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Safe controller changes with additional guaranteed model reference performance improvement for the unknown plant

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***Abstract*— A safe controller change is a change, from a current stabilising controller to a new controller, with guaranteed stability of the real closed loop system (i.e. the closed loop system formed by the new controller and the physical plant), even if the true transfer function of the physical plant is not exactly known. In a previous contribution, we characterised safe controller changes, with guaranteed bounds on the stability margin of the real closed loop system, on the basis of uncertain closed loop models. In this paper, we introduce additional conditions which guarantee safety and also performance improvement of the real closed loop system.**

Notation In the notation we do not distinguish between continuous time and discrete time cases since the following results hold in both settings. We consider MIMO transfer functions. $T(P, C)$ denotes the closed loop transfer function of the feedback connection of a plant P and a controller C. $T(P, C)$ is given by

$$T(P,C) = \begin{bmatrix} P \\ I \end{bmatrix} (I - CP)^{-1} [-C \quad I] \, .$$

The frequency response of $T(P, C)$ is indicated by $T(\omega)$. The symbols $\bar{\sigma}(T, \omega)$, $\kappa(C_0, C_1, \omega)$ and $\rho(P, C, \omega)$ denote respectively the frequency dependent maximum singular value of a matrix transfer function T, the chordal distance between two controllers C_0 and C_1 and the frequency dependent stability margin of the stable closed loop system $[P, C]$ - see [3].

I. Introduction

Let P be a plant, whose exact transfer function is unknown to the designer, and C_0 a feedback controller that stabilises P. A safe controller change is a change of the current controller C_0 to a new controller C_1 with guaranteed bounds on the stability margin of the new closed loop system $[P, C_1]$. In [1] [2], we characterised safe controller changes on the basis of uncertain closed loop models.

Let P_0 be a closed loop model of the plant (resulting from an identification procedure) that satisfies the following assumptions

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A.1 The controller C_0 stabilises both P_0 and P.

A.2 P_0 is such that

$$\bar{\sigma}\left(T(P, C_0) - T(P_0, C_0), \omega\right) \leq \varepsilon_\omega \, \rho(P_0, C_0, \omega) \quad \forall \omega$$

where ε_ω is known and $0 \leq \varepsilon_\omega < 1$.

Then, the following lemma was derived in [1] [2].

Lemma 1: Let P_0, P and C_0 satisfy Assumptions (A1)-(A2) and assume that C_1 is a new controller that also stabilises P_0, then the condition

$$\kappa(C_0, C_1, \omega) \leq (\alpha_\omega - \varepsilon_\omega) \; \rho(P_0, C_0, \omega) \quad \forall \omega \qquad (1)$$

guarantees $[P, C_1]$ is stable with

$$\rho(P, C_1, \omega) \geq (1 - \alpha_\omega) \; \rho(P_0, C_0, \omega) \quad \forall \omega, \qquad (2)$$

where $\alpha_\omega \in [0, 1) \; \forall \omega$ is a pointwise upper bound on the percentage allowable degradation in the robust stability margin of $[P, C_1]$ when compared to that of $[P_0, C_0]$. □

Therefore, a controller change that satisfies (1) for some α_ω, such that $\alpha_\omega > \epsilon_\omega \, \forall \omega$, is a safe controller change with guaranteed stability margin given by (2).

Assume now that $T_* = T(P_0, C_*)$ is the desired closed loop be such that $C_1 = C_*$ does not satisfies (1). Therefore, it is not safe to implement directly C_* on the real plant. Our objective is then to find a controller C_1 which (a) is safe in the sense of (1) and (b) is somehow between C_* and C_0, where point (b) needs to be quantified.

In [1] [2] we took the following approach. To start with, let us denote $T_{*,1}$ a target intermediate reference model and let us assume that we have a method to solve model reference design control problems that satisfies the following condition.

A.3 Given a reference model $T_{*,1}$ and a nominal plant transfer function P_0, the controller C_1 is designed in such a way that $[P_0, C_1]$ is stable and the following inequality is satisfied:

$$\bar{\sigma}\left(T(P_0, C_1) - T_{*,1}, \omega\right) \leq c \, \bar{\sigma}\left(T(P_0, C_0) - T_{*,1}, \omega\right) \quad \forall \omega$$

where $c \in [0\,, 1]$ is known.

Notice, that this assumption is not very restrictive: it is trivially satisfied by choosing $c = 1$ and $C_1 = C_0$. For $c < 1$, it says that $T(P_0, C_1)$ is closer to $T_{*,1}$ than is $T(P_0, C_0)$, i.e. C_1 does a better job of achieving a closed-loop like $T_{*,1}$ than C_0. Then, the following result was derived in [1] [2].

Theorem 2: Let P_0, P, and C_0 satisfy Assumptions (A.1)-(A.2) and $\gamma \in [0, 1]$. If $T_{*,1}$ satisfies the following two conditions:

$$\bar\sigma\left(T(P_0,C_0)-T_{*,1},\omega\right) \le \frac{\alpha_\omega-\varepsilon_\omega}{1+c}\,\rho(P_0,C_0,\omega)\quad\forall\omega \qquad (3)$$

$$\bar\sigma\left(T_{*,1}-T_*,\omega\right) \le \frac{\gamma-c}{1+c}\,\bar\sigma\left(T(P_0,C_0)-T_*,\omega\right)\quad\forall\omega \qquad (4)$$

then a controller C_1, that satisfies Assumption (A.3), satisfies also the inequality

$$\bar\sigma\left(T(P_0,C_1)-T_*,\omega\right) \le \gamma\bar\sigma\left(T(P_0,C_0)-T_*,\omega\right)\quad\forall\omega \qquad (5)$$

and the safety condition (1). □

The above theorem states that if one chooses an intermediate reference model $T_{*,1}$ satisfying conditions (3) and (4) and designs the controller C_1 according to Assumption (A.3) then she/he achieves safety in the sense of (1) and model reference performance improvement in the nominal loop $[P_0, C_1]$, with respect to the desired reference model T_*, quantified by (5). Notice from inequality (4) one must always chose $\gamma > c$. A practical procedure to obtain such intermediate reference models was provided in [1] [2].

II. Guaranteed performance improvement for the unknown plant

In this section and hence in this brief contribution, we present additional conditions under which the unknown physical closed loop system $[P, C_1]$, once again with the guaranteed bound on its stability margin given by (2), is also guaranteed performance improvement in the sense of getting the $T(P, C_i)$ closer to T_* when C_0 changes to C_1, similar to what was previously achieved for the nominal closed-loop system $[P_0, C_i]$ in equation (5). Such a result comes at some cost; not surprisingly, the quality of identification has to be strengthened.

To start with, we have the following result.

Lemma 3: Suppose the hypotheses of Theorem 2 are fulfilled, including the inequality conditions in the theorem statement, and let $\eta_\omega \in (\gamma_\omega, 1]$. If the following additional inequality holds

$$\alpha_\omega \le 1+\frac{\varepsilon_\omega}{2}- \qquad (6)$$
$$\sqrt{\frac{\varepsilon_\omega^2}{4}+\frac{1}{\eta_\omega-\gamma_\omega}\frac{\varepsilon_\omega}{\rho(P_0,C_0,\omega)}\frac{1}{\bar\sigma\left(T(P_0,C_0)-T_*,\omega\right)}}\quad\forall\omega$$

then the controller C_1 satisfies also the inequality

$$\bar\sigma\left(T(P,C_1)-T_*,\omega\right) \le \eta_\omega\bar\sigma\left(T(P_0,C_0)-T_*,\omega\right)\quad\forall\omega\,. \qquad (7)$$

Proof See [2]

Then, in order to quantify guaranteed performance improvement for the unknown plant we use the following lower bound on $\bar\sigma\left(T(P,C_0)-T_*,\omega\right)$:

$$\bar\sigma(T(P,C_0)-T_*,\omega) \ge \bar\sigma(T(P_0,C_0)-T_*,\omega)-\varepsilon_\omega\rho(P_0,C_0,\omega)$$

which can be written also as

$$\bar\sigma(T(P,C_0)-T_*,\omega) \ge \bar\eta_\omega^-\bar\sigma(T(P_0,C_0)-T_*,\omega) \qquad (8)$$

$$\bar\eta_\omega^- = 1-\frac{\varepsilon_\omega\rho(P_0,C_0,\omega)}{\bar\sigma(T(P_0,C_0)-T_*,\omega)}\,.$$

Now, assuming that (7) is satisfied for some η_ω, from (7) and (8) we obtain

$$\bar\sigma\left(T(P,C_1)-T_*,\omega\right) \le \frac{\eta_\omega}{\bar\eta_\omega^-}\bar\sigma\left(T(P,C_0)-T_*,\omega\right)\quad\forall\omega\,. \qquad (9)$$

Notice that on the right hand side of the above inequality now the unknown plant P appears instead of P_0. Therefore, we can state that C_1 attains *guaranteed performance improvement for the unknown plant* if the inequality (7) is satisfied for some $\eta_\omega \le \bar\eta_\omega^-$. In this case the performance improvement is measured by (9).

In the following theorem, we give conditions under which inequality (6) is satisfied, and therefore (7) is also true, for $\eta_\omega \le \bar\eta_\omega^-$.

Theorem 4: Let P_0, P and C_0 satisfy Assumptions A.1-A.2. If the following additional inequality holds

$$1-\varepsilon_\omega \ge \qquad (10)$$
$$\frac{\varepsilon_\omega}{\rho(P_0,C_0,\omega)}\frac{1}{[\bar\sigma(T(P_0,C_0)-T_*,\omega)-\varepsilon_\omega\rho(P_0,C_0,\omega)]}\quad\forall\omega$$

then there exist: $\gamma_\omega \in [0, 1]$ that satisfies

$$\gamma_\omega \le \bar\eta_\omega^- - \frac{\varepsilon_\omega}{1-\varepsilon_\omega}\frac{1}{\rho(P_0,C_0,\omega)}\frac{1}{\bar\sigma(T(P_0,C_0)-T_*,\omega)}\quad\forall\omega\,, \qquad (11)$$

$c_\omega \in [0, \gamma_\omega]$, η_ω that satisfies $\eta_\omega \le \bar\eta_\omega^-$ and

$$\eta \ge \gamma + \frac{\varepsilon_\omega}{1-\varepsilon_\omega}\frac{1}{\rho(P_0,C_0,\omega)}\frac{1}{\bar\sigma(T(P_0,C_0)-T_*,\omega)}\ \forall\omega\,, \qquad (12)$$

and $\alpha_\omega \in [\varepsilon_\omega, 1)$ that satisfies inequality (6) such that the controller C_1 designed under the hypotheses of Theorem 2, including the inequality conditions in the theorem statement, satisfies also the inequality (7) and hence C_1 guarantees performance improvement for the unknown plant given by (9).

Proof See [2]

Condition (10) can be seen as an additional restriction on the Assumption A.2. It can be easily seen that it is an upper bound on ε_ω which is satisfied for ε_ω sufficiently small. Notice from (10) that for performance improvement on the real plant one needs $\bar\sigma(T(P_0,C_0)-T_*,\omega)-\varepsilon_\omega\rho(P_0,C_0,\omega) > 0$, i.e. the distance from the desired target must be greater than the identification error. If this is not the case, only improvement in worst case performance can be guaranteed - see [2].

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