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Safe adaptive controller changes based on reference model adjustments

Andrea Lecchini*
Department of Engineering
University of Cambridge - UK
al394@cam.ac.uk

Alexander Lanzon and Brian D.O. Anderson†
Research School of Information Sciences and Engineering
The Australian National University and National ICT Australia
{Alexander.Lanzon, Brian.Anderson}@anu.edu.au

***Abstract*— A controller change from a current controller which stabilizes the plant to a new controller, designed on the basis of an approximate model of the plant and with guaranteed bounds on the stability properties of the true closed loop, is called a safe controller change. In this paper, we present a model reference approach to the determination of safe controller changes on the basis of approximate closed loop models of the plant and robust stability results in the ν-gap.**

I. Introduction

The identification of an unknown plant in practice always delivers an approximate model. It is a recognized fact that the mismatch between the plant and the identified model is influenced by the experimental conditions under which the identification has been carried out. This fact has been broadly investigated in the last ten years in the context of closed loop identification. For a recent overview on this area the reader is referred to [5].

A typical closed loop identification scenario is as follows. Let P be an unknown plant operating in feedback connection with a controller C_0 and let P_0 be a model identified from data collected under such an operating condition. Let $[P, C_0]$ denote the closed loop system formed by the plant P and the controller C_0. Then, the model P_0 is expected to give rise to a closed loop system $[P_0, C_0]$ which is similar to $[P, C_0]$ and in this sense P_0 approximates P. However, it is not guaranteed that for some different controller C_1 the closed loop systems $[P, C_1]$ and $[P_0, C_1]$ would be similar. In particular, taking the case $C_1 = 0$, it is evident (and well known) that the transfer functions P and P_0 need not be close.

The observation above means that there are practical limitations applying in the redesign of controllers based on identified models. Even if the controller C_1 has been designed on the basis of the model P_0 and $[P_0, C_1]$ has very good performance, since P_0 is only a limited description of P, in general the designer is not assured that also $[P, C_1]$ will have good performance. In fact, in certain situations the closed loop system $[P, C_1]$ could even be unstable.

The observation in effect imposes a need for small controller changes [1]–[4], [9], where "smallness" is a concept which still needs definition. The rationale behind this is intuitive: if the change between C_0 and C_1 is small enough, then also the change between the closed loop transfer functions of $[P, C_0]$ and of $[P, C_1]$ should be small [12]. Thus in principle, by limiting the change in the controller to be sufficiently small, one can bound the degradation of the stability properties that can occur in the actual closed loop. A controller change with guaranteed bounds on the stability properties of the closed loop is called a safe change.

The quantification of safe controller changes can be put on a solid mathematical ground by using the ν-gap metric [12]. The idea of quantifying small controller changes via the ν-gap metric for the purposes of adaptive control was introduced in [3]. In [1], [2], [9], the idea has been applied to multiple model adaptive control in order to assure safe switchings in a set of candidate controllers. In [4], safe controller changes have been connected to the Youla-Kucera parametrization of all stabilizing controllers.

In this contribution, and in the companion paper [10], we develop a model reference approach to the concept of safe controller changes. Let us recall that in model reference control, the objective is to make the designed closed loop as close as possible to a certain desired reference model, since this reference model captures the desired performance specifications. In certain situations, the exact solution of a model reference control problem can be obtained via simple algebraic calculations (as will be the case in this paper). The resulting controller is thus calculated on the basis of the known plant P_0 and we would like to implement this new controller on the unknown plant P since the reference model captures well the desired performance specifications. However, it may be the case that this new controller is a considerable distance away from the pre-change controller and hence the above discussion on safety implies that we should not implement this new controller directly on the unknown plant P in the absence of further information.

Instead, in this paper, we take the following approach. We first find a temporary intermediate reference model that is in some sense between the currently achieved closed-loop transfer function and the ultimately desired reference model. We choose this temporary intermediate reference model such

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that the new controller resulting from the model reference control problem involving this intermediate reference model is a safe distance away from the pre-change controller. Thus, we can implement this resulting new controller on the unknown plant P. Then we repeat the above procedure until the safety region around the currently achieved transfer function does not permit us to move any further in the direction of the ultimately desired reference model. When this happens, we re-identify the unknown plant P to obtain a new plant model P_1 and then all the above is repeated.

The important point to understand here is that this particular procedure for iterative identification and control redesigns ensures safety at each step and hence we are guaranteed that the performance is always improving (in the sense of approaching the ultimate reference model). Note that at each step in the safe controller changes described above, the designer can perform a reduction of the McMillan degree of the controller in order to obtain an approximate but lower order solution to the intermediate model reference problem. This is permitted but care must be exercised since the lower order controller too needs to be a safe distance away from the pre-change controller.

The paper is organized as follows. Some results on robust stability, from [12], are recalled in Section 2. The initial assumptions are stated in Section 3. The guidelines for the choice of a safe intermediate reference model are discussed in Section 4. A particular parametrised set of such intermediate reference models is given in Section 5. The task of controller reduction is considered in Section 6. In Section 7, an illustrative numerical example is proposed. The conclusions, in Section 8, end the paper.

II. VINNICOMBE'S TOOLS FOR ROBUST STABILITY

In this section, we introduce the notation and we recall some robust stability results from [12].

We shall consider MIMO linear time-invariant systems. In the notation, we will not distinguish between the continuous-time and the discrete-time cases. The frequency response of a transfer function T is indicated by $T(\omega)$. If T is a continuous-time transfer function it should be read $T(\omega) = T(j\omega)$. If T is a discrete-time transfer function then $T(\omega) = T(e^{j\omega})$. The symbol $\bar{\sigma}(\cdot)$ denotes the maximum singular value of a matrix. The frequency dependent maximum singular value of a matrix transfer function T is denoted by $\bar{\sigma}(T, \omega)$.

The transfer function of the plant is denoted by P. We denote by $T(P, C)$ the closed loop transfer function of the feedback connection between the plant P and a controller C. It is given by

$$T(P, C) = \begin{bmatrix} P \\ I \end{bmatrix} (I - CP)^{-1} [-C \quad I] \,.$$

The results of Theorems 1 and 2 below provide sufficient conditions on the modification of a current stabilizing controller to a new controller with guaranteed preservation of stability. Firstly, we introduce the following definitions.

Definition 1: (Condition $\mathcal{C}$) Two continuous-time transfer functions C_0 and C_1 satisfy Condition $\mathcal{C}$ if

$$\det(I + C_1(\omega)^* C_0(\omega)) \neq 0 \ \forall \omega \qquad \text{and}$$
$$\text{wno}\left(\det(I + C_1^* C_0)\right) + \eta(C_0) - \bar{\eta}(C_1) = 0,$$

where wno(.) indicates the winding number of the Nyquist diagram of a scalar transfer function, evaluated on a contour along the imaginary axis and indented to the right around any pure imaginary pole, and $\eta(C)$ ($\bar{\eta}(C)$) is the number of open (closed) right-half-plane poles of C.

The statement of Condition $\mathcal{C}$ for discrete-time transfer functions is similar but with the obvious modifications in the wording when considering the z-plane instead of the s-plane.

Definition 2: (Chordal distance) The chordal distance $\kappa(C_0, C_1, \omega)$ is given by

$$\kappa(C_0, C_1, \omega) =$$
$$\bar{\sigma}\left((I + C_1 C_1^*)^{-\frac{1}{2}} (C_1 - C_0) (I + C_0^* C_0)^{-\frac{1}{2}}, \omega \right) .$$

Definition 3: (Frequency dependent stability margin) The frequency dependent stability margin of the stable closed loop system $[P, C]$ is given by

$$\rho(P, C, \omega) = \bar{\sigma}\left(T(P, C), \omega\right)^{-1} .$$

If the closed loop system $[P, C]$ is unstable, we set $\rho(P, C, \omega) = 0$.

Theorem 1: Let $[P, C_0]$ be internally stable and

$$\kappa(C_0, C_1, \omega) < \rho(P, C_0, \omega) \quad \forall \omega \,.$$

Then the closed loop system $[P, C_1]$ is internally stable if and only if the pair C_0, C_1 satisfies Condition $\mathcal{C}$. Further, if C_0, C_1 satisfies Condition $\mathcal{C}$,

$$\rho(P, C_1, \omega) \geq \rho(P, C_0, \omega) - \kappa(C_0, C_1, \omega) \,.$$

Proof See [12, pages 136-137]. □

The theorem below links the modifications of the controller to the corresponding changes which occur in the closed loop transfer function.

Theorem 2: Let $[P, C_0]$ and $[P, C_1]$ be internally stable. Then

$$\kappa(C_0, C_1, \omega) \leq \bar{\sigma}\left(T(P, C_0) - T(P, C_1), \omega\right) \leq \frac{\kappa(C_0, C_1, \omega)}{\rho(P, C_0, \omega) \rho(P, C_1, \omega)}$$

Proof See [12, page 159]. □

III. INITIAL ASSUMPTIONS

We assume that the exact transfer function of the plant P is unknown. However, we assume that the plant is operating in feedback connection with a known stabilizing controller C_0 (as a particular case, if the plant is stable, this controller could be $C_0 = 0$) and that, on the basis of data obtained in this operating condition, a model P_0, which approximates P in a closed loop sense, has been identified. More precisely, we make the following assumptions.

Identification Assumptions

A.1 The controller C_0 stabilizes both P_0 and P.

A.2 P_0 is such that

$$\bar{\sigma}\left(T(P,C_0)-T(P_0,C_0),\omega\right) \leq \varepsilon_\omega\, \rho(P_0,C_0,\omega) \quad \forall\omega$$

where ε_ω is known and $0 \leq \varepsilon_\omega < 1$.

In Assumption (A.2) we basically require that the modelled closed loop $[P_0, C_0]$ captures the stability property and approximate frequency domain behaviour of the real closed loop $[P, C_0]$. If, from the identification procedure, one obtains a bound, say Δ_ω, directly on the closed loop error $\bar{\sigma}\left(T(P,C_0)-T(P_0,C_0),\omega\right)$ ([4], [6], [7]), then, since $\rho(P_0,C_0,\omega)$ is known, ε_ω can be calculated as $\varepsilon_\omega = \frac{\Delta_\omega}{\rho(P_0,C_0,\omega)}$. Notice that a poorly designed C_0 will generally lead to a large value of $\bar{\sigma}(T(P_0,C_0),\omega)$ for some ω, and thus a small value of $\rho(P_0,C_0,\omega)$. Poor designs then require better modelling of P by P_0, by forcing a smaller value for the left side of the inequality in (A.2).

Our objective is to perform the redesign of the controller with safety. We wish to obtain a controller C_1 which gives a designed closed loop $[P_0, C_1]$ that has better performance than $[P_0, C_0]$. At the same time, we want to assume that the new controller C_1 realizes a certain level of the stability margin when it is connected to the unknown plant P.

The key point to obtain a guaranteed stability margin for $[P, C_1]$ is that under the identification assumptions one can compute a lower-bound to the stability margin $\rho(P, C_1, \omega)$ of $[P, C_1]$ as shown next.

Lemma 3: Given P_0, P and C_0 satisfying Assumptions (A.1)–(A.2) and assume that C_1 is a new controller that also stabilises P_0, then the condition

$$\kappa(C_0,C_1,\omega) \leq (\alpha_\omega - \varepsilon_\omega)\; \rho(P_0,C_0,\omega) \quad \forall\omega \tag{1}$$

guarantees

$$\rho(P,C_1,\omega) \geq (1-\alpha_\omega)\; \rho(P_0,C_0,\omega) \quad \forall\omega, \tag{2}$$

where $\alpha_\omega \in [0,1)$ $\forall\omega$ is a pointwise upper bound on the percentage allowable degradation in the robust stability margin of $[P, C_1]$ when compared to that of $[P_0, C_0]$.

Proof See [10]. □

This result is saying that all the controllers C_1 that stabilize P_0 and satisfy (1) are guaranteed to satisfy also (2) (i.e. C_1 stabilises P with a certain guarantee on the achieved robust stability margin).

Note that $\alpha_\omega \in [0,1)$ $\forall\omega$ is a pointwise upper bound on the percentage allowable degradation in the robust stability margin of $[P, C_1]$ when compare to that of $[P_0, C_0]$ because condition (2) can be rewritten as

$$\frac{\rho(P_0,C_0,\omega)-\rho(P,C_1,\omega)}{\rho(P_0,C_0,\omega)} \leq \alpha_\omega \quad \forall\omega.$$

Furthermore, from safety condition (1), it is clear that α_ω should be chosen to be greater than ε_ω at every ω. This means that the allowable percentile degradation in the robust stability margin has to always be greater than the percentile identification error, as one would expect. From condition (1), it can be seen that for fixed ε_ω a larger α_ω will result in a larger controller set whereas from condition (2), it can be seen that a larger α_ω will result in a smaller guaranteed robust stability margin. This is an interesting tradeoff in selecting α_ω.

IV. Safe reference models

We now assume that a controller C_* has been designed using the model P_0 through some design method, and that the closed loop transfer function $T_* = T(P_0, C_*)$ has the desired performance. We also assume that the controller C_* is sufficiently different from C_0 that $C_1 = C_*$ does *not* satisfy the inequality (1). Therefore, it is not safe to implement directly C_* on the real plant.

In this section, we define, on the basis of the knowledge of T_* and the identification assumptions, a simple model reference control problem, with temporary intermediate reference model $T_{*,1}$, such that the solution controller C_1 (a) gives a designed closed loop $[P_0, C_1]$ with better performance than $[P_0, C_0]$ and (b) satisfies the safety condition (1). In saying that $[P_0, C_1]$ has "better performance" than $[P_0, C_0]$, we mean that $T(P_0, C_1)$ represents a more attractive closed-loop, i.e. is closer to the ideal closed-loop T_* than is $T(P_0, C_0)$. This is formally stated in the following definition:

Definition 4 (Performance Improvement): Given an ultimately desired reference model T_* and a nominal plant transfer function P_0, the controller C_1 is said to achieve "performance improvement" when compared to the controller C_0 if the following inequality is satisfied:

$$\bar{\sigma}\left(T(P_0,C_1)-T_*,\omega\right) \leq \gamma\bar{\sigma}\left(T(P_0,C_0)-T_*,\omega\right) \quad \forall\omega, \tag{3}$$

where $\gamma \in [0,1]$ is some scalar chosen at the design stage.

Of course, C_0 and C_1 are actually connected to the real plant P, and it is a separate question as to whether real performance improves, in contrast to the nominal performance computed with P_0.

In our derivation, we will also make use of the following assumption which allows some extra freedom when solving the intermediate model reference problem involving $T_{*,1}$. This extra freedom will be used later for controller order reduction purposes. After the assumption, we describe how $T_{*,1}$ may be chosen.

Control Design Assumption

A.3 Given a temporary intermediate reference model $T_{*,1}$ and a nominal plant transfer function P_0, the controller C_1 is designed in such a way that $[P_0, C_1]$ is stable and the following inequality is satisfied:

$$\bar{\sigma}\left(T(P_0,C_1)-T_{*,1},\omega\right) \leq c\,\bar{\sigma}\left(T(P_0,C_0)-T_{*,1},\omega\right) \;\forall\omega$$

where $c \in [0\,,1]$ is known.

This assumption is not very restrictive: it is trivially satisfied by choosing $c = 1$ and $C_1 = C_0$. For $c < 1$, it says that $T(P_0, C_1)$ is closer to $T_{*,1}$ than is $T(P_0, C_0)$, i.e. C_1 does a better job of achieving a closed-loop like $T_{*,1}$ than C_0.

It is in fact possible to choose $T_{*,1}$ in a parametrised way so that there exists a controller $C_{*,1}$ such that $T_{*,1}$ is exactly attainable for the model P_0, i.e. $T_{*,1} = T(P_0, C_{*,1})$. For example, we can set

$$T_{*,1} = b\,T_* + (1-b)T(P_0, C_0)$$

where b is a scalar parameter in $[0, 1]$. The reader is referred to Theorem 5 to see that such a parametrisation for $T_{*,1}$ is attainable by some controller $C_{*,1}$. Actually, more general parametrisations are possible which ensure $T_{*,1}$ is attainable, as discussed in Section 5.

As a digression, we remark that there are practical advantages in considering situations where $T_{*,1} \neq T(P_0, C_1)$. For instance, it may well be the case that a low order controller C_1 is desired and the degree constraint makes impossible the exact achieving of $T_{*,1}$. One could initially find $C_{*,1}$ with $T_{*,1} = T(P_0, C_{*,1})$ and then find a low order approximation C_1 of $C_{*,1}$, which would need to obey the inequality of Assumption (A.3). There will be further discussion of how to achieve this controller reduction in Section 6.

In the remainder of this section, we focus on the characteristics of those $T_{*,1}$ for which safety in controller change and performance improvement are guaranteed.

Theorem 4: Given P_0, P, C_0 and C_1 that satisfy Assumptions (A.1)–(A.3), then the following two conditions:

$$\bar{\sigma}\left(T_{*,1} - T(P_0, C_0), \omega\right) \leq \frac{\alpha_\omega - \varepsilon_\omega}{1+c}\,\rho(P_0, C_0, \omega) \quad \forall\omega \quad (4)$$

$$\bar{\sigma}\left(T_{*,1} - T_*, \omega\right) \leq \frac{\gamma - c}{1+c}\,\bar{\sigma}\left(T(P_0, C_0) - T_*, \omega\right) \quad \forall\omega \quad (5)$$

together guarantee safety in the controller change and performance improvement in the sense of Definition 4.

Proof See [10]. □

Notice from (5), that one cannot require a smaller upper bound (γ) on performance improvement than the upper bound (c) on the allowed degradation due to the use of $T(P_0, C_1)$ instead of $T_{*,1}$ (i.e. this case usually being for controller order reduction purposes).

V. A SET OF SAFE REFERENCE MODELS

In this section, we will consider a set of possible reference models $T_{*,1}$. To this end, let $T_{*,1}$ be parameterized as:

$$T_{*,1} = B\,T_* + (1-B)T(P_0, C_0) \quad (6)$$

where $B \in \mathscr{RH}_\infty$ is a SISO transfer function.

Notice that for any $T_{*,1}$ given by (6) there always exists a controller $C_{*,1}$ such that $T_{*,1} = T(P_0, C_{*,1})$. Indeed, we have the following result.

Theorem 5: Given a reference model $T_{*,1}$ in the form (6), there exists a controller $C_{*,1}$ such that $T(P_0, C_{*,1}) = T_{*,1}$. Defining $S_0 = [I - C_0 P_0]^{-1}$ and $S_* = [I - C_* P_0]^{-1}$, this controller $C_{*,1}$ is given by:

$$C_{*,1} = [S_0 + B(S_* - S_0)]^{-1}[S_0 C_0 + B(S_* C_* - S_0 C_0)]\,.$$

Proof See [10]. □

Using parametrization (6), the safety and performance improvement conditions (4) and (5) on $T_{*,1}$ can be translated into conditions on B.

In fact, since

$$\bar{\sigma}\left(T_{*,1} - T(P_0, C_0), \omega\right) = |B(\omega)|\bar{\sigma}\left(T_* - T(P_0, C_0), \omega\right)$$
$$\bar{\sigma}\left(T_{*,1} - T_*, \omega\right) = |1 - B(\omega)|\,\bar{\sigma}\left(T_* - T(P_0, C_0), \omega\right)\,,$$

we obtain that (4) and (5) are respectively equivalent to

$$|B(\omega)| \leq \frac{\alpha_\omega - \varepsilon_\omega}{1+c}\,\frac{\rho(P_0, C_0, \omega)}{\bar{\sigma}\left(T_* - T(P_0, C_0), \omega\right)} \;\forall\omega \quad (7)$$

$$|1 - B(\omega)| \leq \frac{\gamma - c}{1+c} \quad \forall\omega\,. \quad (8)$$

In this section, we will illustrate how transfer functions $B \in \mathscr{RH}_\infty$, which satisfy (7) and (8), can be constructed.

To start with, let us introduce a scalar transfer function F with $F, F^{-1} \in \mathscr{RH}_\infty$ such that

$$|F(\omega)|^{-1} \lessapprox \frac{\alpha_\omega - \varepsilon_\omega}{1+c}\,\frac{\rho(P_0, C_0, \omega)}{\bar{\sigma}\left(T_* - T(P_0, C_0), \omega\right)}\,. \quad (9)$$

Notice that such a transfer function F can be easily found with standard techniques. Moreover, by increasing the order of F, the approximation error can be made arbitrarily small. Now, we have that (7) can be equivalently rewritten as

$$|B(\omega)| \leq \left|F^{-1}(\omega)\right| \quad \forall\omega$$

which is equivalent to

$$\|BF\|_\infty \leq 1\,. \quad (10)$$

In a similar way, let us denote

$$\bar{\gamma} = \frac{\gamma - c}{1+c}$$

so that inequality (8) can be equivalently rewritten as

$$\left\|\frac{1}{\bar{\gamma}} - \frac{1}{\bar{\gamma}}B\right\|_\infty \leq 1\,. \quad (11)$$

Now the following condition, while not equivalent to (10) and (11), certainly implies (10) and (11):

$$\left\|\begin{bmatrix} 0 \\ \frac{1}{\bar{\gamma}} \end{bmatrix} + \begin{bmatrix} F \\ -\frac{1}{\bar{\gamma}} \end{bmatrix} B\right\|_\infty \leq 1\,. \quad (12)$$

Any $B \in \mathscr{RH}_\infty$ satisfying (12) defines a $T_{*,1}$ by (6) for which a safe, performance improving $C_{*,1}$ can be found. The problem of finding all the B satisfying (12) is a model matching problem which can be solved as shown in [8].

The solution, when it exists (see condition (13) below), is provided in the following theorem.

Theorem 6: Let F be a scalar transfer function with $F, F^{-1} \in \mathscr{RH}_\infty$ and let $\bar{\gamma} \in [0,1)$ be such that

$$\bar{\gamma}^2 > 1 - \frac{1}{|F(\omega)|^2} \quad \forall \omega \,. \tag{13}$$

Then, the set of all transfer functions $B \in \mathscr{RH}_\infty$ satisfying (12) is given by

$$B = \left[\frac{\bar{\gamma}}{\sqrt{1-\bar{\gamma}^2}} RU + \frac{1}{1-\bar{\gamma}^2} \right]^{-1} \tag{14}$$

where $R, R^{-1} \in \mathscr{RH}_\infty$,

$$R^*(\omega)R(\omega) = \left(\frac{1}{1-\bar{\gamma}^2} - |F(\omega)|^2 \right) \tag{15}$$

and $U \in \mathscr{RH}_\infty$ with $\|U\|_\infty \leq 1$.
Proof See [10]. □

In order to calculate the set of all B satisfying (12), one has to calculate the transfer function R. One way is first to calculate an F satisfying (9) and then solve the equation (15) by spectral factorization. Another possible way is to calculate directly R stable and minimum phase such that

$$|R(\omega)| \lessapprox \sqrt{\frac{1}{1-\bar{\gamma}^2} - \left(\frac{\alpha_\omega - \varepsilon_\omega}{1+c} \, \frac{\rho(P_0, C_0, \omega)}{\bar{\sigma}\left(T_* - T(P_0,C_0),\omega\right)} \right)^{-2}} \,. \tag{16}$$

VI. Controller order reduction

In this section, we give clues on how to find a reduced order controller C_1 starting from $C_{*,1}$ and which is guaranteed to satisfy Assumption (A.3). Let us recall that $C_{*,1}$ is such that $T_{*,1} = T(P_0, C_{*,1})$. In general, any algorithm for controller order reduction can be used to obtain C_1. The important issue is that the final controller C_1 must satisfy the Assumption (A.3) for a given $c \in [0,1]$.

We refer the reader to [11] and [10].

In [11, Section 4.3] an algorithm for controller reduction is given. The algorithm returns a reduced order controller and a bound on the error between the transfer function of the closed loop formed by the initial controller and the plant model and the transfer function of the closed loop formed by the reduced controller and the plant model. The bound depends on the value of a parameter ϵ which is an input parameter, chosen by the user, in the algorithm.

In [10] it is shown how the parameter ϵ must be chosen if one wants to apply the algorithm of [11] to $C_{*,1}$ in order to obtain a controller C_1 that satisfies Assumption (A.3).

VII. Simulation example

Let the plant P (actually unknown) and the current stabilizing controller C_0 be:

$$P(z) = \frac{0.08942z + 0.1788}{z^4 - 1.4z^3 + 1.22z^2 - 0.862z + 0.5785}$$

$$C_0(z) = \frac{0.044303z^2 + 0.19225z - 0.10075}{z^2 - 0.931971z + 0.318336} \,.$$

We assume that a model P_0 has been obtained from closed loop identification together with an upper bound Δ_ω on the closed loop error, i.e $\bar{\sigma}\left(T(P,C_0) - T(P_0,C_0),\omega\right) \leq \Delta_\omega \ \forall\omega$. The model P_0 is given by

$$P_0(z) = \frac{-0.13274z + 0.253774}{z^2 - 1.60447z + 0.892884} \,.$$

The Bode plots of P and of P_0 are displayed in Fig. (1.a). The upper bound Δ_ω and the actual closed loop error $\bar{\sigma}\left(T(P,C_0) - T(P_0,C_0),\omega\right)$ are displayed in Fig. (1.b). Notice that the error is mainly confined to the high frequency region. It can be shown that the model P_0 satisfies the Identification Assumptions (A.1)–(A.2). The values of ε_ω in Assumption (A.2) have been obtained as: $\varepsilon_\omega = \frac{\Delta_\omega}{\rho(P_0,C_0,\omega)}$.

We specify that the required upper bound α_ω on the stability margin degradation, for an update from the current controller C_0 to a new controller C_1 on the plant P, is given by $\alpha_\omega = 0.7 \ \forall\omega$. It can be shown that with this choice we have $\alpha_\omega > \varepsilon_\omega$, which is always necessary.

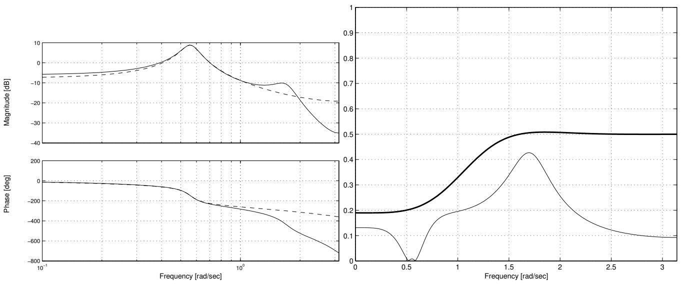
On the basis of the model P_0, the following LQG-optimal controller C_* has been designed (of course, one can actually design C_* through any design methodology)

$$C_*(z) = \frac{0.23045z^2 + 0.0723z}{z^2 - 0.54294z + 0.08411} \,.$$

The designed closed loop is denoted $T_* = T(P_0, C_*)$. However, $C_1 = C_*$ does not satisfy the original safety condition (1): in Fig. (2.a), $\kappa(C_0, C_*, \omega)$ and $\bar{\sigma}\left(T_* - T(P_0,C_0),\omega\right)$ are shown together with $\left(\alpha_\omega - \varepsilon_\omega\right) \rho(P_0, C_0, \omega)$ (i.e. plots are given of the left-hand and right-hand sides of the original safety condition (1) and the alternative safety condition (4) when $c = 0$ and the ultimately desired reference model T_* is being considered). Both safety conditions are violated outside the low frequency region.

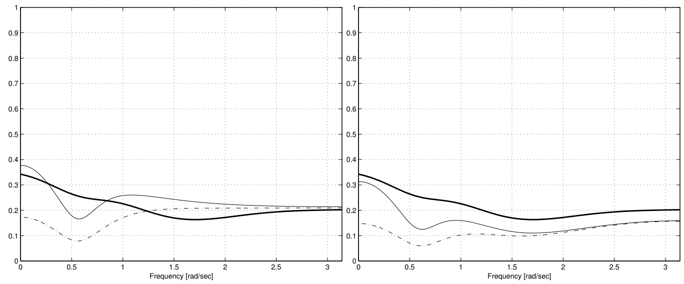
The design of a safe reference model $T_{*,1}$, which satisfies (4) (for safety) and (5) (for performance), has been performed as discussed in this paper. The transfer function $T_{*,1}$ has been chosen in the form (6). The parameters $\gamma = 1$ and $c = 0.1$ have been selected (correspondingly we obtained $\bar{\gamma} = 0.82$). With these values, the parametrization (14) of all the solution of (12) has been constructed (in fact $\gamma = 1$ and $c = 0.1$ satisfy the condition (13) for the existence of (14)). The transfer function R in (14) has McMillan degree equal to 2 and has been designed according to (16).

As for the choice of U in (14), the selection $U = const$ with $const \in [-1,1]$ has been considered. It turned out



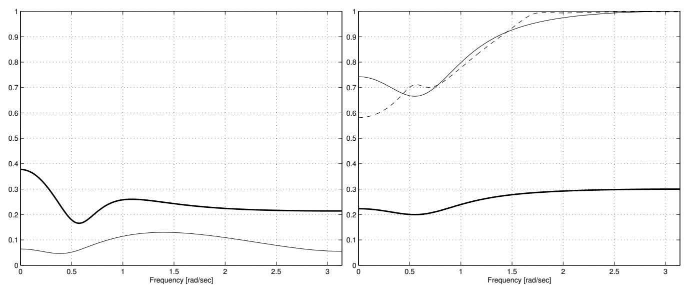
(a) (b)

Fig. 1. (a) Bode plots of $P(z)$ (continuous) and of $P_0(z)$ (dashed). (b) The overbound Δ_ω (bold) and the actual closed loop error $\bar{\sigma}\left(T(P, C_0) - T(P_0, C_0), \omega\right)$ (thin).



(a) (b)

Fig. 2. (a) The safety bound $(\alpha_\omega - \varepsilon_\omega)\ \rho(P_0, C_0, \omega)$ (bold), $\bar{\sigma}\left(T_* - T(P_0, C_0), \omega\right)$ (thin) and the chordal distance $\kappa(C_0, C_*, \omega)$ (dash-dotted). (b) The safety bound (bold), $\bar{\sigma}\left(T(P_0, C_0) - T(P_0, C_1), \omega\right)$ (thin) and the chordal distance $\kappa(C_0, C_1, \omega)$ (dash-dotted).



(a) (b)

Fig. 3. (a) The distance $\bar{\sigma}\left(T(P_0, C_0) - T_*, \omega\right)$ (bold) and the distance $\bar{\sigma}\left(T(P_0, C_1) - T_*, \omega\right)$ (thin). (b) The lower bound $(1 - \alpha_\omega)\ \rho(P_0, C_0, \omega)$ (bold) and the stability margins $\rho(P_0, C_0)$ (thin) and $\rho(P, C_1)$ (dashed).

that the best achievements in performance (i.e. the smallest $\bar{\sigma}\left(T_{*,1} - T_*, \omega\right)$) occurred for negative values of $const$. In the following we illustrate the choice $U = -0.9$.

By choosing $U = -0.9$, we obtained $T_{*,1}$ with McMillan degree equal to 8 and the corresponding $C_{*,1}$ with McMillan degree equal to 6. In this case, by reducing the controller $C_{*,1}$ via the approach of Section VI, we could find a controller C_1 with McMillan degree equal to 2 satisfying the Assumption (A.3) . The controller C_1 is given by

$$C_1(z) = \frac{0.1655z^2 + 0.16378z + 0.06336}{z^2 - 0.32174z + 0.06547}\,.$$

The quantities $\bar{\sigma}\left(T(P_0, C_0) - T(P_0, C_1), \omega\right)$ and $\kappa(C_0, C_1, \omega)$ are shown in Fig. (2.b). The quantity $\bar{\sigma}\left(T(P_0, C_1) - T_*), \omega\right)$ is shown in Fig. (3.a). Finally, the stability margin $\rho(P, C_1, \omega)$ obtained by connecting the controller C_1 to the plant P is shown in Fig. (3.b).

VIII. Conclusions

In this paper, we have proposed a model reference approach to the design of controller changes, that ensures safety (of the real closed loop) and performance improvement (of the nominal closed loop), on the basis of some bounds on the error between the modelled closed loop and the actual closed loop. General guidelines for the design of these controller modifications have been given. A practical procedure to construct them in a particular parametrization has also been provided.

IX. REFERENCES

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