(u_i,2)$, $i \in \{1,2,\ldots,m\}$, are all finite sets because of (4), we have that $\mathcal{S}(v,2)$ is a finite set for each $v \in \mathcal{V}$. Also, $\pi(v) \in \mathcal{S}(v,2)$ for all $v \in \mathcal{V}$. This is clearly true for u_i, $i \in \{1,2,\ldots,m\}$ because of (4). For $i \in \{m+1, m+2, \ldots, n\}$, that $\pi(u_i) \in \mathcal{S}(u_i,1)$ and the position keeping property of f imply $\pi(u_i) \in \mathcal{S}(u_i,2)$. We call the computation of $\mathcal{S}(v,2),\ v \in \mathcal{V}$, the computation of the second sweep of $\mathbb{N}$. It is obvious from (4), (5) and (6) that $\mathcal{S}(v,2) \subset \mathcal{S}(v,1)$ for all $v \in \mathcal{V}$.

Now suppose the kth sweep of $\mathbb{N}$ has been computed, and that for each $v \in \mathcal{V}$, $\pi(v) \in \mathcal{S}(v,k)$ and $\mathcal{S}(v,k)$ is a finite set. The $(k+1)$th sweep is computed as follows. Let $[x] = x_1, x_2, \ldots, x_n$ be a sweep, and let $\mathcal{M}(x_i) = \mathcal{N}(x_i) \cap \{x_1, x_2, \ldots, x_{i-1}\}$ for $i \in \{m+1, m+2, \ldots, n\}$. For $i \in \{1,2,\ldots,m\}$, we define

$$\mathcal{S}(x_i,k+1) = \{\pi(x_i)\} \tag{7}$$

and for $i \in \{m+1, m+2, \ldots, n\}$ we define

$$\mathcal{S}(x_i,k+1) = \mathcal{S}(x_i,k) \bigcap_{w \in \mathcal{M}(x_i)} f(\mathcal{S}(w,k+1), d_{wx_i})\ if\ \mathcal{M}(x_i) \neq \emptyset \tag{8}$$

$$\mathcal{S}(x_i,k+1) = \mathcal{S}(x_i,k)\ if\ \mathcal{M}(x_i) = \emptyset \tag{9}$$

For each $v \in \mathcal{V}$, we have that $\mathcal{S}(v,k+1)$ is a finite set, $\pi(v) \in \mathcal{S}(v,k+1)$ and $\mathcal{S}(v,k+1) \subset \mathcal{S}(v,k)$ by the same reasoning as before.

The preceding shows that if we compute a sequence of sweeps starting with one which is finite position generating, we can generate a sequence of finite sets for each $v \in \mathcal{V}$, i.e. $\mathcal{S}(v,1), \mathcal{S}(v,2), \cdots, \mathcal{S}(v,i), \cdots$, where each set is obtained by means of a finite number of computations and

$$\mathcal{S}(v,1) \supset \mathcal{S}(v,2) \supset \cdots \supset \mathcal{S}(v,i) \cdots$$

and $\pi(v) \in \mathcal{S}(v,i)$ for each i. Thus if we can select a finite number of sweeps, say k, such that for all $v \in \mathcal{V}$, each $\mathcal{S}(v,k)$ will contain just the position $\pi(v)$ of sensor v, then localization will be complete. We call this the *sequential localization* of the network, and we say that the network is *sequentially localizable* in k sweeps. Hence, sequential

localization of a network is carried out in a finite number of steps, each of which is solvable in a straightforward manner. This is in sharp contrast to a direct assault on the localization problem by attempting to solve a large number of simultaneous quadratic equations in $2(n-m)$ variables. In the exposition which follows, we will give the graph properties of some networks that are sequentially localizable in just one or two sweeps. The sweeps are selected by considering properties of the network's graph, hence localizing the network in as few sweeps as possible.

Suppose the set of sensor positions of network $\mathbb{N}$ is a generic configuration in $\mathbb{R}^2$, and let $\mathbb{G}$ be the graph of $\mathbb{N}$. The network $\mathbb{N}$ is said to be *easily localizable* if the vertices of $\mathbb{G}$ can be relabelled as $\{v_1, \ldots, v_n\}$ so that the position of v_i, $i > 3$, can be uniquely determined from just the positions of the vertices in $\mathcal{N}(v_i)$ which precede it together with the distances from all of these vertices to v_i [1], [3]. It is easy to see that $\mathbb{N}$ is easily localizable if and only if $\mathbb{G}$ is a trilateration graph. Obviously, easily localizable networks are also sequentially localizable in one sweep.

Suppose the set of sensor positions of network $\mathbb{N}$ is a generic configuration in $\mathbb{R}^2$, and that $\mathbb{N}$ has at least three anchors. Let $\mathbb{G}$ be the graph of $\mathbb{N}$. We show that $\mathbb{N}$ is sequentially localizable in two sweeps if $\mathbb{G}$ is generically globally rigid in $\mathbb{R}^2$ and a partially acyclic bilateration graph. A number of partially acyclic bilateration graphs that are generically globally rigid in $\mathbb{R}^2$ are also minimally generically globally rigid in $\mathbb{R}^2$. This implies there are localizable networks with just enough edges to ensure localizability that are also sequentially localizable.

Many practical networks are such that the distance between two sensors is known if the sensors are within sensing radius of each other. Suppose $\mathbb{N}$ is such a network, and that the set of sensor positions of $\mathbb{N}$ is a generic configuration in $\mathbb{R}^2$. Let $\mathbb{G}$ be the graph of $\mathbb{N}$. We show that if $\mathbb{G}$ has a spanning subgraph $\bar{\mathbb{G}}$ where $\bar{\mathbb{G}}$ is a "ring graph" with at least three anchor vertices, then $\mathbb{N}$ is sequentially localizable in two sweeps after doubling the sensing radius of each sensor. Where by a *ring graph* we mean a graph whose vertices can be labelled as $\{1, \ldots, n\}$ so that vertex i, $1 < i < n$, is only adjacent to vertices $i-1$ and $i+1$, vertex 1 is only adjacent to vertices 2 and n, and vertex n is only adjacent to vertices $n-1$ and 1. An important consequence of the previous is that if $\mathbb{G}$ is edge 2-connected with at least three anchor vertices, then after doubling the sensing radius of all the sensors, the network $\mathbb{N}$ can be localized by computing a finite number of sweeps in addition to a simple coordinate transformation. Note that if $\mathbb{G}$ has a spanning subgraph that is also a ring graph, then the graph of the resulting network after doubling the sensing radius of all the sensors is a bilateration graph. If $\mathbb{G}$ is edge 2-connected, then the graph of the resulting network after doubling the sensing radius of all the sensors is also a bilateration graph. Rigidity theory, especially as regards to global rigidity, plays an important part in the proofs of these assertions.

The computational complexity of the sequential localization of a network depends on the structure of the network's graph. For some networks, the complexity of sequential localization of the network may be exponential in the number of non-anchor sensors. However, the complexity of localizing certain networks can be reduced by localizing sections of the network in sequence. For example, suppose a network has subnetworks $\mathbb{N}_1, \mathbb{N}_2, \ldots, \mathbb{N}_D$ where each sensor of $\mathbb{N}$ is in at least one of the subnetworks. Suppose each $\mathbb{N}_i$ is sequentially localizable when all sensors occurring in $\mathbb{N}_j$, $j < i$, are also considered to be anchors of $\mathbb{N}_i$. Let $Z(\mathbb{N}_i)$ denote the complexity of sequentially localizing $\mathbb{N}_i$. If the number of non-anchor sensors in each $\mathbb{N}_i$ is small, then $Z(\mathbb{N}_i)$ is not too large even if it is exponential in the number of non-anchor sensors of $\mathbb{N}_i$. Consequently the complexity of localizing the entire network can be rendered acceptable by sequentially localizing all $\mathbb{N}_j$, $j < i$, before sequentially localizing each $\mathbb{N}_i$.

It is an open question if all networks with bilateration graphs that are generically globally rigid in $\mathbb{R}^2$ are sequentially localizable. And while we know that not all bilateration graphs are generically globally rigid in $\mathbb{R}^2$, the necessary and sufficient conditions for a bilateration graph to be generically globally rigid in $\mathbb{R}^2$ are still unknown. Currently, we know that the computational complexity of the sequential localization of a network is determined by the properties of the network's graph. It is part of our future research to precisely quantify this relationship between a network's graph and the complexity of the sequential localization of the network. This will allow us to determine the graph properties of networks that can be efficiently localized using our algorithm.

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