CONTROLLER VALIDATION FOR STABILITY AND PERFORMANCE BASED ON AN UNCERTAINTY REGION DESIGNED FROM AN IDENTIFIED MODEL [1]

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Abstract: This paper focuses on the validation (for stability and for performance) of a controller that has been designed from an unbiased model of the true system, identified either in open-loop or in closed-loop using a prediction error framework. A controller is said to be *validated for stability* if it stabilizes all models defined by an ellipsoidal parametric uncertainty set containing the true system with some prescribed probability. The same controller is said to be validated for performance if the worst case performance achieved by this controller over the plants in the uncertainty region is better than some threshold value. *Copyright © 2000 IFAC*

Keywords: identification, robust stability, robust performance

1. INTRODUCTION

This paper is part of our continuing investigation of identification for control and controller validation based on uncertainty regions defined by prediction error identification methods (Bombois *et al.*, 1999a). Here we consider the case where a nominal model G_{mod} has been identified, together with an uncertainty set $\mathcal{D}$ to which the true system G_0 is known to belong with some prescribed probability. This uncertainty set $\mathcal{D}$ is defined as a set of parametrized rational transfer functions whose parameter vector lies in an ellipsoidal confidence region. This is clearly a non-standard uncertainty set in robust control analysis and design. We focus on the validation of a controller C, designed from G_{mod}, both for robust stability and for robust performance. We present a validation procedure for stability which ensures that the controller C stabilizes all systems in this nonstandard uncertainty set $\mathcal{D}$. We also present a validation procedure for performance in which we compute the worst case performance over all closed loop systems made up of the controller C and all plants in $\mathcal{D}$.

Uncertainty region. Prediction error identification theory (see e.g. (Ljung, 1999)) delivers an estimated model G_{mod} for the true plant G_0 and provides us with tools for the estimation of an uncertainty region. If the parametric structure is

[1] The first three authors acknowledge the Belgian Programme on Inter-university Poles of Attraction, initiated by the Belgian State, Prime Minister's Office for Science, Technology and Culture. The last author acknowledges support of Office of Naval Research, Washington and U.S. Army Research Office Far East, Tokyo. The scientific responsibility rests with its authors.

sufficiently complex to represent the true system, then G_{mod} is asymptotically unbiased and the uncertainty is described by the covariance matrix of the identified model G_{mod}. This covariance matrix allows one to construct a parametric uncertainty region U containing the parameters of the true system G_0 at a certain probability level that we can fix at, say, 95 %. The uncertainty region U in the parameter space defines an equivalent uncertainty region $\mathcal{D}$ in the space of transfer functions. This uncertainty region $\mathcal{D}$ can be obtained for both open-loop identification and indirect closed-loop identification.

Controller validation for stability. Robust stability theory developed in e.g. (Zhou *et al.*, 1995; Hinrichsen and Pritchard, 1988) provides necessary and sufficient conditions for the stabilization, by some given controller C, of all plants in an uncertainty region, provided this uncertainty region is defined in the general LFT (linear fractional transformation) framework for robust stability analysis. Our contribution in the proposed procedure is to show that one can rewrite the closed-loop connection of the controller C and all plants in the uncertainty region $\mathcal{D}$ obtained from both types of identification (open-loop and indirect closed-loop identification) into a particular LFT that takes into account the parametric description of $\mathcal{D}$ (i.e. the uncertainty part of the obtained LFT is a real vector) and whose (real) stability radius is exactly computable, using a result presented in (Hinrichsen and Pritchard, 1988; Rantzer, 1992). The problem of robust stability of all plants in the domain $\mathcal{D}$ of parametrized transfer functions was already addressed in (Bombois *et al.*, 1999a). The solution presented there was to embed the uncertainty region $\mathcal{D}$ into a larger coprime factor uncertainty region, leading therefore only to a sufficient robust stability condition. In the new approach of this paper, we obtain a necessary and sufficient condition, because our new stability results apply directly to the parametrized set $\mathcal{D}$ resulting from the identification step, thereby avoiding the conservativeness resulting from the overbounding of $\mathcal{D}$ by a coprime factor uncertainty set. In the case of open-loop identification of an ARMA structure, the structure of $\mathcal{D}$ has a simpler expression. In (Rantzer, 1992), it is shown that this simpler structure can be expressed as an LFT. In this paper, we give a general formulation of this LFT for all model sets (not just ARMA) and for both open-loop and indirect closed-loop identification. In the case of open-loop identification and an ARX structure, an approach similar to ours and to that presented in (Rantzer, 1992) can also be found in (Kosut and Anderson, 1994).

Controller validation for performance. Our procedure for controller validation for performance is based on the computation of the worst case performance over all closed-loops made up of the controller C and all systems in the uncertainty region $\mathcal{D}$. The performance for a closed loop (see (Zhou *et al.*, 1995)) is often defined via the modulus of the frequency response of one (or several) of the four closed-loop transfer functions. The worst case performance, for each of these four transfer functions, is defined as the maximum of their modulus computed over all plants in the uncertainty region $\mathcal{D}$. These maxima over all plants in $\mathcal{D}$, for a given controller C, define four templates, which are used as performance indicators. A number of standard performance indicators (such as disturbance rejection properties, resonance peak, ...) can be derived from these four indicators. Our contribution is to show that the computation of the worst case performance can be formulated as an LMI-based optimization problem. In fact, we give a general LMI-based optimization problem which allows the computation of the worst case performance for the four different closed-loop transfer functions by an appropriate choice of the weights in the general LMI problem. The LMI formulation of the problem uses the fact that the parametric uncertainty appears linearly in the expression of both the numerator and the denominator of the systems in the uncertainty region $\mathcal{D}$ and, as a consequence, also appears linearly in the expression of the four closed-loop transfer functions.

Paper outline. In Section 2, the general expression of the parametric uncertainty regions delivered by prediction error identification is presented. In Section 3, we show how the closed-loop connections of the systems in the uncertainty region $\mathcal{D}$ and the "to-be-validated controller" can be expressed in the general LFT framework for robust stability analysis. A necessary and sufficient condition for the robust stabilization of all plants in $\mathcal{D}$ is then derived from classical robust stability theory. In Section 4, the worst case performance is defined and the LMI-based optimization problem developed for its computation is given. The procedures for validation for stability and for performance are illustrated by an example in Section 5. Finally, some conclusions are given in the last section.

2. UNCERTAINTY REGION DELIVERED BY PREDICTION ERROR IDENTIFICATION

In this section, we give the general expression of the uncertainty regions delivered by classical prediction error identification, assuming that unbiased model structures are used. This general expression, valid for both open-loop and indirect closed-loop identification, is summarized in the following proposition, where we assume that the true *open-loop* system is linear and time-invariant, with a rational input-output transfer function G_0 such that $y = G_0 u + v$, where v is additive noise. See (Bombois *et al.*, 1999a; Bombois *et al.*, 1999b) for more details.

Proposition 1. Consider $\delta \in \mathbf{R}^{k\times 1}$, the real parameter vector of the parametrized transfer function set; $G_0 = G(\delta_0)$, the true open-loop system, and $G(\hat{\delta})$, the full order identified model obtained either "directly" by open-loop identification or "indirectly" by indirect closed-loop identification. The uncertainty region $\mathcal{D}$ containing $G(\delta_0)$ at a certain probability level has the following general form.

$$\mathcal{D} = \left\{ \begin{array}{l} G(\delta) \mid G(\delta) = \dfrac{e + Z_N\delta}{1 + Z_D\delta} \text{ and} \\ \delta \in U = \{\delta \mid (\delta - \hat{\delta})^T R(\delta - \hat{\delta}) < 1\} \end{array} \right\} \quad (1)$$

where $R \in \mathbf{R}^{k\times k}$ is proportional to the inverse of the covariance matrix of $\hat{\delta}$ and has been scaled so as to obtain 1 on the right hand side, $Z_N(z)$ and $Z_D(z)$ are known row vectors of transfer functions of size k and $e(z)$ is either a known transfer function or is equal to 0.

3. CONTROLLER VALIDATION FOR STABILITY

Consider an identified model $G_{mod} = G(\hat{\delta})$ and the corresponding uncertainty region $\mathcal{D}$ of (1) containing G_0 at some probability level. We say that a controller C, designed from G_{mod}, is *validated for stability* if it stabilizes all models in this uncertainty region $\mathcal{D}$ (and therefore also the true system G_0). Our contribution in this section is to show that the uncertainty region $\mathcal{D}$ is amenable to classical robust stability analysis. Indeed, we present a way to describe the set of closed-loop connections of all plants in $\mathcal{D}$ with the "to be validated controller" C as a set of loops $[M_{\mathcal{D}}(z)\ \phi]$ where the uncertainty part ϕ is a real vector. We also show that the (real) stability radius linked with the set of loops $[M_{\mathcal{D}}(z)\ \phi]$ can be computed exactly and efficiently, using the result presented in (Hinrichsen and Pritchard, 1988; Rantzer, 1992). First, we recall an important result of robust stability analysis (Rantzer, 1992; Hinrichsen and Pritchard, 1988) in the case when the uncertainty is assumed to be a real vector.

3.1 *Robust stability for a real vector uncertainty*

We consider here a set of loops $[M(z)\ \beta]$ that obey the following system of equations:

$$\left\{ \begin{array}{l} p = \beta q \\ q = M(z)p \end{array} \right. \quad (2)$$

In this set of loops, it is assumed that $M(z) \in H_\infty$ is a known *fixed* row vector of size b and that the uncertainty part β is a real vector $\in \mathbf{R}^{b\times 1}$ that varies in the following uncertainty domain : $|\beta|_2 < 1$. $|\beta|_2$ represents the 2-norm of the vector β. The robust stability theorem for the set of loops $[M(z)\ \beta]$ is summarized in the following proposition.

Proposition 2. If $M(z) \in H_\infty$ and $\beta \in \mathbf{R}^{b\times 1}$, then the loops $[M(z)\ \beta]$ given in (2) are internally stable for all $\beta \in \mathbf{R}^{b\times 1}$ such that $|\beta|_2 < 1$ if and only if

$$\max_{\Omega} \mu_\beta(M(e^{j\Omega})) \leq 1 \quad (3)$$

The value $\mu_\beta(M(e^{j\Omega}))$ is called the stability radius of the loop $[M(z)\ \beta]$ at the frequency Ω and is defined below.

Definition 3. For M a known complex matrix $\in \mathbf{C}^{1\times b}$ and $\beta \in \mathbf{R}^{b\times 1}$, the stability radius $\mu_\beta(M)$ is defined as follows if $Im(M) \neq 0$:

$$\mu_\beta(M) = \sqrt{|Re(M)|_2^2 - \frac{(Re(M)Im(M)^T)^2}{|Im(M)|_2^2}} \quad (4)$$

and $\mu_\beta(M) = |M|_2$ if $Im(M) = 0$.

Note that the stability radius is discontinuous only at the frequencies where M is real (Qiu *et al.*, 1995).

3.2 *LFT framework for $\mathcal{D}$ and C*

In order to apply Proposition 2 to check the stabilization of all plants in $\mathcal{D}$ by some controller C, we must show that the closed-loop connections of all plants in $\mathcal{D}$ with C can be described as a particular set of loops $[M(z)\ \beta]$. This first step can be achieved using the following theorem.

Theorem 4. Consider an uncertainty region $\mathcal{D}$ of plant transfer functions given by (1) and a controller $C(z) = X(z)/Y(z)$. The set of closed-loop connections $[G(\delta)\ C]$ for all $G(\delta) \in \mathcal{D}$ are equivalent to the set of loops $[M_{\mathcal{D}}\ \phi]$ which obey the system of equations defined in (2), where the uncertainty part ϕ is a real column vector of size k such that $|\phi|_2 < 1$, and where $M_{\mathcal{D}}(z)$ is a row vector of size k defined as :

$$M_{\mathcal{D}}(z) = \frac{-(Z_D + \frac{X(Z_N - eZ_D)}{Y + eX})T^{-1}}{1 + (Z_D + \frac{X(Z_N - eZ_D)}{Y + eX})\hat{\delta}}, \quad (5)$$

with T a square root of the matrix R defining U in (1) : $R = T^T T$.

Proof. The closed-loop connection of C and a particular plant $G(\delta) = (e + Z_N\delta)/(1 + Z_D\delta)$ in $\mathcal{D}$ (see (1)) can be rewritten as follows by introducing two new signals q and p_1.

$$\begin{cases} \begin{pmatrix} q \\ y \end{pmatrix} = \begin{pmatrix} -Z_D & 1 \\ Z_N - eZ_D & e \end{pmatrix} \begin{pmatrix} p_1 \\ u \end{pmatrix} \\ p_1 = \delta q \\ u = -Cy \end{cases} \tag{6}$$

The variables y and u are now eliminated from (6), yielding the following system of equations:

$$\begin{cases} p_1 = \delta q \\ q = \overbrace{(-Z_D - \frac{C(Z_N - eZ_D)}{1 + eC})}^{M_1(z)} p_1 \end{cases} \tag{7}$$

The system (7) is equivalent with the closed-loop connection of a particular $G(\delta)$ in $\mathcal{D}$ with the controller C. In order to consider the closed-loop connections for all plants in $\mathcal{D}$, we have to consider all $\delta \in \mathbf{R}^{k \times 1}$ lying in the ellipsoid U defined in (1). In order to apply Proposition 2 to the set $[M_1(z)\ \delta]$, $\delta \in U$, we still need a normalization step, as follows. Using $R = T^T T$, define the real vector $\phi \in \mathbf{R}^{k \times 1}$: $\phi \triangleq T(\delta - \hat{\delta})$. Then $\delta \in U \Leftrightarrow |\phi|_2 < 1$. To replace δ by ϕ in (7), we first denote $p \triangleq \phi q$. Since $\delta = \hat{\delta} + T^{-1}\phi$, (7) is equivalent with:

$$\begin{cases} p = \phi q \\ q = \frac{M_1 T^{-1}}{1 - M_1 \hat{\delta}} p = M_{\mathcal{D}}(z) p \end{cases} \tag{8}$$

The result then follows from Proposition 2. □

3.3 *Robust stability condition for $\mathcal{D}$*

Using Proposition 2 and Theorem 4, we can now formulate our main stability theorem.

Theorem 5. Consider an uncertainty region $\mathcal{D}$ of plant transfer functions having the general form given in (1) and let C be a controller that stabilizes the nominal model $G(\hat{\delta})$. All plants in the uncertainty region $\mathcal{D}$ are stabilized by the controller C if and only if

$$\max_{\Omega} \mu_{\phi}(M_{\mathcal{D}}(e^{j\Omega})) \leq 1 \tag{9}$$

where the stability radius μ and $M_{\mathcal{D}}(z)$ are defined in Definition 3 and in (5), respectively.

Proof. $M_{\mathcal{D}}(z)$ lies in H_∞ since its denominator is the denominator of the sensitivity function of the closed loop $[G(\hat{\delta})\ C]$ which is stable by assumption. Therefore, this theorem is a direct consequence of Proposition 2 and Theorem 4. □

4. CONTROLLER VALIDATION FOR PERFORMANCE

In this section, we show that we can evaluate the worst case performance in the uncertainty region $\mathcal{D}$, i.e. the worst level of performance of a closed loop made up of the connection of the considered controller and any plant in $\mathcal{D}$. The worst case performance in $\mathcal{D}$ is of course a lower bound for the closed-loop performance achieved with the true system. We then say that a controller is validated for performance if this worst case performance in $\mathcal{D}$ remains below some threshold. There is no unique way of defining the performance of a closed-loop system. However most commonly used performance criteria can be derived from some norm of a frequency weighted version of the stability matrix $H(G, C)$ of the closed-loop system $[C\ G]$ made up of G in feedback with the controller C.

Definition 6. Given a plant G and a stabilizing controller C, the stability matrix $H(G, C)$ of the closed loop $[C\ G]$ is given by:

$$H(G, C) = \begin{pmatrix} \frac{GC}{1+GC} & \frac{G}{1+GC} \\ \frac{C}{1+GC} & \frac{1}{1+GC} \end{pmatrix}. \tag{10}$$

4.1 *The general criterion measuring the worst case performance*

The worst case performance criterion over all plants in an uncertainty region $\mathcal{D}$ will be similarly defined as the worst possible norm, over all plants in $\mathcal{D}$, of a frequency weighted version of the stability matrix $H(G(\delta), C)$, where $G(\delta)$ is any plant in $\mathcal{D}$ and C is the "to-be-validated" controller C.

General Criterion. Consider an uncertainty region $\mathcal{D}$ given by (1) and containing all systems $G(\delta) = G(z, \delta)$ with $\delta \in U$. Consider also a controller $C(z)$ that is validated for stability. The general criterion J_{WC} measuring the worst case performance level is defined at a frequency Ω as follows:

$$J_{WC}(\mathcal{D}, C, W_l, W_r, \Omega) = \max_{G(z,\delta) \in \mathcal{D}} \sigma_1 \left(W_l H(G(e^{j\Omega}, \delta), C(e^{j\Omega})) W_r \right) \tag{11}$$

where $W_l(z) = diag(W_{l1}, W_{l2})$ and $W_r(z) = diag(W_{r1}, W_{r2})$ are diagonal weights that allow one to define specific worst case performance levels and where $\sigma_1(A)$ denotes the largest singular value of A. Note that J_{WC} is a frequency function : it defines a template.

4.2 *Computation of the general criterion*

We now present a procedure for the computation of the general criterion $J_{WC}(\mathcal{D}, C, W_l, W_r, \Omega)$ at a given frequency Ω.

Theorem 7. Consider an uncertainty region $\mathcal{D}$ defined in (1) and a controller $C(z) = X(z)/Y(z)$. The general criterion J_{WC} defined in (11) is equal to $\sqrt{\gamma_{opt}}$, where γ_{opt} is the optimal value of γ for *the following standard convex optimization problem involving LMI constraints evaluated at the frequency Ω:*

$$\begin{array}{ll} \textit{minimize } \gamma & \\ \textit{over } \quad \gamma, \tau & \\ \textit{subject to } \tau \geq 0 \textit{ and} & \end{array} \quad (12)$$

$$\begin{pmatrix} Re(a_{11}) & Re(a_{12}) \\ Re(a_{12}^*) & Re(a_{22}) \end{pmatrix} - \tau \begin{pmatrix} R & -R\hat{\delta} \\ (-R\hat{\delta})^T & \hat{\delta}^T R\hat{\delta} - 1 \end{pmatrix} < 0$$

where $a_{11} = (Z_N^* W_{l1}^* W_{l1} Z_N + Z_D^* W_{l2}^* W_{l2} Z_D) - \gamma(QZ_1^* Z_1)$, $a_{12} = Z_N^* W_{l1}^* W_{l1} e + W_{l2}^* W_{l2} Z_D^* - \gamma(QZ_1^*(Y + eX))$, $a_{22} = e^* W_{l1}^* W_{l1} e + W_{l2}^* W_{l2} - \gamma(Q(Y + eX)^*(Y + eX))$, $Z_1 = XZ_N + YZ_D$ *and* $Q = 1/(X^* W_{r1}^* W_{r1} X + Y^* W_{r2}^* W_{r2} Y)$.

Proof. Proving this theorem is equivalent to proving that the solution γ_{opt} of the LMI problem (12), evaluated at Ω, is such that: $\gamma_{opt} = \max_{\delta \in U} \lambda_1(H_w(e^{j\Omega}, \delta)^* H_w(e^{j\Omega}, \delta))$, where $H_w(z, \delta) = W_l H(G(\delta), C) W_r$ and $\lambda_1(A)$ denotes the largest eigenvalue of A. An equivalent and convenient way of restating the problem of computing $\max_{\delta \in U} \lambda_1(H_w(e^{j\Omega}, \delta)^* H_w(e^{j\Omega}, \delta))$ is as follows:

minimize γ *such that*
$\lambda_1(H_w(e^{j\Omega}, \delta)^* H_w(e^{j\Omega}, \delta)) - \gamma \leq 0 \; \forall \delta \in U$.

Using the fact $H_w(e^{j\Omega}, \delta)$ has rank one, we have that $\lambda_1(H_w(e^{j\Omega}, \delta)^* H_w(e^{j\Omega}, \delta)) - \gamma \leq 0$ is equivalent with

$$\begin{pmatrix} W_{l1}(e + Z_N\delta) \\ W_{l2}(1 + Z_D\delta) \\ Y + eX + Z_1\delta \end{pmatrix}^* A \begin{pmatrix} W_{l1}(e + Z_N\delta) \\ W_{l2}(1 + Z_D\delta) \\ Y + eX + Z_1\delta \end{pmatrix} \leq 0$$

where $A = diag(I_2, -\gamma Q)$ and Q, Z_1 are defined in (12). This last expression is equivalent to the following constraint on the *real* vector δ:

$$\overbrace{\begin{pmatrix} \delta \\ 1 \end{pmatrix}^T \begin{pmatrix} Re(a_{11}) & Re(a_{12}) \\ Re(a_{12}^*) & Re(a_{22}) \end{pmatrix} \begin{pmatrix} \delta \\ 1 \end{pmatrix}}^{\alpha(\delta)} \leq 0$$

with a_{11}, a_{12} and a_{22} as defined in (12). This last expression is equivalent to stating that $\lambda_1(H_w(e^{j\Omega}, \delta)^* H_w(e^{j\Omega}, \delta)) - \gamma \leq 0$ for a particular δ in U. However, this must be true for all $\delta \in U$. Therefore the last expression must be true for all δ such that

$$\overbrace{\begin{pmatrix} \delta \\ 1 \end{pmatrix}^T \begin{pmatrix} R & -R\hat{\delta} \\ (-R\hat{\delta})^T & \hat{\delta}^T R\hat{\delta} - 1 \end{pmatrix} \begin{pmatrix} \delta \\ 1 \end{pmatrix}}^{\rho(\delta)} < 0$$

which is equivalent to the statement "$\delta \in U$".

Let us now recapitulate. Computing $\max_{\delta \in U} \lambda_1(H_w(e^{j\Omega}, \delta)^* H_w(e^{j\Omega}, \delta))$ is equivalent to finding the smallest γ such that $\alpha(\delta) \leq 0$ for all δ for which $\rho(\delta) \leq 0$. By the $\mathcal{S}$ procedure (Boyd *et al.*, 1994), this problem is equivalent to finding the smallest γ and a positive scalar τ such that $\alpha(\delta) - \tau\rho(\delta) \leq 0$, for all $\delta \in \mathbf{R}^{k \times 1}$, which is precisely (12). □

5. EXAMPLE

To illustrate our results, we present an example of controller validation for a model identified in closed-loop. Let us consider the following true system G_0 with an Output Error structure:

$$y = \overbrace{\frac{0.1047z^{-1} + 0.0872z^{-2}}{1 - 1.5578z^{-1} + 0.5769z^{-2}}}^{G_0} u + e,$$

where e is a unit-variance white noise. The sampling time is 0.05 second. We perform an indirect closed-loop identification of an unbiased closed-loop transfer function $T(\hat{\xi})$ by collecting 1000 reference and output data on the true system in closed loop with an output-feedback controller $u = 3(r - y)$ (see (Bombois *et al.*, 1999a) for details). This controller stabilizes G_0. The open-loop model $G_{mod} = G(\hat{\xi}) = T(\hat{\xi})/(K(1 - T(\hat{\xi})))$ corresponding to $T(\hat{\xi})$ is equal to

$$G_{mod} = \frac{0.1060z^{-1} + 0.0928z^{-2}}{1 - 1.5308z^{-1} + 0.5467z^{-2}}$$

Control design. From the model G_{mod}, we have designed a controller with a phase advance : $C(z) = (1.8464 - 1.3647z^{-1})/(1 - 0.4545z^{-1})$. With this controller, the designed closed-loop $[G_{mod}\ C]$ has a stability margin of 57 degrees and a gain margin of 10dB. The cut-off frequency Ω_c is equal to 0.5 which corresponds to a *real* frequency $\omega_c = 11\ rad/s$. Before applying this controller $C(z)$ to the true system, we verify whether it achieves satisfactory behaviour with all plants in the uncertainty region $\mathcal{D}_{CL}$. The uncertainty region $\mathcal{D}_{CL}$ containing the true system G_0 at a probability level equal to 0.95 is given by

$$\mathcal{D}_{CL} = \{G(\xi) \mid G(\xi) = \frac{T(\xi)}{K(1 - T(\xi))} \quad \textit{and } \xi \in U_{CL}\}$$

where $U_{CL} = \{\xi \mid (\xi - \hat{\xi})^T P_{\xi}^{-1}(\xi - \hat{\xi}) < 12.6\}$ and P_{ξ} is the covariance matrix of $\hat{\xi}$. As shown in (Bombois *et al.*, 1999a; Bombois *et al.*, 1999b), it is easy to prove that $\mathcal{D}_{CL}$ has the general structure (1).

Validation of C for stability. Using the procedure presented in Section 3, we check whether C stabilizes all plants in $\mathcal{D}_{CL}$. For this purpose, we construct the row vector $M_{\mathcal{D}_{CL}}(z)$ defined in Theorem 4 and we compute the corresponding stability radius $\mu_\phi(M_{\mathcal{D}_{CL}}(e^{j\Omega}))$ at all frequencies in $[0\ \pi]$. The maximum over these frequencies is 0.1313. Since this maximum is smaller than 1, we conclude that $C(z)$ stabilizes all plants in $\mathcal{D}_{CL}$ and therefore also the true system G_0.

Validation of C for performance. In order to verify that C gives satisfactory performance with all plants in $\mathcal{D}_{CL}$, we compute at each frequency the worst case modulus $t_{\mathcal{D}_{CL}}(\Omega, S)$ of the sensitivity function "S" achieved by C over all plants in $\mathcal{D}_{CL}$. This can be done by computing $J_{WC}(\mathcal{D}_{CL}, C, W_l, W_r, \Omega)$ using Theorem 7 with the particular weights $W_l = W_r = diag(0, 1)$. The worst case modulus of the sensitivity function over all models in $\mathcal{D}_{CL}$ is represented in Figure 1. In this figure, the worst case performance level $t_{\mathcal{D}_{CL}}(\Omega, S)$ is compared with the sensitivity functions of the designed closed loop $[G_{mod}\ C]$ and of the achieved closed loop $[G_0\ C]$. From $t_{\mathcal{D}_{CL}}(\Omega, S)$, we can find that the worst case static error ($= t_{\mathcal{D}_{CL}}(0, S)$) resulting from a constant disturbance of unit amplitude is equal to 0.1692, whereas this static error is 0.0834 in the designed closed-loop. The achieved static error is 0.1017. Using $t_{\mathcal{D}_{CL}}(\Omega, S)$, we can also see that the bandwidth of $\Omega_c = 0.5$ in the designed closed-loop is preserved for all closed loops with a plant in $\mathcal{D}_{CL}$, since $t_{\mathcal{D}_{CL}}(\Omega, S)$ is equal to 1 at $\Omega_c \simeq 0.5$. The difference between the resonance peak of the designed sensitivity function (i.e. $\max_\Omega \| S(G_{mod}, C) \| = 1.6184$) and the worst case reasonance peak achieved by a plant in $\mathcal{D}_{CL}$ (i.e. $\max_\Omega t_{\mathcal{D}_{CL}}(\Omega, S) = 1.7075$) also remains small. Note that the actually achieved resonance peak (i.e. $\max_\Omega \| S(G_0, C) \|$) is equal to 1.6229.

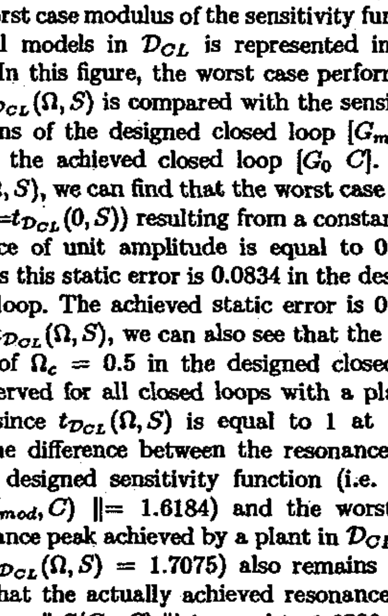

Fig. 1. $t_{\mathcal{D}_{CL}}(\Omega, S)$ (solid), $\| S(G_{mod}, C) \|$ (dashed) and $\| S(G_0, C) \|$ (dashdot)

We may therefore conclude that the controller C is validated for performance since the difference between the nominal and worst case performance level remains very small at every frequency. With such stability and performance analysis results, one would confidently apply the controller to the real system, assuming that the nominal performance is judged to be satisfactory.

6. CONCLUSIONS

We have developed tools for the robust stability and robust performance analysis of a controller designed from a nominal model, when the uncertainty set $\mathcal{D}$ containing the true system is described via ellipsoidal perturbations around the parameter vector of the nominal model. Such ellipsoidal parameter perturbations arise when the nominal model is the result of a prediction error identification procedure using an unbiased model structure. Our solution to the validation for stability problem is in the form of a necessary and sufficient condition for the stabilization of all models in this parametric uncertainty set $\mathcal{D}$ by a given controller C. Our solution to the validation for performance problem takes the form of the exact computation of the worst case performance of the controller C in closed loop with all models in the uncertainty set $\mathcal{D}$.

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