Additive, Multiplicative and Inverse Multiplicative Robust Stability: Connections with the $\nu$-Gap Metric

Brian D. O. Anderson
Thomas Brinsmead

Abstract

Existing Robust Stability Results based on the $\nu$-gap metric can be extended by the introduction of various weighting functions. By appropriately specialising these weighting functions it is possible to recover earlier robust stability results based on additive, multiplicative and inverse multiplicative uncertainty models. In this way it is demonstrated that some of these classical robustness conditions are a special case of the extended $\nu$-gap Robust Stability tests.

1 Introduction

This work indicates that earlier "classical" robust stability results for additive and multiplicative perturbations, can be recovered by specialising stability robustness results derived using the so-called $\nu$-gap or Vinnicombe distance between plants [8, 9].

Stability robustness analysis investigates the closed-loop stability of the interconnection of a particular controller and any member of an entire set of plants (as opposed to a particular plant). Earlier last century, stability robustness results focussed on perturbations in the nominal open loop gain or the phase lag that might be introduced by a time delay. These results led to design robustness specifications involving gain margin and phase margin [6]. Much later however, in the early 1980s, additive, multiplicative and inverse multiplicative perturbations of the nominal plant were investigated for their robustness properties [4, 5, 3, 6, 7, 10].

Recently, another type of stability robustness result, one that exploits the so-called $\nu$-gap or Vinnicombe metric distance between plants, has become available [8, 9]. The main contribution of this extended abstract is to advertise results indicating that although these results superficially appear to be quite different, in fact, they are not. By incorporating certain limiting weights in the $\nu$-gap robustness results, the earlier results of [3, 4, 5, 6, 7, 10] can be obtained.

Section 2 reviews the various aforementioned approaches to robustness. Sections 3 and 4 show how these results are related. Note that this extended abstract only presents the main results and full proofs are available in [1].

2 Existing Robustness Results

2.1 Additive and Multiplicative Perturbations

The stability robustness of a stable closed-loop comprising a plant and controller can be investigated by analysing whether stability is retained when the plant is perturbed. (It is also possible to investigate controller perturbation.) Many such results are obtained by investigating particular classes of plant variation or uncertainty. Such perturbation classes are typically given names such as additive uncertainty, multiplicative input or output uncertainty or inverse multiplicative output uncertainty: see [3, 4, 5, 6, 7, 10].

More precisely, with a nominal plant $P$, and $W_1$, $W_2$ two given weighting transfer functions (both of which are both stable and stably invertible), one can consider perturbed plant sets of the type $P + W_1 \Delta W_2$, $P(I + W_1 \Delta W_2)$, $(I + W_1 \Delta W_2)P$, $P( I + W_1 \Delta W_2)^{-1}$ and so on, where the perturbation parameter $\Delta \in \mathcal{RH}_\infty$ and $\|\Delta\| < 1$. (Here, as usual, $\mathcal{RH}_\infty$ denotes the set of stable, proper, rational transfer functions.)

Sufficient conditions for robustness are given which involve the weights, $W_1$ and $W_2$, and one of the input or output sensitivities ($S_i$ and $S_o$), or com-

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2 Department of Systems Engineering, Research School of Information Sciences and Engineering, Canberra, ACT, 0200, Australia.
Suppose that the interconnection of $C$ with $P$ is stable, and that one seeks to ensure that the stability of the interconnection of $C$ with any plant in the perturbed plant set. Table 1, drawn from [10], lists some sufficient conditions. These conditions are also necessary in the case that the perturbations $\Delta$ are stable but otherwise unstructured.

The requirements that $W_1, \Delta$ and $W_2$ each be stable is somewhat restrictive from the perspective of practical utility. The table only captures the possibility that the perturbation is stable, even if the plant $P$ that undergoes an additive perturbation is unstable. Perturbation of the location of an unstable pole in $P$, for example, is ruled out. The restriction has in fact been removed for some time: see, for example [7].

In the following, we drop the restriction that $\Delta \in \mathcal{RH}_\infty$, and simply require that $\Delta \in \mathcal{L}_\infty$ with $\|\Delta\|_\infty < 1$, that is, the maximum singular value of $\Delta$ is less than unity on the $j\omega$-axis. For any transfer function $X(s)$ that has poles as its only singularities in $Re[s] > 0$, let $\eta(X)$ denote number of right-half plane poles of $X(s)$, that is the McMillan degree of the transfer function matrix $X(s)$, where $X = X_+(s) + X_-(s)$, and $X_-(s)$ has no singularities in $Re[s] > 0$. For a square, nonsingular $X$ (for which $X^{-1}$ is not necessarily proper), define $Z(X)$ to be $\eta(X^{-1})$, the count excluding any poles of $X^{-1}$ at $s = \infty$. Similarly we define $Z(X)$ to be $\eta(X^{-1})$, the count including poles of $X^{-1}$ at $s = \infty$. Then the following results of Table 2, which are somewhat harder to derive than those in Table 1, apply: see also [8]. Note that for brevity we have omitted the cases involving input sensitivities $S_i$ and $T_i$.

In fact, Table 2 sets out sufficient and necessary conditions for arbitrary unstructured, $\mathcal{L}_\infty$-norm bounded perturbations $\Delta$. That is, $[P_1, C]$ is stable for all $P_1$ in the perturbed model set if and only if $[P, C]$ is stable and the corresponding stability test inequality holds.

### 2.2 The Vinnicombe-Gap and Robustness

Robust stability results are also found in [8, 9]. These results are based on the $\nu$-gap metric (which, incidentally, induces the weakest topology on operator space in which stability is a robust property). We review some of these results in this section. In the feedback system of Figure 1, the transfer function from $[r^T_1 \ r^T_2]$ to $[y^T \ u^T]^T$ is given by

$$T(P, C) = \begin{bmatrix} P(I + CP)^{-1}C & P(I + CP)^{-1} \\
(I + CP)^{-1}C & (I + CP)^{-1} \end{bmatrix} \begin{bmatrix} P \\
I \end{bmatrix} (I + CP)^{-1} \begin{bmatrix} C & I \end{bmatrix}.$$ 

One can define a generalized stability margin by

$$b_{P, C} = \begin{cases} \|T(P, C)\|_\infty^{-1} & \text{if } (P, C) \text{ is stable,} \\
0 & \text{otherwise.} \end{cases}$$
We now describe how to obtain a distance measure between two plants $P_1$ and $P_2$ with real rational transfer functions and the same input and output dimensions. In the sequel, for a real rational $X(s)$, we define $X(s)^* = X^T(-s)$. For real rational $\Phi(s)$ with $\Phi(s) = \Phi(s)^*$ and $\Phi(j\omega) \geq 0$ for all $\omega$, the notation $\Phi(s)^{\frac{1}{2}}$ denotes a (possibly but not necessarily stable minimum-phase) spectral factor of $\Phi(s)$. Set

$$\kappa[P_1(j\omega), P_2(j\omega)] = \lim_{\omega \to -\infty} \sigma\{[I + P_2(j\omega)P_1(j\omega)]^{-\frac{1}{2}} \times (P_2(j\omega) - P_1(j\omega))[I + P_1(j\omega)^*P_1(j\omega)]^{-\frac{1}{2}}\}.$$  \hspace{1cm} (3)

(In cases where $j\omega$ is a pole of either $P_1$ or $P_2$ the limiting process assures that the above quantity is well-defined). The $\nu$-gap distance between $P_1$ and $P_2$ is further defined by

$$\delta_\nu(P_1, P_2) = \sup_\omega \kappa[P_1(j\omega), P_2(j\omega)]$$

provided that

$$\text{wno} \{\det(I + P_2^*P_1)\} + \eta(P_1) - \eta(P_2) = 0. \hspace{1cm} (4)$$

Here, $\text{wno} \{\det(I + P_2^*P_1)\}$ is the number of encirclements of the origin made by $\det[I + P_2^*(s)P_1(s)]$ as $s$ follows the standard Nyquist contour, indented into the right half plane around any imaginary axis pole or zero. As before, $\eta(P_1)$ is the number of poles of $P_1$ in the open right half plane $\text{Re}[s] > 0$, while $\bar{\eta}(P_2)$ is the number of poles of $P_2$ in the closed right half plane $\text{Re}[s] \geq 0$. In the event that the winding number condition (4) does not hold, then one sets

$$\delta_\nu(P_1, P_2) = 1.$$  

There are two robust stability results to record. Firstly, suppose that $(P_1, C)$ is stable. Then for any $\beta < b_{P_1, C}$, the set $\{P_2 : \delta_\nu(P_1, P_2) \leq \beta\}$ is stabilized by $C$, while if $\beta > b_{P_1, C}$, there exists at least one $P_2$ in the set which is not stabilized by $C$.

Second, suppose again that $(P_1, C)$ is stable. Suppose also that for all $\omega$, there holds

$$\kappa[P_1(j\omega), P_2(j\omega)]\sigma[T(P_1(j\omega), C(j\omega))] < 1. \hspace{1cm} (5)$$

Then $[P_2, C]$ is stable if and only if the winding number condition (4) holds.

There is of course a connection between these results. Notice that if $\delta_\nu(P_1, P_2) \leq \beta < b_{P_1, C}$, then

$$\kappa[P_1(j\omega), P_2(j\omega)] \leq \delta_\nu(P_1, P_2) < \frac{1}{b_{P_1, C}} \frac{1}{\sup_\omega \sigma[T(P_1(j\omega), C(j\omega))]},$$

that is, equation (5) holds.

The winding number condition (4) is a rather complicated condition involving poles and zeros. (Notice that for a nonsingular $X$, $\text{wno} \{\det(X)\} = Z(X) - \bar{Z}(X)$). As such, it corresponds to (but generalizes) the pole or zero counting conditions of Table 2.

For future use, let us note that from (3), it follows easily that for invertible $P_1$ and $P_2$,

$$\kappa[P_1(j\omega), P_2(j\omega)] = \kappa[P_1^{-1}(j\omega), P_2^{-1}(j\omega)].$$

Also note that the winding number condition (4) is easily seen to be equivalent to

$$\text{wno} \{\det(I + P_2^{-*}P_1^{-1})\} + Z(P_1) - \bar{Z}(P_2) = 0.$$

3 Extending $\nu$-Gap Robustness Results

We aim to demonstrate that a connection exists between the $\nu$-gap robustness results, and the earlier "classical" results in Table 2. The older results are actually consequences of "limiting" versions of the $\nu$-gap result.

Our starting point is the simple observation that $(P, C)$ is stable if and only if, with $V_1, V_2, V_1^{-1}$ and $V_2^{-1}$ all stable, $(V_1PV_2, V_1^{-1}CV_2^{-1})$ is also stable. Consequently, the following simple variant on the robust stability condition is associated with (5).

Lemma 3.1 Let $(P_1, C)$ be stable. Suppose that for some $V_1, V_2$ with $V_1$, $V_2$, $V_1^{-1}$ and $V_2^{-1}$ each stable, there holds for all $\omega$

$$\kappa[V_1P_1V_2(j\omega), V_1P_2V_2(j\omega)] \sigma[T(V_1P_1V_2(j\omega), C(j\omega)\bar{V}_2^{-1}(j\omega))] < 1. \hspace{1cm} (6)$$

Then $(P_2, C)$ is stable if and only if the following winding number condition holds:

$$\text{wno} \{\det(I + P_2^{-*}V_2^{-1}V_1^{-1})\} + \eta(P_1) - \bar{\eta}(P_2) = 0. \hspace{1cm} (7)$$
The lemma suggests the possibility, given a plant pair \( P_1 \) and \( P_2 \) and a controller \( C \), of choosing weights \( V_1 \) and \( V_2 \) in order to secure (6), and then checking that (7) indeed holds. This is roughly what we shall do to recover the traditional robust stability results from the \( \delta \) results, in particular the results associated with (5).

By way of motivation for the choice of weights, let us observe what happens in the scalar case. Use lower case symbols \( p_1, p_2 \) and \( c \) for this special case, and let \( v \) denote the multiplicative combination of \( V_1 \) and \( V_2 \). Then at each \( \omega \),

\[
\kappa(v_{p_1}, v_{p_2}) = \frac{|v(p_1 - p_2)|}{\sqrt{1 + |v_{p_1}|^2} \sqrt{1 + |v_{p_2}|^2}},
\]

\[
\sigma[T(v_{p_1}, v^{-1} c)] = \frac{|v_{p_1} - p_1 c|}{\sqrt{1 + |v_{p_1}|^2} \sqrt{1 + |v^{-1} c|^2}},
\]

and

\[
\kappa(v_{p_1}, v_{p_2}) \sigma[T(v_{p_1}, v^{-1} c)] = \frac{|v_{p_1} - p_1 c| (|v|^2 + |c|^2)}{1 + |p_1 c|}.
\]

The function

\[
g(x) = \frac{x + |c|^2}{1 + x |p_2|^2}
\]

has a monotonic derivative on \( x \geq 0 \). Consequently

\[
\arg \min_{x \geq 0} g(x) = \begin{cases} 0 & \text{if } |c|^2 < |p_2|^{-2} \\ \infty & \text{if } |c|^2 > |p_2|^{-2} \\ [0, \infty) & \text{if } |c|^2 = |p_2|^{-2} \end{cases}
\]

with \( \min_{x \geq 0} g(x) = \min(|c|^2, |p_2|^{-2}) \).

In the scalar case then, the left side of (6) is minimized by letting \( |v| \) tend to either zero or infinity, according to whether \( |p_2 c| < 1 \) or \( |p_2 c| > 1 \). A choice of \( v \) which would be optimal in the sense of minimizing the left side of (6) might then have \( |v| \) very small over some frequencies and very large over others. Such a choice would be likely to make the investigation of (7) rather difficult.

**4 Connecting “Classical” and \( \nu \)-gap Robustness Conditions**

In this section we will demonstrate that by taking \( |v| \) either vanishingly small or arbitrarily large at all frequencies, we can recover the earlier robustness results follow on as a consequence. Let us therefore focus on equations (6) and (7) in these two circumstances.

**4.1 Vanishingly Small Weights**

Although we have briefly looked at scalar plants to motivate the analysis, we now investigate the more general multivariable case. Suppose that we set \( V_2 = \epsilon W_{2}^{-1} \) and \( V_1 = \epsilon W_{1}^{-1} \), where \( W_1 \) and \( W_2 \) together with their inverses are stable and \( \epsilon \) is a positive scalar (which will become very small below). Then

\[
T[V_1 P_1 V_2(j \omega), V_2^{-1} C V_1(j \omega)] = [W_1^{-1} P_1(I + CP_1)^{-1} C W_1 \quad \epsilon^2 W_1^{-1} P_1(I + CP_1)^{-1} W_2^{-1} \quad \epsilon^{-2} W_2(I + CP_1)^{-1} C W_1].
\]

At each \( \omega \), the maximum singular value is dominated by the 2–1 block for \( \epsilon \) sufficiently small:

\[
\delta[T[\epsilon^2 W_1^{-1} P_1 W_2^{-1}, \epsilon^{-2} W_2 C W_1]] \simeq \epsilon^2 \delta[W_1(I + CP_1)^{-1} C W_1].
\]

Also, for all \( \omega \):

\[
\kappa[\epsilon^2 W_1^{-1} P_1 W_2^{-1}, \epsilon^2 W_1^{-1} P_2 W_2^{-1}] = \delta \left( I + \epsilon^2 W_1^{-1} P_2 W_2^{-1} W_1^{-1} P_1 W_2^{-1} \right)^{\frac{1}{2}} \cdot \epsilon^2 W_1^{-1} (P_2 - P_1) W_2^{-1} \cdot \left( I + \epsilon^4 W_2^{-1} P_1 W_2^{-1} W_1^{-1} P_2 W_1^{-1} \right)^{-\frac{1}{2}}
\]

It can be shown (although the calculations are somewhat tedious), that if \( j \omega_0 \) is a pole of \( P_1 - P_2 \), then \( \kappa \) is not continuous at \( \epsilon = 0 \). If \( P_1 - P_2 \) has no \( j \omega \)-axis poles (a sufficient condition being that neither \( P_1 \) nor \( P_2 \) has any \( j \omega \)-axis poles), it can be verified quite easily that for very small \( \epsilon \),

\[
\kappa[\epsilon^2 W_1^{-1} P_1 W_2^{-1}, \epsilon^2 W_1^{-1} P_2 W_2^{-1}] \simeq \epsilon^2 \delta[W_1^{-1} (P_2 - P_1) W_2^{-1}](j \omega)
\]

Hence, given that \( P_1 - P_2 \) possesses no \( j \omega \)-axis poles, then with the substitutions \( V_1 = \epsilon W_{1}^{-1} \) and \( V_2 = \epsilon W_{2}^{-1} \) and with \( 0 < \epsilon << 1 \), the inequality (6) of Lemma 3.1, becomes,

\[
\delta[W_1^{-1} (P_2 - P_1) W_2^{-1}](j \omega)
\]

Next, we investigate the winding number part of the stability condition. (We shall continue to demand that \( P_1 - P_2 \) has no \( j \omega \)-axis poles and that \( \epsilon \) is suitably small.) To this end, we shall use the following lemma.
Lemma 4.1 Let $\Delta \in \mathcal{RL}_\infty$, that is with $\Delta$ rational, and with the same dimensions as some prescribed proper real rational $\Phi$. Then for scalar $\epsilon < 1$ such that $\|\epsilon^2 \Delta \|_\infty < 1$, there holds

$$\text{wno}[\det(I + \epsilon^4 \Phi^* \Phi + \epsilon^4 \Delta^* \Phi)] = \tilde{\eta}(\Phi) - \eta(\Phi).$$

Proof: See [1].

Set $P_2 - P_1 = W_1 \Delta W_2$, so that because $P_2 - P_1$ has no imaginary axis poles it follows that $\Delta \in \mathcal{L}_\infty$. Also, set $\Phi = W_1^{-1} P_1 W_2^{-1}$. Condition (7) becomes

$$\text{wno}[\det(I + \epsilon^4 \Phi^* \Phi + \epsilon^4 \Delta^* \Phi)] + \eta(P_1) - \tilde{\eta}(P_2) = 0.$$  

Using Lemma 4.1, and noting the equalities $\eta(\Phi) = \tilde{\eta}(P_1)$ and $\eta(\Phi) = \tilde{\eta}(P_2)$, we see that (7) is equivalent to $\eta(P_1) = \tilde{\eta}(P_2)$. Since $P_2 - P_1 \in \mathcal{L}_\infty$, this is also equivalent to

$$\eta(P_1) = \tilde{\eta}(P_2).$$  

(11)

In summary, we have established that if $P_2 - P_1$ has no $j\omega$-axis poles, and if (9) and (11) hold, then (6) and (7) hold with $V_1 = \epsilon W_1^{-1}$ and $\epsilon$ suitably small for some stable and stably invertible $W_1$ and $W_2$. Accordingly the stability of $(P_1, C)$ implies the stability of $(P_2, C)$.

The connection between this result and the first entry on Table 2 should now be clear. In (9), we are modelling the difference between $P_2$ and $P_1$ by $P_2 = P_1 + W_1 \Delta W_2$ where $\Delta \in \mathcal{L}_\infty$ and $P_1$ and $P_2$ have the same right half plane pole count, as in Table 2. However, in (9), we are simply requiring the frequency by frequency condition that

$$\tilde{\sigma}(\Delta) \tilde{\sigma}[W_2 C(I + P_1 C)^{-1} W_1] < 1$$  

(12)

Table 2 requires the apparently stronger conditions that $\|\Delta\|_\infty < 1$ and $\|W_2 C(I + P_1 C)^{-1} W_1\|_\infty \leq 1$. However, because of the freedom in the choice of the perturbation parameter, the conditions in Table 2 are actually equivalent to the condition that equation (12) holds. It is certainly easy to see that equation (12) is implied by the conditions $\|\Delta\|_\infty < 1$ and $\|W_2 C(I + P_1 C)^{-1} W_1\|_\infty \leq 1$.

Let us emphasise that we are describing a robust stability result. We are not advocating a design method where the closed loop transfer function $T[V_1 P V_2(j\omega), V_2^{-1} C V_1^{-1}(j\omega)]$ is implemented.

Such a design strategy would not be prudent, since $\tilde{\sigma}(\Delta)$ goes to infinity as $\epsilon$ goes to zero.

It is possible to make connections with the second entry of Table 2 using similar ideas. Let us suppose that $\Delta \in \mathcal{L}_\infty$, and temporarily suppose that $P_1$ has no imaginary axis pole. We consider

$$P_2 = (I + W_1 \Delta W_2) P_1$$

and suppose that (with all $\Delta \in \mathcal{L}_\infty$ with $\|\Delta\|_\infty \leq 1$) there holds

$$\eta(P_2) = \eta(P_1).$$  

(13)

Define $W_{2\epsilon}$ to be the stable and stably invertible spectral factor of $\epsilon I + (W_2 P_1)^*(W_2 P_1)$ for small $\epsilon$. Select

$$V_1 = \epsilon W_1^{-1},$$

$$V_2 = \epsilon W_{2\epsilon}^{-1}.$$  

Using similar methods to the previous case (see [1] for details) we have that for small $\epsilon$

$$\kappa[V_1 P_1 V_2(j\omega), V_1 P_2 V_2(j\omega)] \simeq \epsilon^2 \tilde{\sigma}(\Delta)$$

and equation (6) becomes

$$\tilde{\sigma}(\Delta) \tilde{\sigma}[W_{2\epsilon} P_1 (I + C P_1)^{-1} C W_1] < 1.$$  

(14)

The winding number condition is equivalent to (13), and so stability of $(P_1, C)$ will imply stability of $(P_2, C)$ if both (13) and (14) hold.

This result can also be extended to allow $P_1$ to possess imaginary axis poles so that (13) and (14) yield a robust stability result also for the case: again see [1].

4.2 Arbitrarily Large Weights

By suitably specialising the Vinnicombe form of the robustness conditions it is also possible to derive the third result of Table 2. We restrict attention to square nonsingular $P$ for simplicity of exposition. (Nonsingular rectangular $P$ may be dealt with by a "squaring-up" operation that will result in a square, nonsingular $P$.) In the formulas for $T[V_1 P_1 V_2(j\omega), V_2^{-1} C V_1^{-1}(j\omega)]$, we shall choose,

$$V_1 = M W_2,$$

$$V_2 = M \tilde{W}_1,$$

where $M$ is a scalar which we will allow to become arbitrarily large, and $\tilde{W}_1$ is a minimum phase and
stable spectral factor such that (under the temporary assumption that $P_1$ has no extended $j\omega$-axis pole or zero)

$$\tilde{W}_{1}^{-1}\tilde{W}_{1}^{-1} = (W_{1}^{-1}P_1)(W_{1}^{-1}P_1).$$

For each $\omega$, the maximum singular value of $T$ is determined by the 1-2 block. For very large $M$

$$\tilde{\sigma}\{T[M^2W_2P_1\tilde{W}_1,M^{-2}\tilde{W}_1^{-1}CW_2^{-1}]) \simeq M^2\sigma[W_2(I + P_1C)^{-1}P_1\tilde{W}_1] =$$

$$M^2\sigma[W_2(I + P_1C)^{-1}W_1].$$

We can show, using the invertibility of the $P_1$ and results from Section 2 that

$$\kappa[V_1P_1V_2,V_1P_2V_2] = \kappa[V_2^{-1}P_1^{-1}V_1^{-1},V_2^{-1}P_2^{-1}V_1^{-1}].$$

and hence for very large $M$, (see [1] for details)

$$\kappa[V_1P_1V_2,V_1P_2V_2] \simeq M^{-2}\sigma(\Delta).$$

Equation (6) then becomes

$$\tilde{\sigma}(\Delta)\tilde{\sigma}[W_2(I + P_1C)^{-1}W_1] < 1.$$

The winding number condition (Section 2) is,

$$\text{wno}\{\det (I + M^{-4}W_2^{*}P_2^{-*}\tilde{W}_1^{*}P_1^{-1}W_2^{1}) + \eta(P_1^{-1}) - \eta(\tilde{W}_1^{-1}P_2^{-1}W_2^{1})\} = 0.$$

It can also be shown that for very large $M$, condition (17) becomes

$$\mathcal{Z}(P_1) = \mathcal{Z}(P_2) = \mathcal{Z}[(I + W_1\Delta W_2)^{-1}P_1].$$

These results can be extended to the case where $P_1$ has poles or zeros on the $j\omega$-axis by perturbation arguments similar to those introduced previously.

5 Summary

The main contribution of this extended abstract is to show that earlier "classical" robust stability results of can be obtained by using specialized weighting functions in the more recent $\nu$-gap robust stability theory [8, 9]. The pole or zero counts preservation conditions in the earlier results are specialisations (and thus restrictions) of winding number conditions required for the application of the $\nu$-gap robustness results.

In the course of demonstrating the results, we showed how the introduction of specialized weights into the generalized sensitivity matrix, can isolate particular input or output sensitivity or complementary sensitivity functions. Of course, similar analysis could be performed for any of those sensitivity functions. For scalar $P$ and $C$, as has been observed in [2], the choice $|V_1V_2| = |C|/|P|$ yields at each frequency a minimum for $\tilde{\sigma}(T)$, of $|(I + CP)^{-1}| + |P(I + CP)^{-1}C|$. These observations suggest the possibility of obtaining results in addition to those for robust stability.

References


