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An $\mathscr{H}_\infty$ model referencing design utilizing a two degree of freedom controller scheme

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***Abstract*— In this paper, we propose an $\mathscr{H}_\infty$ controller design method which improves model referencing feature and extends the applicability of the Internal Model Control (IMC) design method to the generic class of LTI systems (SISO, MIMO, stable, unstable) by incorporating the ideas of $\mathscr{H}_\infty$ loop-shaping and utilizing Youla-Kucera parameterization in a two-degree-of-freedom scheme to achieve robust model referencing and high performance design while ensuring a sensible robust stability margin.**

***Index Terms*— $\mathscr{H}_\infty$ Control, Robust Control, Internal Model Control, Adaptive Control**

I. Introduction

Our prime aim in this paper is broadly to propose an $\mathscr{H}_\infty$ controller design method which achieves a closed-loop transfer function equal or otherwise sensibly close to a desired transfer function (Section II) while ensuring a level of robustness. The Internal Model Control (IMC) design method of [20] exploits the well-known Youla-Kucera parameterization of all stabilizing controllers for stable plants [11], [26], [28] in a specialized way, and in principle allows—for stable plants with no $j\omega$-axis zeros—the design of a controller that achieves a closed-loop magnitude response exactly equal to that of a desired transfer function known as the IMC filter F (see Section II) containing a free parameter tuning the closed-loop bandwidth. This important bonus of offering instantaneous and direct tuning of the bandwidth of T_{yr_1} in Fig. 1 via setting the bandwidth of F is utilized in the area of adaptive robust control to progressively increase the closed-loop bandwidth [1], [17] in identification and controller re-design [3], [4]. Nonetheless, the IMC design method is restrictive and may result in an unacceptable design [5]. It even has to be almost abandoned when the plant is unstable since the design method becomes much more involved, to say the least, and there are a set of interpolation constraints to satisfy (Section II). Even worse is that the above-mentioned single design parameter no longer directly tunes the closed-loop bandwidth [2], [20]. Research directed at finding solutions for the above-mentioned problems has resulted in design methods which are application-specific (e.g. excluding unstable plants with unstable zeros) [2], [17].

Even for stable plants, there are circumstances where the IMC design method can result in undesirable or even unacceptable design which are detailed in [5], [7]. The related disadvantages of the IMC design method are partly due to the fact that the design method deals specifically with the complementary sensitivity, viz. $T_{yr_1} = P(1 + PC)^{-1}C$ in Fig. 1, by setting its magnitude response (Section II), but it does not explicitly handle the other transfer functions (T_{yr_2},T_{ur_1},T_{ur_2}) and hence it may fail to ensure that their magnitudes are acceptable. However, the other three transfer functions do relate to certain input-output properties of a feedback loop [29] as discussed in Section II. The aforementioned difficulties are also partly related to the associated design method which has a single-degree-of-freedom expressed in terms of a single Youla-Kucera parameter Q (Section II), and well known fundamental design compromises associated with such systems which have to be made between internal stability requirements, robust stability and performance specifications [8], [9]. These trade-offs may not be possible to achieve and may even result in an impossible design (Section II).

To alleviate the above-mentioned concerns, two-degree-of-freedom configurations are usually considered in which a separation is made between disturbance attenuation and model reference objectives [24], [27]. The two degrees of freedom in configurations of this type may be parameterized in terms of two stable but otherwise free parameters Q_1 and Q_2 (Section II-B).

We propose a two-degree-of-freedom controller design method, outlined in Section III, which inherits the model referencing feature of the IMC design method but addresses difficulties and disadvantages of the IMC (see [7] and Section II) in a coherent framework. Our proposed $\mathscr{H}_\infty$ design method relies on a two-degree-of-freedom scheme of Section II-B which utilizes Youla-Kucera parameterization in an elegant way to link with the work in the area of robust tracking and the well-received $\mathscr{H}_\infty$ loop-shaping ideas of [19], [21]. The design method of Section IV can be used for stable/unstable and SISO/MIMO systems, and can achieve a desired magnitude response for T_{yr_1} with a free parameter tuning its bandwidth, even for unstable systems. The step-by-step presented design method of Section IV offers robust tracking and high performance design while

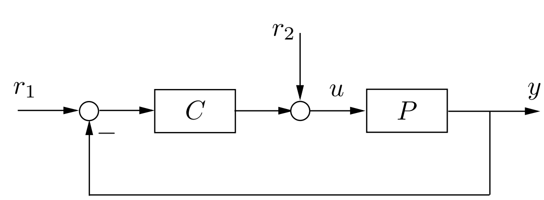

Fig. 1. Standard Feedback Configuration

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ensuring a sensible robust stability margin, and relies on an $\mathscr{H}_\infty$ control problem which can be easily solved using standard software.

II. From Youla-Kucera Parameterization to Robust Model Matching

A. Background

Let us consider a standard feedback arrangement shown in Fig. 1 and begin with stating the well-known Youla-Kucera parameterization [11], [26] of all stabilizing controllers for a given linear time-invariant plant.

Let $P = NM^{-1} = \tilde{M}^{-1}\tilde{N}$, with $\{M, N\}$ a right and $\{\tilde{M}, \tilde{N}\}$ a left coprime factorization of P over $\mathscr{RH}_\infty$ respectively, and let $C_0 = UV^{-1} = \tilde{V}^{-1}\tilde{U}$ be a stabilizing controller. Then every controller that internally stabilizes the feedback system in Fig. 1 is parameterized by

$$C = (\tilde{V} - Q\tilde{N})^{-1}(\tilde{U} + Q\tilde{M}) \qquad (1)$$

for any $Q \in \mathscr{RH}_\infty$, where $\tilde{V}$ and $\tilde{U}$ can be chosen to satisfy the Bezout identity $\tilde{U}N + \tilde{V}M = I$, see [24], [29].

The parameterized controller C of (1) can be implemented in a standard feedback structure as shown in Fig. 2.

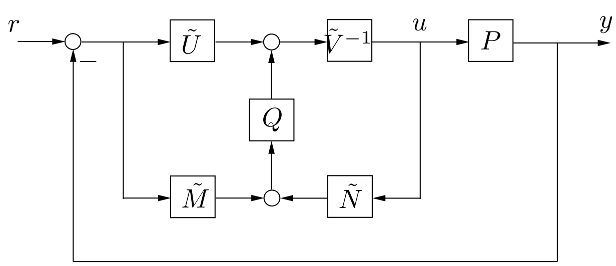

Fig. 2. Youla-Kucera Controller Parameterization

For stable plants, $P \in \mathscr{RH}_\infty$, choosing $\tilde{N} = P$, $\tilde{M} = I$, $\tilde{U} = 0$ and $\tilde{V} = I$ in (1) will result in the parameterization

$$C = (I - QP)^{-1}Q = Q(I - PQ)^{-1} \qquad (2)$$

which is the controller parameterization exploited in the area of process control and termed "Internal Model Control"[1] [20]. Notice that in this case when $P \in \mathscr{RH}_\infty$, one can easily recover the standard IMC structure of [20] in Fig. 3, in which P_t denotes the true plant, from Fig. 2.

One may seek to cast this parameterization to have the transfer function T_{yr} in Fig. 2 equal or otherwise reasonably close to a desired transfer function even for MIMO/SISO, stable/unstable plants, and plants with lightly-damped stable/unstable poles and zeros.

It is not hard to verify that using the controller parameterization in Equation (2) will result in a closed-loop transfer function, T_{yr_1} in Fig. 1 which is affine in the free stable transfer function Q, viz. $T_{yr_1} = PQ$ when $P = P_t$. Following the IMC design method of [20], stable plants, which are simpler, are treated separately from unstable plants. Recall the feedback system of Fig. 1 and suppose $P \in \mathscr{RH}_\infty$ with no $j\omega$-axis zero. One decomposes P into an inner-outer factorization of the form $P = P_a P_m$ with $P_a \in \mathscr{RH}_\infty, P_a^\sim P_a = I$ and P_m stable minimum-phase (all zeros in $Re[s] < 0$). Then the goal of obtaining a closed-loop transfer function with a desired magnitude is achieved by choosing an "IMC filter transfer function" $F(s)$ such that with an appropriate controller—which is easily found by setting the Q-parameter in (2) to $Q = P_m^{-1}F$—it achieves $|T_{yr_1}| = |F|$. For the stable plant case, a common choice for the filter is $F = [\lambda/(s+\lambda)]^n$ for some λ which specifies the desired bandwidth for T_{yr_1}. Evidently $Q \in \mathscr{RH}_\infty$ provided that $F \in \mathscr{RH}_\infty$ and n, the relative degree of F, is at least equal to the relative degree of P_m.

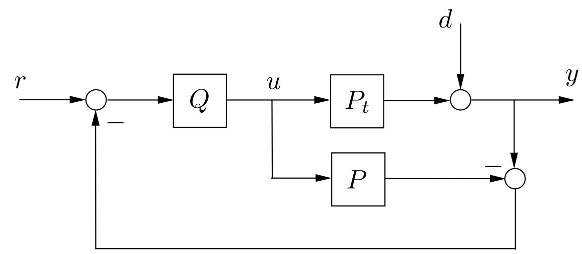

Fig. 3. Standard Internal Model Control Structure

The IMC design method for unstable plants, however, requires substantial adjustment [20]. Let us again seek a controller parameterization for the design of C as in (2) and develop the requirements on C that ensure internal stability in Fig. 1. The closed-loop mapping of Fig. 1, i.e. $\left[\begin{smallmatrix} r_1 \\ r_2 \end{smallmatrix}\right] \mapsto \left[\begin{smallmatrix} y \\ u \end{smallmatrix}\right]$, is usually denoted by $H(P, C)$ and one needs to ensure that $H(P, C) \in \mathscr{RH}_\infty$, see [29], which is equivalent to $\left[\begin{smallmatrix} PQ & P(I-QP) \\ Q & I-QP \end{smallmatrix}\right] \in \mathscr{RH}_\infty$ where C is substituted for as in (2). Thus, i. $Q \in \mathscr{RH}_\infty$, ii. the closed right half-plane poles of P must be cancelled by the zeros of Q, and iii. the closed right half-plane poles of P must be cancelled by the zeros of $(1 - PQ)$ in order to satisfy the internal stability requirements. Thus, the parameterization in (2) ought to be evolved to meet these constraints on Q. Clearly, these interpolation constraints will complicate the design and hence we cannot expect that the choice F as in the stable plant case will meet the simple requirements [5], [20]. A different transfer function for the filter F is given in [20] but the filter parameter λ—unlike the stable case—does not directly adjust the bandwidth of the closed-loop frequency response even though $|T_{yr_1}| = |F|$. Another approach for the choice of F is proposed in [2] but it is nonetheless very application specific (e.g. excluding unstable plants with unstable zeros) and requires additional parameter tuning to trade-off the magnitude of the overshoot and the settling time in the step response.

Even for the stable plant case, the IMC design method has limitations, depends on restrictive assumptions and gives rise to certain open problems which range from achieving a non-robust and unsensible design to having an improper controller, or even to dealing with an impossible design. The reader is referred to [5]–[7] for a comprehensive study of these difficulties and shortcomings. These problems

[1]The term internal model control is used because the controller can be viewed as a combination of two elements, one being a model of the plant.

are partly associated with the fact that the IMC design method only deals with $T_{yr_1} = P(I+PC)^{-1}C$, which is the complementary sensitivity and is clearly important for model matching, by setting its magnitude response but only ensures that the other transfer functions in $H(P,C)$ of Fig. 1 are stable but does not explicitly handle their size. These shortcomings are also related to insisting on using the standard IMC structure in Fig. 3 with the controller parameterization given in (2) and the particular choice of $Q = P_m^{-1}F$ discussed earlier. Obviously, we shall seek a different structure and controller parameterization in order to tackle the difficulties mentioned above while maintaining the desired model referencing feature of the IMC.

B. A Proposed Generalized Structure for Robust Tracking

Let us now consider the structure in Fig. 4 which is a rearrangement of the controller implementation in Fig. 2 and where the reference signal r enters into the structure from a different place. A similar approach in the controller implementation is utilized in [30] in the area of fault-tolerant control but the closed-loop mapping is different.

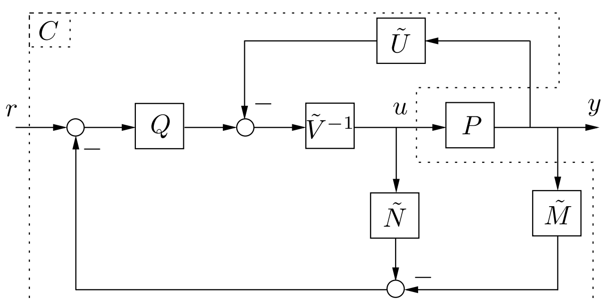

Fig. 4. Youla-Kucera Parameterization and IMC

The closed-loop scheme of Fig. 4 offers advantages over the standard IMC approach while ensuring the model referencing properties of the IMC. In this representation, Q is only required to be stable and there is no need for Q to satisfy a set of interpolation constraints discussed in Section II-A since we have a full Youla-Kucera parameterization as in (1). Moreover, when $P_t = P$, i.e. a perfect model of the true plant is available, the structure in Fig. 4 reduces to a closed-loop scheme for robust reference tracking design—where the objective is to ensure that the output of the plant follows a reference trajectory in the face of disturbances and uncertainty—which is discussed in detail in [24], [25] and is shown in Fig. 5.

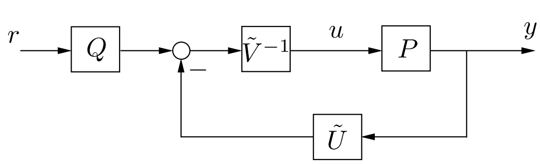

Fig. 5. A Reference Tracking Structure

For the scheme of Fig. 5, it is immediately seen that $y = NQ\,r$ and hence the model reference problem is dealt with separately from the disturbance rejection problem [24] since $\tilde{V}$ and $\tilde{U}$ deal with the disturbance rejection.

If $\tilde{V}$ and $\tilde{U}$ are chosen as simply stabilizing, however, in the structure of Fig. 4 the single free parameter Q needs to be designed such that robustness with respect to model uncertainty, disturbance rejection, performance objectives, and robust tracking are achieved simultaneously, which may be hard to reach a trade-off [8], [9]. Thus a separation of tasks is required.

The two-degree-of-freedom controller structure depicted in Fig. 6 is a replica of the structure discussed above and shown in Fig. 4, but has more appealing properties, one being the separation of model referencing and robustness, which are discussed in the sequel.

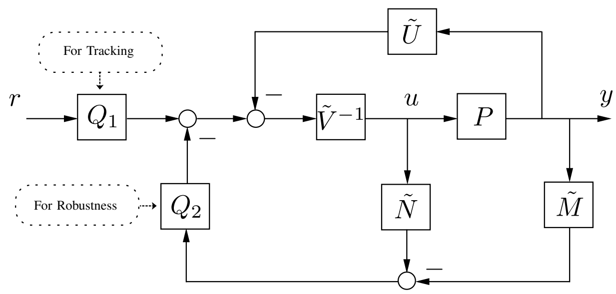

Fig. 6. A two-degree-of-freedom controller structure

In terms of the standard two-degree-of-freedom controller scheme, the structure in Fig. 6 can be redrawn as Fig. 7 and the set of all stabilizing controllers is given by

$$\begin{aligned} C_1 &= (\tilde{V} + Q_2\tilde{N})^{-1}Q_1 \\ C_2 &= (\tilde{V} + Q_2\tilde{N})^{-1}(\tilde{U} - Q_2\tilde{M}) \end{aligned} \tag{3}$$

for any $Q_1, Q_2 \in \mathscr{RH}_\infty$, where $\tilde{V}$ and $\tilde{U}$ are chosen to satisfy the Bezout identities $\tilde{U}N + \tilde{V}M = I$. There are parameterizations in the literature [18], [23], [24], [30] using Q_1 and Q_2 but the proposed structure as it appears in this specialized way in Fig. 6 does not seem to appear in those works.

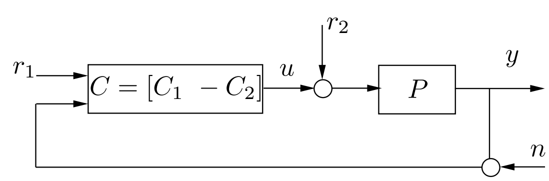

Fig. 7. Standard two-degree-of-freedom Structure

In satisfying internal stability requirements in Fig. 7, one needs to enure the closed-loop mapping $\left[\begin{smallmatrix} r_1 \\ n \\ r_2 \end{smallmatrix}\right] \mapsto \left[\begin{smallmatrix} y \\ u \end{smallmatrix}\right]$ is stable. That is to ensure

$$\Psi(P,C) = \begin{bmatrix} P \\ I \end{bmatrix} (I + C_2P)^{-1} \begin{bmatrix} C & & I \end{bmatrix} \quad \in \mathscr{RH}_\infty \tag{4}$$

To better highlight some important advantages of considering the configuration of Fig. 6, or the similar two-degree-of-freedom structure in Fig. 7, substitute for $C = [C_1 \;\; -C_2]$

given in (3) into $\Psi(P,C)$ of (4) to yield

$$\Psi(P,C)=\begin{bmatrix} NQ_1 & -N(\tilde{U}-Q_2\tilde{M}) & N(\tilde{V}+Q_2\tilde{N}) \\ MQ_1 & -M(\tilde{U}-Q_2\tilde{M}) & M(\tilde{V}+Q_2\tilde{N}) \end{bmatrix} \quad (5)$$

Clearly, the first column of $\Psi(P,C)$, T_{yr_1} and T_{ur_1}, is affine in Q_1 and handles model reference behavior, while the other two columns are $H(P,C_2)$ and are all affine in Q_2 which deals with disturbance rejection and robustness. We shall discuss the choice of Q_1 first.

Returning to our model reference design to have the transfer function from reference to the plant output $T_{yr_1} = \Psi_{11} = NQ_1$ equal or close to a desired transfer function F, we need to find the free parameter Q_1 to achieve this. That is to find Q_1 which makes $\|NQ_1 - F\|_\infty$ zero or otherwise small. Evident in the first column of Equation (5) is that any choice of Q_1 will also automatically lock the transfer function from reference to the controller output, $T_{ur_1} = \Psi_{21} = MQ_1$. Obviously, T_{yr_1} and T_{ur_1} are closely related and one requires MQ_1 to be somehow well behaved, e.g. not having too high bandwidth or its size not being too large, etc. If such a choice of Q_1 to make perfect model referencing is available, then one is left to deal with disturbance rejection and robustness utilizing the other free parameter Q_2.

It may however not be straightforward to choose Q_1. If the plant P has lightly-damped zeros in the left half plane and within the closed-loop passband, they will appear as lightly-damped zeros of N and can result in poor design and model referencing as Q_1 will try to cancel those zeros—provided that they are not part of the model reference F—by placing poles at the exact locations and hence $T_{ur_1} = MQ_1$ will be large near the frequencies of those zeros undesirably. Obviously, T_{ur_1} is the transfer function from reference input to control signal and must be kept below a certain size for a sensible design to avoid control actuator saturation, high power consumption and high energy control action.

Note that N normally inherits all right-half plane zeros of P, and since $Q_1 \in \mathscr{RH}_\infty$, it cannot cancel them when solving $\|NQ_1 - F\|_\infty$. This observation supported by the discussion above reveals that lightly-damped stable zeros must be treated in a specialized way to prevent poor design and performance.

For the choice of Q_2, or equivalently C_2, clearly the second and third columns of $\Psi(P,C)$ in Equation (5) are actually $H(P,C_2)$ and C_2 can be obtained via an $\mathscr{H}_\infty$ loop-shaping weighting scheme discussed in the sequel. This will enable us to readily deal with the problems of disturbance rejection and robustness, and also obtain a guaranteed level of robust stability and robust performance via $\|H(P,C_2)\|_\infty^{-1} = b_{P,C}$ provided that $[P,C_2]$ is stable [25], [29]. This is referred to as the generalized robust stability margin [25] and it corresponds to the smallest size of (coprime factor) uncertainty that can perturb P without destabilizing the loop [29]. Thus, we clearly wish not only to have all the transfer functions in $H(P,C_2)$ small or below certain size, viz. $\|H(P,C_2)\|_\infty$ small, but also to have all the transfer functions in $\Psi(P,C)$ small after ensuring that a good model referencing is obtained.

In the following section, we shall introduce a new controller design method that achieves robust tracking with a guaranteed level of robust stability and robust performance.

III. The Proposed $\mathscr{H}_\infty$ Controller Design Method

To capture our objectives—viz., to achieve a closed-loop transfer function equal or otherwise close in magnitude to that of a desired transfer function, to extend the applicability of the design method to a wider class of plants by addressing the difficulties discussed above and in Section II in a coherent framework while ensuring a guaranteed level of robust stability and robust performance—we shall introduce a new $\mathscr{H}_\infty$ controller design problem which is based on a two-degree-of-freedom structure discussed in Section II-B and incorporates loop-shaping ideas of [19], [21] in the sense of limiting the size of $H(P,C_2)$ and in part the weighting structure.

The discussion at the end of previous section must have convinced the reader that at the end of the day, one is facing a trade-off between achieving a good reference tracking and sensible performance and robustness. Put another way, one needs to trade-off between keeping the size of the transfer functions in the last two columns of (5) below certain values and minimizing the error between the desirable transfer function, F, and the achievable, NQ_1, for T_{yr_1}. This may be hard, for example, in the presence of lightly-damped stable poles and/or zeros in P as discussed earlier. As hinted in the last section, the trick is to deal with these lightly-damped stable poles/zeros as if they were in the right half plane. This is elegantly done by solving a model matching problem on a shifted $j\omega$-axis. That is to solve

$$\xi = \inf_{\hat{Q}_1 \in \mathscr{RH}_\infty} \left\| \hat{N}\hat{Q}_1 - \hat{F} \right\|_\infty \quad (6)$$

where $\hat{F} := F(s-\alpha)$, α is the amount by which we shift the $j\omega$-axis to the left and will be discussed in Section III-B, and $\hat{P}(s) := P(s-\alpha) = \hat{N}\hat{M}^{-1}$. Then Q_1 is easily found to be $Q_1 := \hat{Q}_1(s+\alpha)$.

Then the admissible controllers $C = [C_1 \ -C_2]$ are given by solving the $\mathscr{H}_\infty$ controller design problem

$$\gamma = \inf_{C \in \underline{\mathcal{C}}} \left\| \begin{pmatrix} W_2 & 0 \\ 0 & W_1^{-1} \end{pmatrix} \cdot \left(\begin{bmatrix} P \\ I \end{bmatrix} (I + C_2 P)^{-1} [C_1 \ -C_2 \ I] - \begin{bmatrix} NQ_1 & 0 & 0 \\ MQ_1 & 0 & 0 \end{bmatrix} \right) \cdot \begin{pmatrix} W_3 & 0 & 0 \\ 0 & W_2^{-1} & 0 \\ 0 & 0 & W_1 \end{pmatrix} \right\|_\infty \quad (7)$$

where $N(s) := \hat{N}(s+\alpha)$ and $M(s) := \hat{M}(s+\alpha)$ and $\underline{\mathcal{C}}$ denotes the set of all proper stabilizing controllers for the plant P.

The frequency weights W_1, W_2 and W_3 are stable, minimum-phase and proper weights which enforce a trade-off between model reference with $\left[\begin{smallmatrix} NQ_1 \\ MQ_1 \end{smallmatrix}\right]$ and limiting the size of other transfer functions as will be discussed in Section III-A. Abstractly, W_1 and W_2 are the frequency weights designed as in the standard $\mathscr{H}_\infty$ loop-shaping literature [19] and W_3 is introduced to trade-off model referencing with robust performance of the closed-loop.

Note that the proposed design procedure outlined above achieves our objectives set out at the beginning of this section and addresses all the difficulties discussed in Sections II and II-B. Let us set down a few key points about our proposed $\mathscr{H}_\infty$ design method above. First, we have evidently achieved the desired decomposition for P as shown above with N having all its poles to the left of the shifted axis, the $-\alpha$ axis, which ensures the stability of N with the some margin α. Likewise M is stable with the same margin α. Second, $\{\hat{N}, \hat{M}\}$ are right coprime over $\mathscr{RH}_\infty$ and hence $\binom{\hat{N}}{\hat{M}}$ has full column rank for $Re[s] > 0$ which in turn means that $\binom{N}{M}$ not only has full column rank for $Re[s] > 0$ but for the shifted open right half plane; i.e. $Re[s] > -\alpha$. Third, if ξ in (6) is small, then the norm for the unshifted model matching problem, viz. $\|NQ_1 - F\|_\infty$, is even smaller by the maximum modulus theorem [29]. Fourth, if one considers the unshifted model matching problem $\inf_{\hat{Q}_1 \in \mathscr{RH}_\infty} \|NQ_1 - F\|_\infty$, with NQ_1 and F as achievable and desired T_{yr_1} respectively, instead of the shifted one given in (6), problems will occur. This is because Q_1, being restricted to have all its poles to the left of $-\alpha$ as $\tilde{Q}_1$, cannot cancel the lightly damped zeros in $N(s)$ as otherwise MQ_1 will be large at the frequencies where Q_1 would have been large (due to the cancellation of the zeros of N by Q_1).

A. Choice of W_1, W_2 and W_3

Weighting functions W_1, W_2 and W_3 were introduced as a part of the $\mathcal{H}_\infty$ index in (7) to achieve the desired effect. As discussed before we use the $\mathscr{H}_\infty$ loop-shaping design procedure proposed in [19] as it has proven to be an effective method for designing robust controllers and has found its application in many control problems [21]. We briefly give guidelines for choosing weight functions W_1, W_2 and W_3.

Following the standard literature on $\mathscr{H}_\infty$ loop-shaping [19], [21] in the context of our proposed design procedure, one chooses W_1 to have high gain in the low frequency region (to reduce sensitivity, for example), to roll up around the closed-loop bandwidth λ (to help stability by introducing phase lead and hence improving $b_{P,C}$), and to have flat low gain in the frequency region beyond $5\,\lambda$. W_2 is chosen to have the shape of a low-pass filter with a bandwidth of around $5\,\lambda$ and unity gain in low frequency region (below λ). W_2 should start decaying soon after λ.

Note that an algorithm that automatically designs weights W_1 and W_2 in the standard $\mathscr{H}_\infty$ loop-shaping context has been proposed in [12], [13]. This algorithm may be used here to facilitate the selection of these weight functions. Further qualitative discussion on effects of the adjustment of these weights in $\mathscr{H}_\infty$ control designs is presented in [14].

Now we need to choose W_3 such that γ reflects good tracking too, viz. the smaller γ is the better model matching we achieve. Note that for a good design the acceptable range for γ, assuming that a perfect model matching is obtained, is $1 \leq \gamma \leq 3.3\,(= \frac{1}{b_{P,C}})$ [22]. Clearly we wish to put weight on the first column of our index in (7), i.e. $\left[\begin{smallmatrix} W_2(\Psi_{11}-NQ_1)W_3 \\ W_1^{-1}(\Psi_{21}-MQ_1)W_3 \end{smallmatrix}\right]$, such that with a $\gamma \leq 3.3$ we obtain a good model matching. Hence, one chooses W_3 to have high gain in low frequencies below the closed-loop bandwidth and it should start decaying above the bandwidth λ. One should not choose W_3 to have very high gain in the low frequencies as it will result in achieving a perfect model matching at the expense of having a non-robust design as γ becomes greater than 3.3. Neither should one choose W_3 to be zero or very small as this will result in a poor model matching but a robust ($\gamma \leq 3.3$) design, and hence a trade-off must be reached.

B. Guidelines for choosing α

We discussed earlier and based our design on the approach that we shall shift the $j\omega$-axis in order to place lightly-damped stable poles and also lightly-damped stable zeros to the right of the shifted axis. In this section we shall develop a mechanism for choosing α, the amount by which we shift the $j\omega$-axis. One can choose α to be slightly smaller than the smallest real part of any lightly-damped stable pole or zero, but it is required (Section III) that the bandwidth of the filter F (if it has a $\lambda/(s+\lambda)$ form) is chosen such that $\alpha < \lambda$ to ensure stability of $\hat{F}$.

Clearly each pair of lightly-damped poles has a characteristic equation, with its roots $s_{1,2} = -\zeta\omega_n \pm j\,\omega_n\sqrt{1-\zeta^2}$, which is similar to that of a second-order system $G(s) = \omega_n^2/(s^2 + 2\zeta\omega_n s + \omega_n^2)$ where ω_n is the natural frequency and ζ is the damping ratio. One can readily verify that $\bar{\sigma}\,[G(j\omega)] \leq (2\zeta\sqrt{1-\zeta^2})^{-1} \quad \forall\omega$. If we consider only those lightly-damped poles which make $\bar{\sigma}\,[G(j\omega)] \geq 3\,dB \quad \forall\omega$, then we are looking for those with $0 \leq \zeta \leq 0.3832$.

One can extend the above discussion to consider lightly-damped stable zeros of P. Note, however, that the lightly-damped poles and zeros which lie in the category defined in the last paragraph have $0 \leq \zeta \leq 0.3832$ but may have a real part, $-\zeta\omega_n$ which is far into the left-half plane, i.e. outside the closed-loop bandwidth. We just include those lightly-damped poles and zeros with $0 \leq \zeta \leq 0.3832$ and $\zeta\omega_n < \lambda$ and then α is set to be bigger than the largest $\zeta\omega_n$.

IV. The Proposed $\mathscr{H}_\infty$ Control Design Procedure

In this section we shall summarize the proposed $\mathscr{H}_\infty$ control design method.

- **Step 1.** Given $P(s)$, set α according to the rules given in Section III-B and define $\hat{P}(s) := P(s-\alpha)$ and then perform the following factorization $\hat{P} = \hat{N}\hat{M}^{-1}$ with $\hat{N}$ and $\hat{M}$ normalized right coprime factors; i.e. $\{\hat{N}, \hat{M}\}$ right coprime over $\mathscr{RH}_\infty$ and $\hat{M}^*\hat{M} + \hat{N}^*\hat{N} = I$. Now define $N(s) := \hat{N}(s+\alpha)$ and $M(s) := \hat{M}(s+\alpha)$.
- **Step 2.** Choose an appropriate transfer function for the filter F which represents the *desired* T_{yr} in Fig. 6;

- **Step 3.** Solve the $\mathscr{H}_\infty$ problem given in (6) and obtain $\hat{Q}_1$; viz. $\xi = \inf_{\hat{Q}_1 \in \mathscr{RH}_\infty} \left\| \hat{N}\hat{Q}_1 - \hat{F} \right\|_\infty$ with the solution given in [10] and where $\hat{F} := F(s-\alpha)$. Then $Q_1 := \hat{Q}_1(s+\alpha)$;
- **Step 4.** Design the frequency weights W_1, W_2 and W_3 according to the rules given in Section III-A;
- **Step 5.** Solve the $\mathscr{H}_\infty$ controller design problem given in (7) and obtain γ and the admissible controller C;
- **Step 6.** If $1 \leq \gamma \leq 3.3 (= \frac{1}{b_{P,C}})$, the obtained controller C achieves the desired model reference objectives and ensures a sensible robust stability margin.

The above-stated procedure shows the practicality and easy-to-use features of our proposed $\mathscr{H}_\infty$ design method.

V. Conclusions

We have introduced an $\mathscr{H}_\infty$ controller design method based on a proposed two-degree-of-freedom scheme which achieves a closed-loop transfer function T_{yr_1} in Fig. 7 equal or close to a desired transfer function F. Our proposed $\mathscr{H}_\infty$ design method generalizes the IMC design method to be used for MIMO/SISO, stable/unstable plants, plants with lightly-damped poles/zeros or plants with $j\omega$-axis zeros. Moreover, the design method has incorporated the $\mathscr{H}_\infty$ loop-shaping ideas of [19] into a two-degree-of-freedom scheme to ensure that a guaranteed level of robust stability and robust performance is obtained while achieving robust model referencing. The proposed model referencing design method can now be readily utilized in the area of iterative control and identification [5] with the prospect of being applicable for controller changes in adaptive control schemes [15], [16].

These all have been achieved despite the existence of distinct limitations, shortcomings and restrictive assumptions detailed in [7] and mentioned in Section II and II-B.

VI. ACKNOWLEDGMENTS

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