Transactions

The Positive Side of Filters: A Summary

by Luca Benvenuti, Lorenzo Farina, and Brian D. O. Anderson

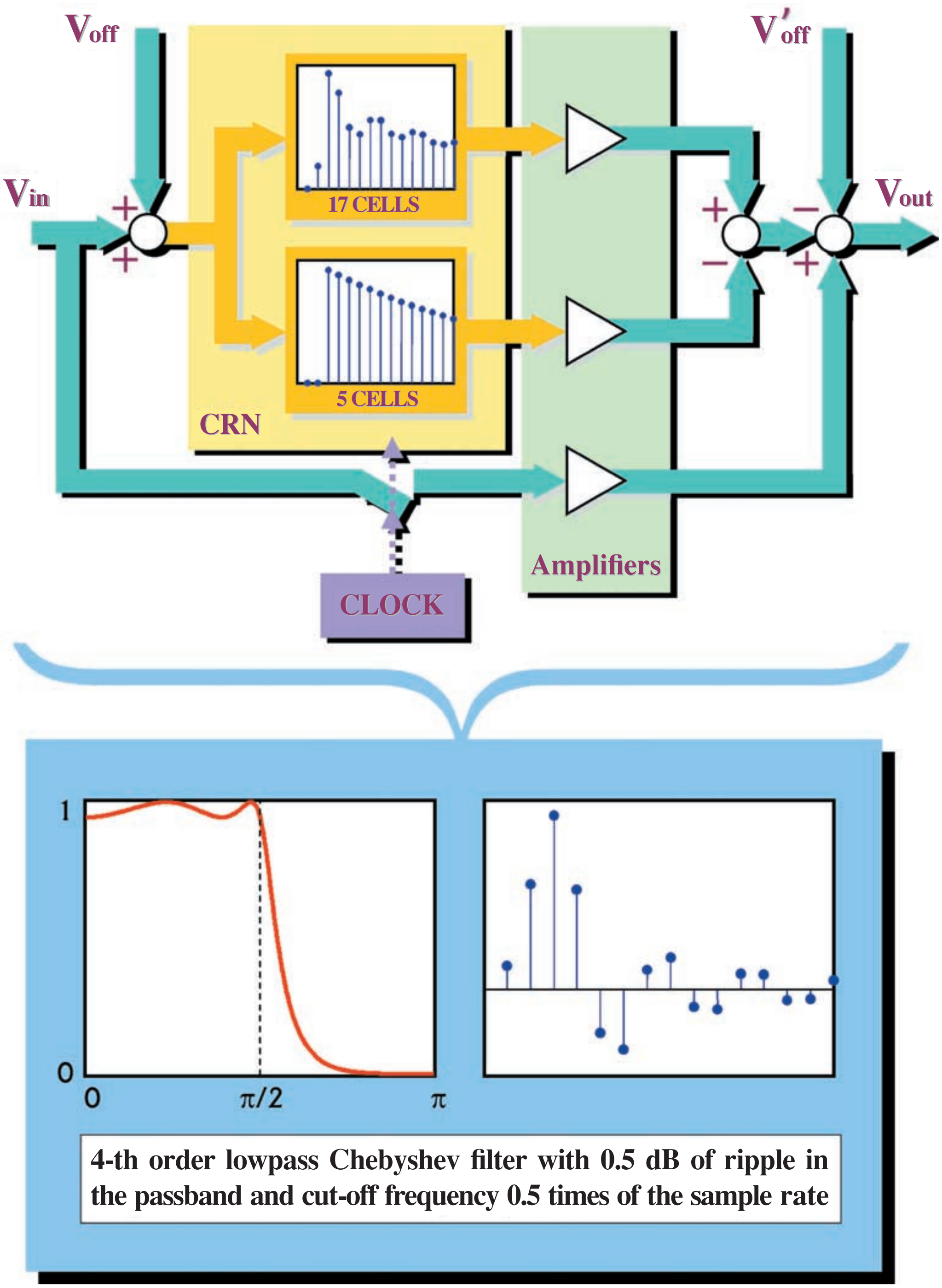

Figure 1. Realizing a Chebyshev filter using CRNs.

"Filtering through Combination of Positive Filters Criteria"

Luca Benvenuti, Lorenzo Farina, and Brian D. O. Anderson

Abstract—The linear filters characterized by a state-variable realization given by matrices with nonnegative entries (called positive filters) are heavily restricted in their achievable performance. Nevertheless, such filters are the only choice when dealing with the charged coupled device MOS technology of charge routing networks (CRNs), because nonnegativity is a consequence of the underlying physical mechanism. In order to exploit the advantages offered by this technology, the authors try to overcome the above-mentioned limitation by realizing an arbitrary transfer function as a difference of two positive filters.

IEEE Transactions on Circuits and Systems, Part I: Fundamental Theory and Applications, December 1999, pp. 1431–1440.

Summary

What is the positive side of filters? Obviously to many, but maybe not all, we are talking about positivity meaning that of the filter impulse response. As a matter of fact, the linear filters characterized by a positive impulse response (called positive filters), are heavily restricted in their achievable performance, *e.g.* as low-pass filters. In fact, the most widely used filters (Butterworth, Chebyshev, etc...) have no sign limitation on their impulse response. As a consequence, positive filters cannot have arbitrary pole patterns. Nevertheless, positive filters are the only choice when dealing with a charged coupled device technology such as *Charge Routing Networks* (CRNs). In order to overcome such limitations the paper considers the possibility of realizing an arbitrary filter as a difference of two positive filters. Then the basic question is: *Is it always possible to implement an arbitrary filter as a difference of two positive filters, as for CRNs?*

The class of CRN filters, introduced by Gersho and Gopinath at the Bell Labs in 1979, is based on a family of functional solid-state electronic devices using MOS technology. More precisely, a CRN consists of a collection of *storage cells*, locations where a packet of charge can be stored and

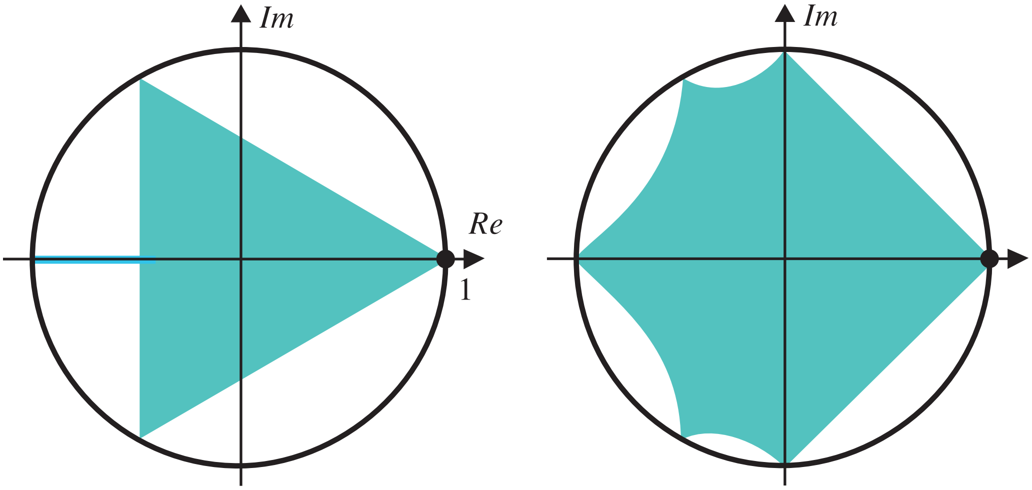

Figure 2. Allowed pole patterns for 2-phase CRNs with 6 and 8 internal cells.

The Positive Side of Filters

Figure 3. CNR layout.

maintained isolated from the others, and of a specific periodically repeating *routing procedure*, operations involving the packets of charge stored in the cells. The basic operations consist in applying a charge packet to a storage cell such that the packet's size is proportional to a given positive voltage amplitude (*injection*), in splitting the charge packet of a cell into positive components and transferring these parts into distinct cells previously empty (*splitting and transfer*), in combining charge packets from different cells and transferring them simultaneously into the same cell (*addition*) and in emptying a cell by removing its charge packet from the network while generating a voltage amplitude proportional to the size of the extracted packet (*extraction*).

From the above follows that every CRN can be completely described by a directed graph whose nodes represent cells and whose branches indicate routes of charge transfer between cells. Each directed branch has a positive weight value which indicates the fraction of the charge of the starting node transferred along that route, so that the impulse response of the CRN is positive.

Using this basic mechanism, these devices can perform a wide range of electronic functions including image sensing, data storage, signal processing or logic operations. CRNs offer the possibility of achieving discrete-time signal processing on an MOS integrated circuit chip with the advantage of lighter weight, smaller size, lower power consumption and improved reliability (with respect to an equivalent digital implementation). Moreover, they can perform many sampled-data filtering functions directly in the analog domain. In fact, CRNs are inherently analog, and as such they are ideally suited to a number of sampled data signal processing functions. In a sense, CRNs combine the features of digital and analog techniques: like digital filters, CRNs are controlled by a master clock and their characteristic is as stable as the master oscillator but the requirement for analog-to-digital conversion is eliminated and all functions are performed in the analog domain. It is worth noting that the celebrated CCD devices are nothing but CRNs with a single route!

As regards the original question whether it is possible to realize an arbitrary filter as the difference of two

CRNs, the paper shows that the answer is positive. In fact, when considering 2-phase CRNs, an arbitrary filter of order n can be realized as the difference of an N_c-cell positive filter and a 5-cell one, for some $N_c > n$. This answer bears another question: *Can one give some a priori bound for the number* $5 + N_c$ *of nodes (cells) needed to implement such positive filters?*

As regards the original question whether it is possible to realize an arbitrary filter as the difference of two CRNs, the paper shows that the answer is positive. In fact, when considering 2-phase CRNs, an arbitrary filter of order n can be realized as the difference of an N_c-cell positive filter and a 5-cell one, for some $N_c > n$.

This question is far from trivial; in fact, it may well be the case that N_c *must* be much larger than the filter order n.

In order to gain partial insight into this problem, the case of first and second order filters is initially considered. Then, the case of a filter of order n, whose system function $H(z)$ has simple poles, is tackled by decomposing it into a sum of first and second order filters so that an *a priori* upper bound for N_c is given. In the case of 2-phase CRNs, this upper bound is given by the formula

$$N_c = 3 + 2\left(n + N_2 + \sum_{i \geq 3} (i-2)N_i\right)$$

where N_2 is the sum of the number of negative real poles and of nonnegative real poles with negative residues of $H(z)$, N_3 is the number of pairs of complex poles of $H(z)$ belonging to $\mathcal{P}_3$, and N_i $(i > 3)$ is the number of pairs of complex poles of $H(z)$ belonging to the set

$$\mathcal{P}_i / \bigcup_{j=3}^{i-1} \mathcal{P}_j$$

where $\mathcal{P}_i$, with $i \geq 3$, denotes the set of points in the complex plane that lie in

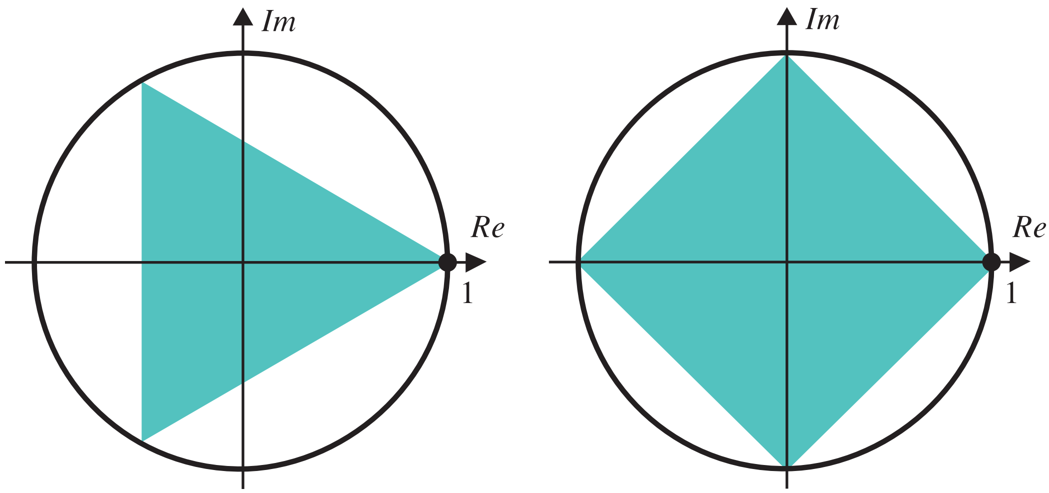

Figure 4. The sets $\mathcal{P}_3$ *and* $\mathcal{P}_4$.

the interior of the regular polygon with i edges having one vertex at the point $1 + j0$ and inscribed in the unit disk centered at the origin of the complex plane.

Furthermore, the paper provides an introductory description of CRN behavior and implementation; and, on the basis of its results, a design procedure to implement an arbitrary filter as the difference of two CRNs is given.

As a final comment, the result of this paper may be used for example to implement optical filters in which the signals are modulated as intensity variations on optical carriers whose coherence time is less than the shortest relevant time delay in the system. In fact, in this case, the signals are positive and add on an intensity basis so that only positive filters can be implemented.

What else? Well, the moral of the story is... *positive filters make the difference!*

Luca Benvenuti was born in Rome, Italy, on February 8, 1966. He received the “Laurea” degree in electrical engineering (*summa cum laude*) and the Ph.D. degree in systems engineering from the University of Rome “La Sapienza”, Rome, Italy, in 1992 and 1995, respectively. He was a visiting graduate student at the University of California, Berkeley, in 1995. In 1997 he was scientific consultant for Magneti Marelli, from 1997 to 1999 he had a postdoctoral position at the Department of Electrical Engineering, University of L’Aquila, L’Aquila, Italy, and from 1997 to 2000 he was scientific consultant at PARADES (Project on Advanced Research on Architectures and Design of Electronic Systems), a research laboratory supported by Cadence Design Systems, Magneti Marelli, and STMicroelectronics. He is currently assistant professor at the Department of Computer and System Science, University of Rome “La Sapienza”, Rome, Italy.

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