On the Performance Assessment of Scalar Nonminimum-Phase Plants

Leonardo C. Kammer[†], Brian D. O. Anderson[‡], Robert R. Bitmead[♯]

[†]Pulp and Paper Research Institute of Canada
3800 Wesbrook Mall
Vancouver, BC V6S 2L9, Canada

[‡] Department of Systems Engineering, RSISE
The Australian National University
Canberra ACT 0200, Australia

[♯] Department of Mechanical & Aerospace Engineering
University of California at San Diego
La Jolla CA 92093-0411, USA

Abstract: The problem of minimum variance performance assessment of scalar nonminimum-phase plants requires almost full knowledge of the plant. Tyler and Morari (1995) claimed to have found a solution to this problem, which is still being cited in the literature (Huang and Shah 1999, Harris and Seppala 2001, Horch and Isaksson 2001, Eikens 2001, Ko and Edgar 2000), even though no one has ever reported using that result. This paper shows that that work is incomplete since the delay and the location of the "unstable" zeros are not enough to assess the minimum variance performance of a scalar nonminimum-phase plant.

1 Introduction

Performance Assessment attempts to answer a mainly economic question: *Suppose we were to invest money to achieve optimal regulation of a particular process, what is the level of achieved performance that we might expect?* That is, the focus is on assessing the best achievable performance of the process and in doing this **before** deciding to design the optimal controller. For minimum variance (MV) performance of scalar minimum-phase plants, Harris (1989) provides a simple and elegant answer, which relies on knowledge of the process delay only (once it is established that the process is indeed minimum-phase): the minimum achievable variance is given by the summed squared impulse response coefficients (of the transfer function from the noise source to the process output) up to the process delay. These coefficients are invariant to and can be estimated with any stabilizing feedback controller in place. The utility of this result is that it can be used to determine the likely benefits of control design and to provide a point of comparison for the current performance.

The problem of assessing the minimum variance performance of scalar nonminimum-phase plants (that is, those with "unstable" zeros) was tackled by Tyler and Morari (1995), and their conclusion was that MV performance assessment of this kind of plants depends on knowing only the plant delay and the position of the "unstable" zeros of the plant. The multivariable case was studied by Huang and Shah (1999), with the conclusion that the generalized unitary interactor matrix of the plant must be known in order to assess minimum variance performance from routine operating data. That book missed the opportunity to address the complexity of the scalar problem and assumed that the solution provided by Tyler and Morari (1995) was complete. Unfortunately the solution to the scalar case is not so simple, since almost all the plant model has to be known, as we argue here.

Consider a generic scalar linear plant described by the auto-regressive moving-average exogenous

0-7803-7896-2/03/$17.00 ©2003 IEEE

Proceedings of the American Control Conference
Denver, Colorado June 4-6, 2003

input (ARMAX) model

$$A(q^{-1})\,y(t) = B(q^{-1})\,u(t) + C(q^{-1})\,e(t), \qquad (1)$$

where $\{e(t)\}$ is a zero-mean white-noise process of variance σ_e^2 and

$$\begin{aligned}
A(q^{-1}) &= 1 + a_1\,q^{-1} + \cdots + a_{n_a}\,q^{-n_a},\\
B(q^{-1}) &= B_-(q^{-1})\,B_+(q^{-1})\,q^{-d},\\
&\quad B_-(q^{-1}) = b_{n_b^-} + b_{n_b^- -1}\,q^{-1} + \cdots +\\
&\qquad\qquad b_{1-}\,q^{-n_b^- -1} + q^{-n_b^-},\\
&\quad B_+(q^{-1}) = b_{1+} + b_{2+}\,q^{-1} + \cdots + b_{n_b^+}\,q^{-n_b^+},\\
C(q^{-1}) &= 1 + c_1\,q^{-1} + \cdots + c_{n_c}\,q^{-n_c}.
\end{aligned}$$

The roots of $B_+(q^{-1})$ are precisely the minimum-phase zeros of the plant, which is assumed to have no unit circle zeros. The polynomial $B_-(q^{-1})$ contains all the roots of $B(q^{-1})$ that are located on or inside the unit circle—the "unstable" zeros of the plant.

When the plant contains no zeros on or inside the unit circle ($n_b^- = 0$), the minimum variance control law is given by

$$u(t) = -\frac{M(q^{-1})}{B_+(q^{-1})\,F(q^{-1})}\,y(t),$$

where $F(q^{-1})$ and $M(q^{-1})$ are polynomials obtained from

$$C(q^{-1}) = A(q^{-1})\,F(q^{-1}) + q^{-d}\,M(q^{-1}),$$

with

$$\begin{aligned}
F(q^{-1}) &= 1 + f_1\,q^{-1} + \cdots + f_{d-1}\,q^{-d+1},\\
M(q^{-1}) &= m_1 + m_2\,q^{-1} + \cdots + m_{n_m}\,q^{-n_m+1},\\
n_m &= \max(n_c - d + 1, n_a).
\end{aligned}$$

Under this control action, the output of the plant becomes

$$y(t) = F(q^{-1})\,e(t). \qquad (2)$$

That is, $\{y(t)\}$ is a moving average process of order $d-1$. It is important to emphasize that, because of the delay in the plant, the first d elements of the closed-loop system impulse response are actually invariant to the control law. For MV systems, the only non-zero elements are these first d ones.

This powerful feature, relevant for MV performance monitoring and assessment (Harris 1989), is absent when the plant contains "unstable" zeros. Minimum variance systems with this kind of plant have at least one closed-loop pole away from the origin, therefore their closed-loop impulse responses are infinite in extent—as they also are under linear quadratic optimality. This is the main reason why the extension of Harris's result under a quadratic cost function is so convoluted (Kammer *et al.* 1998, Kammer 1998).

2 Nonminimum-Phase Plants

The minimum variance control law of plants with "unstable" zeros is derived in Åström and Wittenmark (1990), and modified here for our specific model description:

$$u(t) = -\frac{D(q^{-1})}{B_+(q^{-1})\,E(q^{-1})}\,y(t),$$

where $E(q^{-1})$ and $D(q^{-1})$ are obtained from

$$\begin{aligned}
&C(q^{-1})\,B_-{}^*(q^{-1}) =\\
&\qquad A(q^{-1})\,E(q^{-1}) + q^{-d}\,B_-(q^{-1})\,D(q^{-1}),
\end{aligned}$$

with

$$\begin{aligned}
B_-{}^*(q^{-1}) &= 1 + b_{1-}\,q^{-1} + \cdots + b_{n_b^-}\,q^{-n_b^-},\\
E(q^{-1}) &= 1 + e_1\,q^{-1} + \cdots + e_{n_e}\,q^{-n_e},\\
n_e &= d + n_{b^-} - 1,\\
D(q^{-1}) &= d_1 + d_2\,q^{-1} + \cdots + d_{n_d}\,q^{-n_d+1},\\
n_d &= \max(n_c - d + 1, n_a).
\end{aligned}$$

The output of the plant thus becomes

$$y(t) = \frac{E(q^{-1})}{B_-{}^*(q^{-1})}\,e(t). \qquad (3)$$

The part of the closed-loop impulse response in (3) that is invariant to the control law, i.e. the first d elements of the impulse response, can be isolated:

$$\begin{aligned}
y(t) &= \frac{C(q^{-1})}{A(q^{-1})}\,e(t) - \frac{q^{-d}\,B_-(q^{-1})\,D(q^{-1})}{A(q^{-1})\,B_-{}^*(q^{-1})}\,e(t)\\
&= F(q^{-1})\,e(t) +\\
&\qquad \frac{q^{-d}[B_-{}^*(q^{-1})\,M(q^{-1}) - B_-(q^{-1})\,D(q^{-1})]}{A(q^{-1})\,B_-{}^*(q^{-1})}\,e(t).
\end{aligned} \qquad (4)$$

Proceedings of the American Control Conference
Denver, Colorado June 4-6, 2003

From the analysis of

$$\frac{E(q^{-1})}{B_-{}^*(q^{-1})} = F(q^{-1}) + \frac{q^{-d}[B_-{}^*(q^{-1})\, M(q^{-1}) - B_-(q^{-1})\, D(q^{-1})]}{A(q^{-1})\, B_-{}^*(q^{-1})}$$

it can be concluded that

$$B_-{}^*(q^{-1})\, M(q^{-1}) - B_-(q^{-1})\, D(q^{-1}) = q^d\, L(q^{-1})\, A(q^{-1}), \quad (5)$$

where

$$L(q^{-1}) = E(q^{-1}) - F(q^{-1})\, B_-{}^*(q^{-1}),$$

with

$$L(q^{-1}) = l_1\, q^{-d} + l_2\, q^{-d-1} + \cdots + l_{n_l}\, q^{-d-n_l+1},$$
$$n_l = n_b^- + \max(0, n_c - n_a - d + 1).$$

The substitution of (5) into (4) results

$$y(t) = F(q^{-1})\, e(t) + \frac{L(q^{-1})}{B_-{}^*(q^{-1})}\, e(t). \quad (6)$$

This last expression highlights three features of the impulse response of the closed-loop system:

- The initial d terms are invariant to the control law.
- The terms from $d+1$ up to $d+n_l$ depend on $L(q^{-1})$ and $B_-(q^{-1})$, where $L(q^{-1})$ depends on $A(q^{-1})$ and $C(q^{-1})$, among others. Note that $n_l \geq 1$ is guaranteed in nonminimum-phase plants, given that $n_b^- \geq 1$.
- The following terms are determined from $B_-(q^{-1})$ and previous terms of the impulse response.

If it were not for the terms from $d+1$ to $d+n_l$, minimum variance performance monitoring and assessment of scalar nonminimum-phase plants would just require the extra knowledge of $B_-(q^{-1})$. Unfortunately the only information about the model that is not needed for MV performance assessment of nonminimum-phase plants is $B_+(q^{-1})$, i.e. the "stable" zeros of the plant.

3 Examples

Example 1: The nonminimum-phase ARMAX plant

$$(1 - 1.82\, q^{-1} + 0.828\, q^{-2})\, y(t) = -0.021333\, (1 - 1.3\, q^{-1})\, q^{-5}\, u(t) + (1 - 0.8\, q^{-1})\, e(t),$$

with $\sigma_e^2 = 1$, produces an open-loop variance in $y(t)$ of 17.1 units. Under Minimum Variance Control the variance of $y(t)$ reduces to 11.5 units. Figure 1 shows the responses of $y(t)$ to an impulse in $e(t)$ for these two cases and also for Tyler and Morari's conjecture: the impulse response decays at a ratio given by the inverse of the "unstable" zero of the plant, beginning immediately after the first 5 samples that are invariant to any control action (due to the delay). If feasible, this latter situation would have achieved a variance in $y(t)$ of 6.7 units.

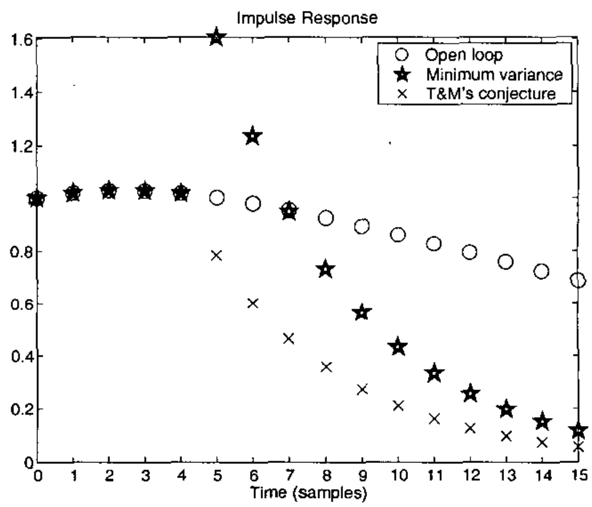

Figure 1: Responses of $y(t)$ to an impulse in $e(t)$ (Example 1).

Example 2: The continuous-time first-order plus dead-time plant

$$G(s) = \frac{0.84\, \mathrm{e}^{-1.26s}}{8.6s + 1}$$

produces a discrete-time model with an unstable inverse when sampled at 0.5 time units:

$$G_d(q^{-1}) = \frac{0.0231(q^{-3} + 1.05\, q^{-4})}{1 - 0.944\, q^{-1}}.$$

Proceedings of the American Control Conference
Denver, Colorado June 4-6, 2003

Let us assume that the noise model is

$$H_d(q^{-1}) = \frac{1}{1 - 0.99\,q^{-1}}.$$

The conversion of the system $\{G_d(q^{-1}), H_d(q^{-1})\}$ into an ARMAX format is straightforward. Figure 2 shows the responses of $y(t)$ to an impulse in $e(t)$ for the three cases considered in the previous example ($\sigma_e^2 = 1$). The open-loop variance in $y(t)$ is 50.3 units; under Minimum Variance Control the variance of $y(t)$ reduces to 2.96 units, and the variance assessed under Tyler and Morari's mechanism is 11.9 units.

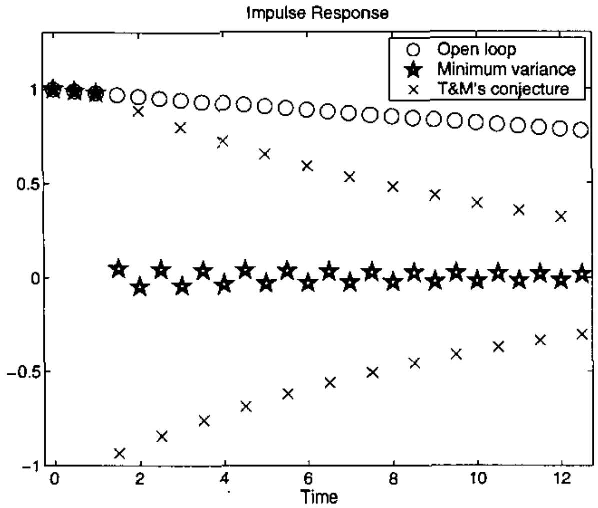

Figure 2: Responses of $y(t)$ to an impulse in $e(t)$ (Example 2).

4 Conclusion

Contrary to popular belief, in order to assess the minimum achievable output variance of a scalar nonminimum-phase process, it is necessary to know almost the whole model of the process. The only information that is not needed is the location of the "stable" zeros of the process.

References

Åström, K. J. and B. Wittenmark (1990). *Computer Controlled Systems: Theory and Design.* 2nd ed.. Prentice Hall. Englewood Cliffs, USA.

Eikens, B. (2001). Control loops - designing, assessment and analysis. *Wochenblatt Für Papierfabrikation* **129**(3), 130–134.

Harris, T. J. (1989). Assessment of control loop performance. *The Canadian Journal of Chemical Engineering* **67**, 856–861.

Harris, T. J. and C. T. Seppala (2001). Recent developments in controller performance monitoring and assessment techniques. In: *Chemical Process Control VI.* Tuscon, USA.

Horch, A. and A. J. Isaksson (2001). Assessment of the sampling rate in control systems. *Control Engineering Practice* **9**(5), 533–544.

Huang, B. and S.L. Shah (1999). *Performance Assessment of Control Loops: Theory and Applications.* Springer Verlag.

Kammer, L. C. (1998). Performance Monitoring and Assessment of Optimality under a Linear Quadratic Criterion. PhD thesis. The Australian National University. Canberra, Australia.

Kammer, L. C., R. R. Bitmead and P. L. Bartlett (1998). Optimal controllers properties from closed-loop experiments. *Automatica* **34**(1), 83–91.

Ko, B.-S. and T. F. Edgar (2000). Performance assessment of cascade control loops. *AIChE Journal* **46**(2), 281–291.

Tyler, M. L. and M. Morari (1995). Performance assessment for unstable and nonminimum-phase systems. In: *Preprints of IFAC Workshop on On-line Fault Detection and Supervision in the Chemical Process Industries.* Newcastle-upon-Tyne, UK. pp. 200–205.