Proceedings of the 41st IEEE
Conference on Decision and Control
Las Vegas, Nevada USA, December 2002

Robust input disturbance suppression for nonlinear systems based on multiple model adaptive control

Steven W. Su*†, Brian D. O. Anderson†, Thomas S. Brinsmead†

† *RSISE Australian Nat. Uni. and National ICT Australia Acton ACT, 0200, Australia*

* *email: steven.su@anu.edu.au*

Abstract

In this paper, we make initial steps in the direction of nonlinear multiple model adaptive control by focusing on a contrived example in which an unknown parameter has a nominal value in one of the two intervals $[-0.3, -0.1]$ and $[0.1, 0.3]$ and can switch between them. The design should suppress a constant input disturbance. We discuss the use of a multi-estimator and multi-controller to achieve the goal, with the construction of the stable multi-estimator being based on stable kernel representation of the plant. Simulation results indicate that satisfactory performance is achieved.

1 Introduction

This brief paper serves to marry two ideas: controller design to secure constant input disturbance rejection for a nonlinear system, and multiple model adaptive control. To illustrate the ideas, we shall work with an underlying nonlinear plant, containing a parameter which can take values in one of two non-overlapping intervals (and the parameter can switch intervals but not extremely frequently). Robust control design is required, with an adaptive overlay, taken here to be based on multiple model adaptive control. Paper [3] actually provides a way to achieve robust (constant) disturbance suppression and constant reference tracking for a linear SISO plant based on supervised control system. The main disturbance suppression methodology, not unsurprisingly, is to integrate the reference tracking error by including an integrator in the controller.

This paper is a first step in the direction of extending some of these ideas to nonlinear plants. The notion of constant disturbance suppression for nonlinear systems is reasonably straightforward, see [4]. The key issue is to explore how to achieve an MMAC capability, and this is in turn rests on having a so-called stable multi-estimator. The stable multi-estimator for a possibly unstable nonlinear plant is constructed based on stable kernel representations. This is one way an extension of the linear system ideas in the papers [1] and [3] can be achieved.

0 * This research was supported by the US Office of Naval Research, grant number N00014-97-1-0946.

2 Multi-estimator design for nonlinear plants

The structure of the linear multi-estimator presented in [1] is as follows. For N nominal models P_i, we identify their transfer functions $P_i = \frac{n_i(s)}{d_i(s)}$, with $n_i(s)$ and $d_i(s)$ coprime polynomials. For a stable polynomial $D(s)$, the part of the multi-estimator linking $[y, u]^T$ to y_i is

$$y_i = [\frac{D(s) - d_i(s)}{D(s)} \vdots \frac{n_i(s)}{D(s)}][y\, u]^T. \tag{1}$$

In order to ensure the error signals are comparable for different P_i, it appears more logical to use different normalising stable polynomials $D_i(s)$ instead of a single stable polynomial $D(s)$. Set $z_i = y_i - y$, we have

$$z_i = [-\frac{d_i(s)}{D_i(s)} \vdots \frac{n_i(s)}{D_i(s)}][y\, u]^T. \tag{2}$$

This is equivalent to saying that the multi-estimator implements a stable kernel representation of the system corresponding to the nominal model P_i: thus $[-\frac{d_i(s)}{D_i(s)} \vdots \frac{n_i(s)}{D_i(s)}]$ is a stable transfer matrix with "inputs" y and u, and "output" z, while the input-output behaviour of the system corresponding to P_i is the set of all pairs (y, u) which are mapped by $[-\frac{d_i(s)}{D_i(s)} \vdots \frac{n_i(s)}{D_i(s)}]$ onto $z = 0$. Now let us extend this to the nonlinear case, based on ideas of [2]. For simplicity we consider only a nominal plant P_i that is affine in the control:

$$P_i \left\{ \begin{array}{rcll} \dot{x} & = & f(x) + g(x)u, & x \in \Re^n, u \in \Re^m, \\ y & = & h(x), & y \in \Re^p. \end{array} \right. \tag{3}$$

Here, $f(\cdot)$, $g(\cdot)$ and $h(\cdot)$ are sufficiently smooth functions ensuring that a well-defined response exists for all $u(\cdot)$ in some suitable class, and $f(0) = 0$, $h(0) = 0$.

Theorem 1 *Consider equation (3). Assume there exists a scalar solution $W(x) \geq 0$ to the following Hamilton-Jacobi equation*

$$W_x(x)f(x) + \frac{1}{2}W_x(x)g(x)g^T(x)W_x^T(x) - \frac{1}{2}h^T(x)h(x) = 0, \tag{4}$$

and a solution $k(x)$ to

$$W_x(x)k(x) = h^T(x).$$

Then the system

$$K_{P_i} \left\{ \begin{array}{rcl} \dot{x} & = & [f(x) - k(x)h(x)] + g(x)u + k(x)y \\ z & = & y - h(x), \end{array} \right. \tag{5}$$

is a normalised stable kernel representation of P_i which has $\mathcal{L}_2$-gain equal 1.

We note that in the linear case, a certain co-inner transfer function matrix is defined by (5), and this explains the word "normalised". Theorem 1 can be directly used to construct a multi-estimator for a collection of nonlinear nominal models. For performance assessment, we still look at an $\mathcal{L}_2$ norm [1] as in the linear case

$$\mu_i = \int_0^t e^{-\lambda(t-\tau)} z_i^2 d\tau, \tag{6}$$

where λ is a positive smoothing constant.

3 An Example

In this section, a simple admittedly academic example of MMAC for the nonlinear plant is presented to highlight some fundamental issues of the robust disturbance suppression for the nonlinear case. The nonlinear plant model is given as follows.

$$P \left\{ \begin{array}{rcl} \dot{x} & = & \theta x^3 + u \\ y & = & x^3, \end{array} \right. \tag{7}$$

where θ is a piecewise constant. Its nominal value is -0.2 ± 0.1, but it can suddenly change its sign to 0.2 ± 0.1. Sign changes are assumed to be infrequent, in the sense that after a change, the adaptive system should be able to learn that change before the next one. Assume two nominal models are given as the following form.

$$P \left\{ \begin{array}{rcl} \dot{\xi} & = & \theta_i \xi^3 + u \\ y & = & \xi^3, \end{array} \right. \tag{8}$$

where $i = 1, 2$, $\theta_1 = 0.22$ and $\theta_2 = -0.18$. Based on the method given in last section, a multi-estimator can be written as follows.

$$K_{P_i} \left\{ \begin{array}{rcl} \dot{\xi}_i & = & \theta_i \xi_i^3 - \frac{1}{\sqrt{1+\theta_i^2} - \theta_i} \xi_i^3 + \frac{y}{\sqrt{1+\theta_i^2} - \theta_i} + u \\ z & = & y - \xi_i^3. \end{array} \right. \tag{9}$$

We design the stabilising controller as follows.

$$u_i = -\theta_i x^3 - 2x.$$

Furthermore, we augment each controller with a small gain integrator. Of course this controller used on P_i would be satisfactory. We implemented our design simulation by Matlab Simulink. During simulation, the plant is stable for the controller u_1 at $t < 20_{sec}$, and unstable with that controller afterwards. So, it is necessary to switch controller properly to ensure stability. The simulation results are in Figure 1. The top figure in Figure 1 is the output y of the plant. The other two figures are the output of Performance assessment μ_1 and μ_2 respectively (See equation (6)). The integration of equation (6) is reseted to zero every 1 seconds. ¿From Figure 1, we can see that the multi-estimator accurately identified the variation of plant very fast. The whole control system can track a constant reference input under constant disturbance and switching model variant.

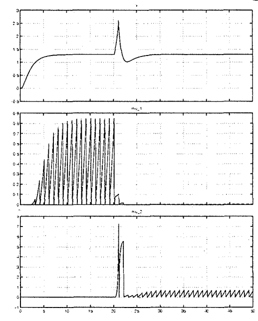

Figure 1: The output of the plant and multi-estimator.

4 Conclusion

This paper presents a modest extension of the Multiple Model Adaptive Control method to solve a robust constant disturbance rejection problem for nonlinear systems.

References

[1] B.D.O. Anderson, T. Brinsmead, D. Liberzon, and A.S. Morse. Multiple model adaptive control with safe switching. *Special issue of the International Journal of Robust and Nonlinear Control on Adaptive Control with Confidence*, 8:446–470, 2001.

[2] A. J. Van der Schaft. *$\mathcal{L}_2$ Gain and Passivity Techniques in Nonlinear Control.* Springer Verlag, New York, 1996.

[3] A.S. Morse. A bound for the disturbance-to-tracking-error gain of a supervised set-point control system. In D. Normand Cyrot, editor, *Perspectives in Control, Theory and Applications*, pages 23–41. Springer-Verlag, 1998.

[4] S.W. Su, B.D.O. Anderson, and T.S. Brinsmead. Constant disturbance rejection and zero steady state tracking error for nonlinear systems design. In Biswa Datta, editor, *ACCSC'2001*, pages 1–30. Kluwer.